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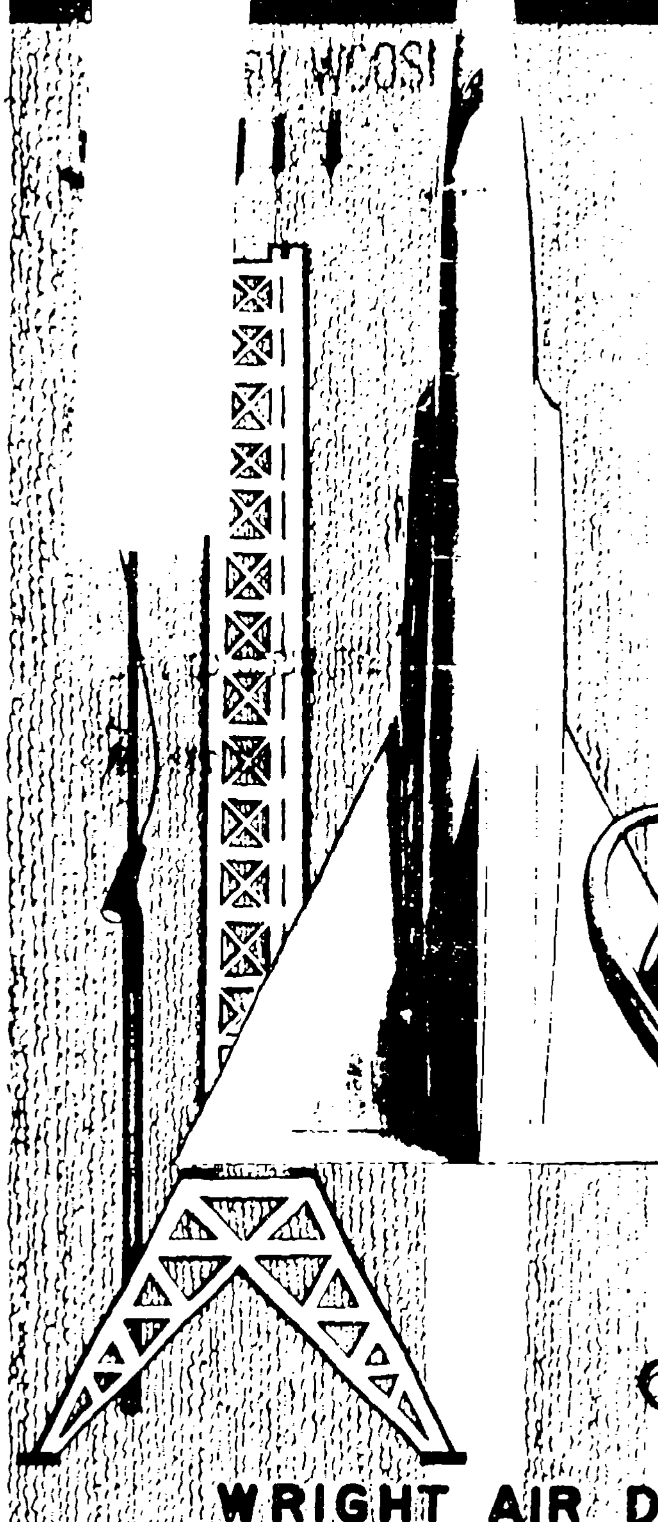
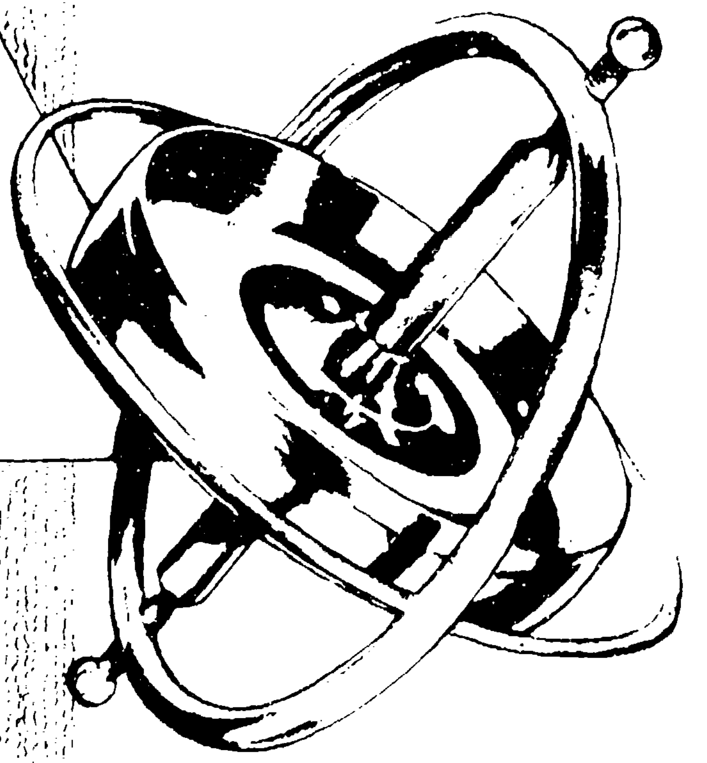
WADC TR 59-49

PROCEEDINGS OF

THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS SYMPOSIUM....

an ARDC presentation





WRIGHT AIR DEVELOPMENT CENTER

13 & 14 JAN 1959

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WADC TECHNICAL REPORT 59-49
ASTIA DOCUMENT NO. AD 209389

PROCEEDINGS OF THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS SYMPOSIUM

Lt P. C. Gregory, Editor
FLIGHT CONTROL LABORATORY

MARCH 1959

Flight Control Laboratory
Project 8225, Task 82181

WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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34-718-600-5-1-59

FOREWORD

In compliance with the direction of Headquarters, Air Research and Development Command, a Symposium on Self Adaptive Flight Control Systems was held at Wright Air Development Center on 13-14 January 1959.

The preparations for this symposium were handled jointly by the Control Requirements Section of the Control Synthesis Branch, Flight Control Laboratory and the office of the Deputy Chief of Staff for Plans and Operations. The efforts of the following persons, members of the symposium committee, helped to make the event a success: Mr. A. J. Cannon, Mr. A. S. Drysdale, Mr. C. E. Sondergelt, Mr. L. A. Ferguson, Mr. R. E. Hendrickson, Capt R. V. Simon, M/Sgt W. A. Wood, and M/Sgt H. T. Blaine. The proceedings of the symposium were transcribed by Mr. P. Lund.

The selection of papers and speakers for the symposium was accomplished by Capt R. R. Rath. Assistance in the editing of the proceedings of this symposium was given by Mr. L. A. Ferguson. Miss Annie R. Richardson prepared a large part of the final typed manuscript.

The success of the symposium and the value of these proceedings are both due entirely to the efforts of the contributors of the papers. Their cooperation is gratefully acknowledged.

Lt P. C. Gregory

ABSTRACT

This report gives an account of the presentations at the Symposium on Self Adaptive Flight Control Systems held at Wright Air Development Center on 13-14 January 1959. Papers presented described the " state of the art" of adaptive control systems. Various self adaptive control philosophies and the mechanization and flight test of these philosophies are presented. Future plans and programs for self adaptive systems are discussed. Individual conclusions reached indicate that present flight control systems in operational vehicles could have a self adaptive capability.

PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or conclusions contained herein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

F B Carlson

F. B. CARLSON
Colonel, USAF
Chief, Flight Control Laboratory

AD-No. 213561

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INTRODUCTION

During the past several years there has been a growing realization that development programs for tailoring flight control systems to operational aircraft are not producing the optimum flight control systems. This condition exists largely because of the greater extremes of conditions through which the aircraft are operating. These varying conditions cause changes in the aircraft characteristics which require changes in the control system parameters if the response of the aircraft-control systems combination is to remain a constant. At present the required changes in the control system are made in a predetermined fashion based upon instantaneous air data measurements. Thus, there is no guarantee of true relationship between these changes in parameters and the aircraft stability.

Changing the control parameters in this manner is often referred to as an open-loop adjustment accomplished by a process of omphaloskepsis. Several important points about these open loop adjustments should be emphasized. First, accurate and detailed information about the aircraft characteristics is required for the entire flight regime. Second, the capability must exist for measuring air data for all flight conditions. Third, the calculation of the required adjustments is a long process and must be confirmed by flight test data, and fourth, subsequent changes in airplane configuration often require more autopilot testing and adjustment.

Because of the above-mentioned factors, the Air Research and Development Command (ARDC) initiated a program to determine methods of adjusting automatic flight control systems in a closed-loop fashion requiring no air data measurements, using some sort of aircraft stability criteria. This program reached a point where it was felt that in the best interests of the Air Force the information now known should be presented to industry to encourage applications. Therefore, a symposium was held with the immediate objectives of:

(1) Acquainting Industry with the progress thus far realized in adaptive systems.

(2) Exchanging technical information between all who have done work or have ideas in this field.

(3) Stimulating the thinking of military and industrial personnel with work in fields in which adaptive controls are applicable.

This report is a composite of the papers presented. Because of the speakers' time limitations, they were permitted to publish a more comprehensive paper in these transactions than that which was presented. However, in most cases the paper herein is very similar to the oral presentation.

Throughout the symposium, there was considerable confusion regarding the definition of an adaptive, self adaptive, or self optimizing system. Speakers used the three terms interchangeably and presented conflicting views regarding them. It is felt that the confusion could be best cleared up by recourse to semantics. All too often, engineers working in a new field assign a unique meaning to a stock English word for technical purposes. This practice leads to much confusion when others try to understand the new field.

In the present case, consultation of the Webster Dictionary reveals that the term adaptive means " tending to or showing adaption" . Adaption, in turn, is defined as " adjustment to environmental conditions" . Thus, an adaptive system would be one which has the capability of adjusting in some manner to changing environmental conditions.

The key to this definition lies in the requirement for adjustment. Thus, if a system's parameters are not adjusted or changed in some manner the system is not adaptive. Under this definition present flight control systems with an air data programmed gain change would be adaptive.

" Webster" defines the prefix self as " the agent that of itself acts in a manner implied by the word with which it is joined" . Thus, a system that changed its own parameters to adapt to a changing environment would be a self-adaptive system.

An optimalizing or optimizing system would be one which operates at the best or most favorable condition. A self-optimizing system should be one which adjusts its own parameters to achieve an optimum condition and could be considered as a particular self-adaptive system.

As Dr. Aseltine points out later in this volume, there is a three-step process consisting of measurement, evaluation, and then adjustment required for adaption. In order for a system to be self-adaptive, it must perform these three steps itself.

A self-adaptive system will be defined as one which has the capability of changing its parameters through an internal process of measurement, evaluation, and adjustment to adapt to a changing environment, either external or internal to the vehicle under control.

If the above definitions are accepted, then, for example, a conventional linear system with fixed feedback values cannot be considered self-adaptive or adaptive, even though it may provide satisfactory response over a wide range of operating conditions. However, a system designed by the use of non-linear techniques will be an adaptive system but not a self-adaptive system because no parameter is changed when the environment changes.

The author would like to express his appreciation to Mr. M. F. Marx, General Electric Company, who stimulated this definition through remarks made at a discussion following the symposium.

The open forum discussions are included in their entirety based upon a transcript taken by the symposium recorder.

The speakers' introductions and other comments dealing with the operation of the symposium have been omitted for the sake of brevity.

THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS
SYMPOSIUM

PROGRAM

WADC AUDITORIUM

TUESDAY MORNING, 13 JANUARY 1959

SESSION I

Capt R. R. Rath, Chairman
Flight Control Laboratory

WELCOME

Colonel W. R. Grohs, Vice Commander
Wright Air Development Center
Wright-Patterson Air Force Base, Ohio

Good morning, gentlemen. I would like to extend to each one of you a very sincere welcome to the Wright Air Development Center. The Center has the honor of assisting the Air Research and Development Command in presenting this symposium. I am sorry that General Wray was not able to be here to greet you this morning, but he is on one of his very rare trips to the West Coast. As you know, this symposium deals with Adaptive Flight Control Systems. I know that those of you who have looked over the agenda recognize that this will be a very informative and interesting meeting. Within our somewhat limited confines here, we have attempted to assure that this symposium will be held under conditions which will permit your maximum absorption of the material being presented. The most modern and advanced air and space vehicles or weapons systems become useless if their control systems fail to do their jobs. The severe environment that future vehicles must operate in will require that these vehicles have control systems that are superior to present operational systems.

We are meeting here today and tomorrow to present to you people our ideas regarding the " state of the art" and the techniques associated with flight control which have the capability of providing the required characteristics. It is hoped that upon leaving here you will utilize these techniques to provide the military with the control systems that are necessary for our advanced vehicles.

At this time, I would like to introduce to you, General L. I. Davis, Deputy Commander for Research, Air Research and Development Command.

INTRODUCTION

Major General L. I. Davis
Deputy Commander for Research
Air Research and Development Command

It is a pleasure to be here today, especially because I see so many people in the audience. I am very surprised to see such a large turnout. I hope that most of you are here for the same reason that I am, and that is to learn something of the background of this effort in adaptive control systems. In my former tour in the Headquarters of the Air Research and Development Command, I put a lot of emphasis on automatic controls, control systems of various sorts. In coming back to the Headquarters again in August, I requested some information about the progress of the research in adaptive control systems, and perhaps this group this morning, who are going to meet here tomorrow also, will be able to educate Davis.

My interest, of course, stems from the very fundamental relationships that exist in all our military weapon systems. I like to use the analogy of the three-legged milking stool with the seat representing the warhead; one leg representing aerodynamics; another leg, propulsion systems; and the third leg representing guidance and control. Without any of those three legs you don't have an effective military weapon.

You all know, of course, of the tremendous amount of emphasis placed on aerodynamics, the billions of dollars that are invested in wind tunnels, space flight rockets, and things of that sort. You also know of the billions of dollars that are involved in our propulsion systems, both in research and development, and in industry as well as in the service. It is my feeling that we don't have a corresponding amount of effort on this other leg of the stool representing guidance and control and including the adaptive control systems.

I did not see the last launching of the Atlas, the one that went into orbit, but I have been very impressed by the great degree of stability shown by our ballistic missiles in the initial part of their flight path. It is amazing to note the performance of the automatic controls, but this is just the first part of the flight, the part of the flight you might say, which is still under pilot control. The later portions are the more precise portions. Those dealing with putting something into orbit, or perhaps controlling it more precisely upon re-entry will require a much greater degree of sophistication and accuracy. I bring up this subject of the Atlas flight because in this instance, you are using a radio control system to guide it in trajectory. Although I was not there, it was reported by those people who did see the flight that one of the reasons it did not go into higher orbit was that the control system was rather sloppy. An appreciable amount of energy was used in the gyrations along the path.

This is just an example of what still needs to be done in all our control systems. So far as adaptive controls are concerned, when you consider the wide range of acceleration, the rapid change of mass, and the wide range of atmospheric density upon re-entry, you see that a missile is a system that is pretty hard to prepare on the ground for all the necessary adjustments before the flight. It has to be self adjusting, or should be.

Another reason why we are very interested in this adaptive control system is philosophy. I think that before we get too involved in serious matters, I ought to mention a little story that Jimmy Doolittle brought back from Europe. It deals with a famous old circus over there, well known throughout Europe. One of its best acts was a tiger-taming act. This had been a good act for many years, but as all things happen sooner or later the old tiger died and they got a new young tiger that was very fierce and had sharp teeth. At the next showing, the young man stepped into the cage with his pistol, chair, and whip. When the young tiger saw him, he jumped across the cage and knocked the chair down, took a slap at the pistol and almost removed the tamer's clothes. The young tiger tamer backed out of the cage. This was very serious because it was a famous old circus and they had a great tradition. The show must go on and they had another performance in about two hours. So the manager called all the troopers together and asked for volunteers. There was a silence and finally a little girl in a Bikini bathing suit, who was in a tight rope act, volunteered. She was raised in the circus, born in a trunk and all that sort of thing. After some consideration they decided to let her go on. At the three o'clock matinee performance she stepped in the cage with a chair, pistol, and whip. They opened the other gate and in dashed this fierce tiger. She cracked the whip and he just sat there. She cracked the whip again and he went down on his stomach and crawled across the cage. He licked the girl on the toe, then he licked her calf, then her thigh, and he finally ended up with his head on her busom purring gently and the crowd went crazy hollering "Bravo" and so forth. The manager turned to the young tiger tamer and said, "Why can't you do that?" The tiger tamer said, "You get that damned tiger out of there and I'll show you!"

AIR RESEARCH AND DEVELOPMENT COMMAND
PLANS AND PROGRAMS

Lt P. C. Gregory
Flight Control Laboratory
Wright Air Development Center

With such a distinguished group as this, it might seem inconsequential to try to establish the problem of why we are here, and what we are doing. However, certainly there are a few people here who are not familiar with self adaptive controls. Most of you know that with the advent a few years ago of hypersonic and supersonic aircraft, the Air Force was faced with a control problem. This problem was two-fold; one, it was taking a great deal of time to develop a flight control system; and two, the systems in existence were not capable of fulfilling future Air Force requirements. These systems lacked the ability to control the aircraft satisfactorily under all operating conditions. Because of the non-linear aerodynamic link between the control surfaces and the stability of the aircraft, the flight control systems had the capability of working only where we could predict they would be operating. Obviously, if we always know what this link is we can program it into our control system and work with it, but if we don't know what it is or if our control system undergoes unexpected changes during flight then we experience control difficulties.

Approximately three years ago, the Air Force became interested in . oving the effects of this non-linear link from the flight control system by the use of self adaptive techniques. I will not at this time attempt to describe to you our past programs. That is largely the purpose of this symposium. I will, however, mention two programs that are being initiated. A contract has been signed with Lear, Incorporated to mechanize and flight test a version of the MIT system which will be presented here this morning. This will be a three-axis system installed in an F-101A and will be flight tested sometime this summer. This system includes control stick steering, G-limiting, and an angle of attack warning system. Another contract will be signed soon for the installation of a three-axis self adaptive system in an X-15 test vehicle. Flight test should start during the summer of 1961.

The Air Force approach to the problem of self adaptive controls has been to try to determine techniques that would work, to prove the feasibility of these techniques through simulation and study, and then flight test systems designed by these techniques to achieve a degree of confidence in them. We have many plans for the future of our systems. I think there is one general statement that we can make about most of our systems and that is, they work, but why do they work? In the future we intend to try to establish the basic fundamentals of why our systems work and how we can analyze them better.

We intend to try to categorize our techniques, break them down into different philosophies and different methods of solution. We also intend to continue with our flight test program. We have several techniques that have not been flight tested and we have other techniques under development. Several of our present techniques that are approaching operational capability have been developed from existing flight control systems. There are undoubtedly a great many other ways by which present systems could be modified so that they would have a self adaptive capability; however, the Air Force feels that more would be gained by using the basic fundamental physics of the problem to develop new self adaptive control systems. Our future plans reflect this thinking. We intend to devote more of our program to the development of the basic fundamentals.

We intend to try to apply our present techniques in new ways. We have several " in the house" efforts working on control problems that are of interest to the Air Force. Our policy regarding these " in the house" efforts is that we will attempt to establish the basic fundamentals regarding the problem and to define the problem in such a manner that a procurement can be written. Then other commercial research groups are contacted to do most of the work and finish the problem. We do not attempt to find a complete solution through the " house" efforts. Some of the areas in which we have been working are aeroelasticity, its effects on self adaptive control systems, the integration of aerodynamic and reaction controls, with the implication of space vehicle re-entry and exit; the possibility of the re-entry of a vehicle maintaining constant temperature at some point, a truly universal autopilot, and several other imaginary or visionary concepts.

Along with Wright Air Development Center, another Air Force group that has been working on self adaptive controls is the Air Force Office of Scientific Research. They have sponsored several projects, one of which is being done by the University of California, Los Angeles and will be reported on later. UCLA has been doing basic research regarding self adaptive control systems theory. AFOSR has a contract with Stanford, where Mrs. Flugge-Lotz is conducting an extension of her previous work. She is studying means of reducing non-linear transients in switching contacts. Rennselar Polytechnic Institute is studying the non-linear effects of on-off control devises in the presence of viscous damping. Bell Aircraft has a program underway to study the effects of structural damping on flutter. Westinghouse is studying the control requirements imposed by orbital transfer under perturbations experienced in space flight.

The purpose of the Air Force Office of Scientific Research is to establish the basic fundamentals regarding origins or sources of non-linear effects in self adaptive flight control systems, determine the parametric relationships regarding these non-linear effects, and develop procedures or techniques which will solve these relationships.

It should perhaps be emphasized at this point that there is no conflict between the Office of Scientific Research and the WADC effort. They are starting from basic fundamentals and working toward a development aspect. We started on the development end because of the necessity of proving that these systems would work and we are now working to establish basic theories.

Another related area in which we hope to sponsor some work is component development. As you will note from the following papers, most of our systems have a gain changing device. Presently this is done mechanically or electro-mechanically. We would like to see this replaced by a solid state gain changing device. This should increase both the reliability and the rate of adjustment. We would also like to see more work being done on special action devices such as hot gas servos and pulse-modulated servos. Another area in which we feel significant improvement could be made is in the design of devices to measure higher order derivatives such as acceleration and perhaps jerk.

With the aid of our contractors, and with the aid of a number of people from industry, Wright Air Development Center has been attempting to monitor the " state of the art" of self adaptive control systems. This has been very successful to date, due to the cooperation of this group. However, the program has expanded to the point now where there are people working on self adaptive ideas who have not been in touch with us. So I would like at this time to make an appeal to those of you who are working in this area to let us know what you are doing when you make any significant advances. This information will, of course, be treated as you desire. Most of the organizations with which we have been in contact do not regard any of their developments as proprietary in nature; however, if you feel that your development is proprietary we will regard it as such. If we are in contact with your organization, we will then be able to say that someone is working in this area, and we can refer interested parties to you. This procedure has the obvious advantage of rapid dissemination of information to avoid duplication of results and speed the application of new ideas.

I would like to make one other point in closing. We in ARDC feel that the self adaptive control system is here to stay. It is the future means of control for Air Force vehicles. As General Davis has mentioned, we have a whole field of application to missiles yet to investigate. We would like to apply these techniques to missile control. There are other areas where we know they will be helpful. We would like to impress upon you, during these two days, the abilities of self adaptive control systems. We think we have established some operating techniques and now we are hopeful that you will apply them. Thank you.

U.S. ARMY SIGNAL RESEARCH AND DEVELOPMENT LABORATORY FLIGHT CONTROL PROGRAMS - PAST, PRESENT, AND FUTURE

By Lt J. P. Gilmore
Project Engineer, Avionics Division
U. S. Army Signal Research and Development Laboratory
Fort Monmouth, New Jersey

The U. S. Army Signal Corps is responsible for the research and development of electronic flight aids for Army Aviation. In connection with this activity the Signal Research and Development Laboratory has been actively engaged in providing engineering support in the field of automatic flight control systems for Army aircraft.

The initial needs for flight control equipment for immediate army applications were urgent; the Laboratory, therefore, evaluated a number of existent military and commercial autopilots for army fixed and rotary wing aircraft. Thus, several interim army flight control systems were generated. These systems ranged from a modified F-5 fixed wing autopilot to a U. S. Navy helicopter stabilization system. As new tactical concepts developed, the scope of Army aviation missions expanded, and the need for larger quantities and varied types of more advanced flight control systems became evident. The Signal Research and Development Laboratory recognized these needs and considered the problems that would be associated with the procurement, supply, and maintenance of these equipments. Thus, in early 1956, a study was begun to determine the feasibility of the development of a universally adaptable type of autopilot.

The objectives of this study were to determine the feasibility of the development of a set of modular components which when combined in various configurations would provide a variety of types of flight control systems. These systems would be readily adaptable to present and foreseeable Army fixed and rotary wing aircraft. Fixed wing and helicopter autopilot requirement study programs were initiated. The studies evolved a set of normalized autopilot system parameters for helicopters and fixed wing aircraft. Technical requirements were then generated for the development of an adaptable set of lightweight autopilot components. This concept which was nicknamed the " Universal Autopilot" received final task approval in 1957 and early 1958, the design of the " Universal Autopilot" by the Sperry Phoenix Company under the cognizance of the U. S. Army Signal Research and Development Laboratory was begun. This system as such is not a self adaptive autopilot since it possesses no gain changing features; it achieves adaptability through its mechanization (i.e., the method of combining the modular parts). The equipment is adapted to each aircraft's flight characteristics by means of plug-in calibration cards. It is obvious that this type of system does not obviate the need for aircraft aerodynamic information. However, this equipment is intended for use in

relatively low performance fixed and rotary wing aircraft. These aircraft do not exhibit extensive aerodynamic changes in flight and therefore flight control systems with elaborate gain scheduling provisions are not required.

Figure 1 depicts each of the basic components of the " Universal Autopilot". Note the absence of a centralized amplifier computer. The number of components shown in this figure are those which are required to form either a complete 5-axis helicopter displacement system with control stick steering or a 3-axis fixed wing displacement system. The legend denotes those units which are common to both types of system (HF), and those which are required only for the fixed wing (F), or rotary wing systems (H). Note the relatively low weights which are projected for these systems.

The control panel provides the necessary autopilot engagement switches and attitude synchronizers. The navigational coupler provides all the necessary circuitry to enable the coupling of various navigational systems (ILS, VOR, doppler navigator, etc) to the autopilot. The components under the sensor column of Figure 1 (i.e. vertical gyro, force link, accelerometer, etc) provide aircraft attitude, altitude, and pilot stick force sensing functions that are required to effect the various control modes. The power unit is a self contained servo-loop. It provides all those computing, amplifying, and prime mover functions which are required for a single flight control axis.

Thus, it can be seen that a variety of control systems can be readily mechanized by the combination of the appropriate number of sensors and power units (i.e. a single axis damper to a five axis helicopter displacement autopilot). Figure 2 illustrates a typical application of this equipment as a 3-axis displacement system in an aircraft.

A unique mechanization technique enables the use of the power unit (illustrated in the fixed wing system form) as a differential series actuator for helicopter applications. This is achieved by removing the power unit's capstan and interconnecting a flexible shaft to the differential linkage (illustrated in Figure 1). The differential linkage is installed in series with the existing helicopter push rods forward of the helicopter hydraulic power boost. It is essentially an irreversible screw jack mechanism which transforms the rotary motion of the power units to linear motion. Thus by means of this technique the autopilot signals are mixed differentially with all pilot cyclic stick inputs and are not reflected back to the pilot's stick.

The equipment described is intended for application in the present Army fixed and rotary wing aircraft, and foreseeable low performance Army vehicles. It is still in the equipment design and fabrication stages of development and may be subject to some minor modifications. It is recognized that this system cannot cope with the problems associated with higher performance aircraft which require high power irreversible hydraulic servos, and gain scheduling provisions.

The desirability of achieving self adaptivity of the equipment to each aircraft without requiring the necessity of developing gain scheduling computers dependent upon extensive aerodynamic information is an obvious objective. Thus, the need arises for the development of a sound adaptive technique. It is envisioned that the universality concept can be extended by the development of a modular adaptive controller and associated specialized sensors. If we imagine that Figure 2 represents a hypothetical VTOL installation (assume control stick steering is incorporated and the maneuver controller is deleted), it is obvious that the adaptive controller would be inserted downstream of the couplers, sensors, and control panel. It would provide all the necessary automatic adapting which would be necessary for this type of aircraft. The use of this controller in all aircraft applications in place of the calibration cards now being used would simplify flight calibration and logistic supply problems. However, this consideration must be weighed by equipment reliability considerations and equipment cost, weight, and size.

The Signal Research and Development Laboratory is preparing a technical requirement for the development of such a controller. It is anticipated that development of the adaptive controller will be initiated in the near future.

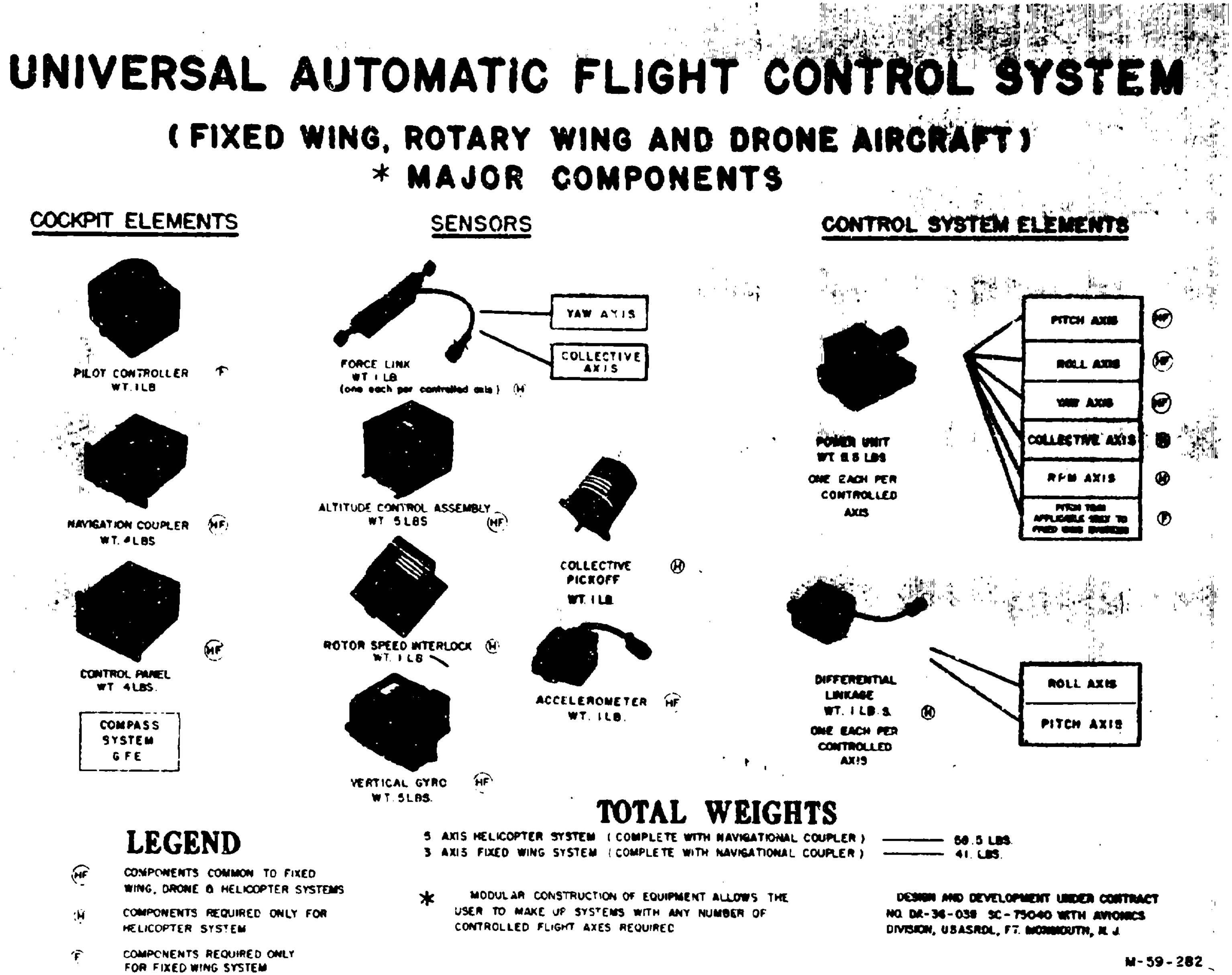
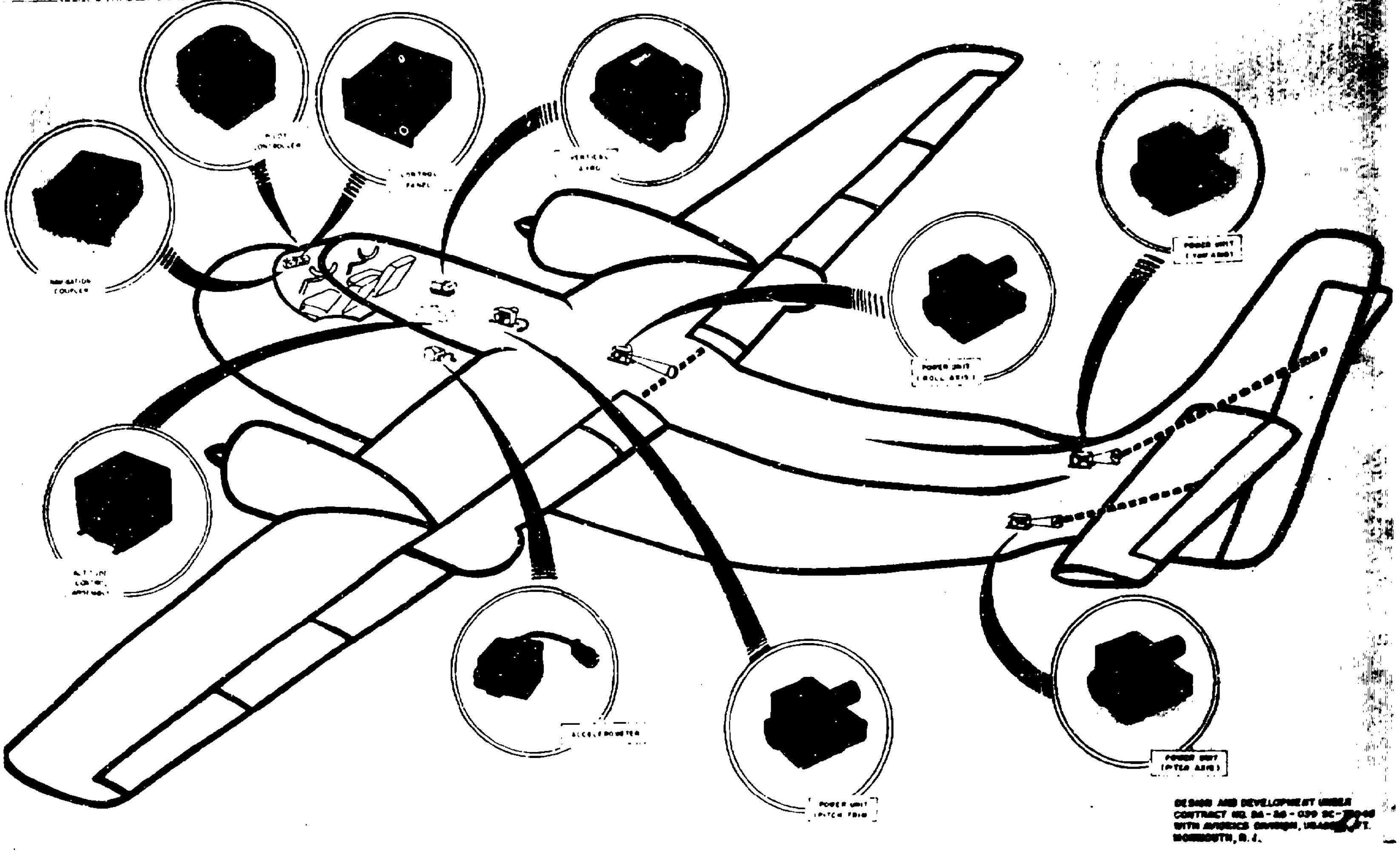

UNIVERSAL AUTOMATIC FLIGHT CONTROL SYSTEM
(FIXED WING, ROTARY WING AND DRONE AIRCRAFT)
* MAJOR COMPONENTS
COCKPIT ELEMENTS
SENSORS
CONTROL SYSTEM ELEMENTS
PILOT CONTROLLER
WT. 1 LB
NAVIGATION COUPLER
WT. 4 LBS
CONTROL PANEL
WT 4 LBS.
COMPASS
SYSTEM
GFE
FORCE LINK
WT 1 LB
(one each per controlled axis)
YAW AXIS
COLLECTIVE AXIS
ALTITUDE CONTROL ASSEMBLY
WT 5 LBS
ROTOR SPEED INTERLOCK
WT 1 LB
VERTICAL GYRO
WT 5 LBS.
COLLECTIVE
PICKOFF
WT 1 LB
ACCELEROMETER
WT. 1 LB.
POWER UNIT
ONE EACH PER
CONTROLLED
AXIS
PITCH AXIS
ROLL AXIS
YAW AXIS
COLLECTIVE AXIS
RPM AXIS
DIFFERENTIAL
LINKAGE
WT. 1 LB. S
ONE EACH PER
CONTROLLED
AXIS
ROLL AXIS
PITCH AXIS
TOTAL WEIGHTS
5 AXIS HELICOPTER SYSTEM (COMPLETE WITH NAVIGATIONAL COUPLER) — 58.5 LBS.
3 AXIS FIXED WING SYSTEM (COMPLETE WITH NAVIGATIONAL COUPLER) — 41. LBS.
LEGEND
HF COMPONENTS COMMON TO FIXED WING, DRONE & HELICOPTER SYSTEMS
H COMPONENTS REQUIRED ONLY FOR HELICOPTER SYSTEM
F COMPONENTS REQUIRED ONLY FOR FIXED WING SYSTEM
* MODULAR CONSTRUCTION OF EQUIPMENT ALLOWS THE USER TO MAKE UP SYSTEMS WITH ANY NUMBER OF CONTROLLED FLIGHT AXES REQUIRED
DESIGN AND DEVELOPMENT UNDER CONTRACT NO. DA-36-039 SC-75040 WITH AVIONICS DIVISION, USASRDL, FT. MONMOUTH, N.J.
M-59-282

Fig 1

UNIVERSAL AUTOMATIC FLIGHT CONTROL SYSTEM

(FIXED WING, ROTARY WING AND DRONE AIRCRAFT)
TYPICAL INSTALLATION IN ARMY AIRCRAFT



THE SYSTEMS DYNAMICS RESEARCH AIRPLANE

E. C. Foudriat and S. A. Sjoberg
National Aeronautics and Space Administration
Langley Air Force Base, Virginia

It is the purpose of the next two talks to acquaint you with a flight research airplane, which is now being equipped, and which will be used for flight research on airborne systems and for simulation studies of advanced airplanes. The airplane with its equipment is well suited to the study of adaptive control techniques and it is planned that one of the first uses of this vehicle will be in this field. In this talk the various pieces of equipment that go to make up this research airplane will be described and then the next talk will go into the research capabilities.

The first slide is a schematic which indicates the main pieces of equipment to be installed in an F-101 airplane. I' ll not attempt to go through this flow diagram. The main pieces of equipment are general purpose analog and digital computers, electrical input flight control systems - indicated by the series servo, a stable platform included with the sensors and instrumentation, a pilot' s display - which indicates computed rather than actual flight conditions, and an air data computer which is not indicated on this slide.

The equipment that makes this vehicle unique and which provides good versatility for systems and airplane dynamics studies is the on-board computers.

The usual procedure in conducting flight research on airborne systems is to design equipment to study a specific problem. This specific problem approach is time consuming and usually results in hardware that is not applicable to other studies without considerable modification. Using the airborne analog computer, airborne systems can be simulated and thus good versatility for in-flight systems studies is obtained. Let' s now get into some details on this airborne analog unit.

DESCRIPTION OF EQUIPMENT

Analog Computer

The analog computer which is now under development by the Autonetics Division of North American Aviation is a transistorized differential analyzer. To give you an idea of the computer capacity a list of the computing components is shown on the next slide. As you can see from the slide, the computer has a reasonably good capacity, including 100 operational amplifiers and a significant amount of non-linear equipment. Besides being transistorized, the computer has been miniaturized so as to meet the space limitations.

The miniaturization has caused the basic accuracy of this airborne unit to be reduced from that of general purpose ground based computers. The inherent accuracy of most of the individual components is about 0.1 percent.

The next slide lists some other pertinent features of the analog computer equipment.

Significant non-linear function generation can be accomplished since 20 tapped pots with 19 taps per pot and 4 diode function generators are available. In addition 10 diode bridge limiters and 24 free diodes are available for switching, limiting, and other types of function generation.

As in the ground based computer, the inputs to and outputs of the computer components are brought to a patching system so problems can be changed easily.

Two types of systems are available for generation of time varying parameters. A punch tape timer system for discrete problem changes and a magnetic tape playback system for continuous changes. In addition the magnetic tape makes possible the insertion of noise voltages into the problem by taping the noise signal on the ground and playing it back in the air.

A multi-recording system is also available. The majority of the recording will be done on a 50 channel oscillograph. In addition 7 channels of magnetic tape recordings are available. In order to determine problem malfunctions in flight, an events recorder shows when and where any amplifier overload occurred. The computer will weigh about 900 pounds, including recording and pilot display equipment, and occupies a volume of about 21 cubic feet.

While on the subject of analog computers, it should be mentioned that a ground based computer will be used to simulate the F-101 airplane on the ground. This will make possible the studying and checking out of test problems on the ground prior to the actual flight tests. Thus the flight test time can be used most efficiently.

Digital Computer

Next, let's go into the digital computer. The digital computer, being built by the Burroughs Corporation, is a completely independent computing system. However, it has features which allow it to be integrated with the analog computer system providing a combination which affords flexibility, high accuracy and rapid calculations.

A block diagram of the digital computer is shown on the next slide. The system consists of a magnetic drum for storage of programs and constants, a random access memory, a three register arithmetic unit, input and output

systems which provide analog-to-digital and digital-to-analog conversion and a problem control unit.

The magnetic drum is used to store the computer programs, constants used in the computation, and initial conditions.

The arithmetic unit consists of three registers. Within these are performed the arithmetic operations. Addition and subtraction require about 25 microseconds to complete. Multiplication and division take about 75 microseconds. These times are about twice those of an IBM 704.

The random access memory is a ferrite core unit with 128 word storage. The input-output system is one of the unique features of the computer. There are available 32 input and 10 output channels. Both the analog-to-digital and digital-to-analog conversions can be completed in 75 microseconds. The digital computer program is formulated in the ground and loaded into the drum by a punch tape system through the control unit. The programming procedure is quite straightforward.

It is the combination of the digital and analog computers which give good flexibility and no-drift high accuracy computing capabilities to the system. It is expected that the analog will do the rapid summation, integration and solution of the basic differential equations since these type processes are quite slow on a digital machine. However, the digital machine can do the long term high accuracy calculations such as coordinate transformations and space position calculations. The combination of analog and digital computers should provide an excellent airborne computation and simulation system.

Electrical Input Flight Control System

Next, let's go into some of the details of the electrical input flight control system to be used in this research airplane. The control systems have been developed by the Sperry Gyroscope Company and the next slide lists some of the control system characteristics. The stabilizer, rudder, and the two aileron channles are electro-hydraulic in operation and are installed in series with the conventional controls through summing links. The input signals can come from the computers, the sensors or the pilot's side controller stick. Full authority has been designed into the system but the authority can be limited by ground adjustable stops.

As to the dynamic performance, the specifications call for the phase lag between input and output to be less than 3 degrees at one cycle per second and less than 45 degrees at 10 cycles per second. The amplitude ratio at any frequency is specified to be less than 1.3 times the static amplitude ratio.

The dynamic performance of the electrical input control systems is

high as compared to the power actuators so they will not be a limiting factor.

For safety considerations, all of the electrical input control systems are dualed. The dualed systems operate from the same input signal. If for any reason the dualed servos do not operate together within preset limits the electrical input servos are shut off and the pilot assumes control through the conventional control systems. In addition to the dualed electrical input control systems, limit sensors are used to prevent structural overloading of the airplane.

A side located control stick is used by the pilot for generating control signals, and a torque servo is used to provide feel forces to the pilot's side controller. The torque servo accepts electrical signals from the various sensors or computers and its output is a force proportional to the sum of the input signals.

Stable Platform

Another important piece of equipment is the stable platform and its computer. It has also been built by the Sperry Gyroscope Company. The platform is of the 4 gimbal type and the inertial angle and velocity data it provides will be used in target simulation and navigation studies.

Sensor System and Flight Instrumentation

A complete set of sensors and flight instrumentation is installed to measure the linear and angular motions of the aircraft, the motions of the control surfaces, and the flight conditions. An air data computer developed by Bendix provides analog signals representing Mach number, true airspeed, altitude and dynamic pressure.

Pilot's Display

A portion of the instrument panel in the F-101 airplane has been made available for a pilot's display of test problems. It consists of various indicating instruments which depend on the program being run. These instruments will receive their information from the computers and will indicate computed flight conditions. In addition a 4 input channel oscilloscope is available for target and other displays.

RESEARCH CAPABILITIES

Now that the equipment has been described, I would like to tell you about the research capabilities of the vehicle. To do this, I will outline some of the research areas of interest today, where the research vehicle will be used. Then, to give you a better idea of how the equipment will be employed, I will outline how we might simulate a self-adaptive controller in conjunction with an

advanced aircraft design.

The first slide lists some of the research areas of interest where this research vehicle has capabilities. The areas are divided into two categories; the first having to do with the studies of systems and system techniques and the second with the simulation of advanced aircraft and re-entry vehicles.

Under the first category is control systems. The most important development here has been self-adaptive control techniques. The research airplane is well suited to the study of adaptive control principles.

As examples, the relay amplifiers included with the analog equipment can be used in the study of non-linear switching techniques like those proposed by Mrs. Flugge-Lotz and by the Minneapolis-Honeywell Regulator Company.

The technique proposed by the Sperry Gyroscope Company employs a mechanism which senses damping ratio changes of the high frequency control system poles and modifies the system accordingly. These changes could be sensed using a digital amplitude comparitor logic system which would compare the amplitude peaks in the time interval after excitation.

A third system, the most difficult for the airborne simulator to handle, is a technique using cross correlation which has been proposed by Aeronutronics Systems, Inc. One method of instrumenting this technique uses a random flip-flop to produce binary white noise and a time delay on the input signal to obtain a comparison between input shifted in time and the output. This shift register may be accomplished in the digital computer by sampling the input and using a digital program to shift this sampled input. The delayed signals may be reproduced by using the digital-to-analog converter. With the equipment available, 10 such delayed signals could be reproduced since there are 10 digital-to-analog converter channels. It is possible to simulate other proposed adaptive techniques as well and also there are other methods for simulating the techniques outlined above. What I have tried to do by citing these examples is to give you an idea of the research vehicle's versatility with the on board computing equipment.

The research vehicle has capabilities for studies of systems other than self-adaptive controllers. The stable platform-digital computer combination provides the essentials of a high performance navigation system and also provides the axis reference for target simulation. Thus studies can be conducted on digital autopilot and navigation systems, automatic weapons delivery including fire control systems, and automatic on-board landing systems. The target simulation in some cases allows for studies in speed ranges greater than that of the F-101 airplane because the target is not actual but is represented by simulation.

In the category on the right hand side of the slide, studies of advanced aircraft such as the B-70 or F-108 can be done in actual flight. Because of the on-board computers continuous flight operations can be accomplished where Mach number and altitude are changing. This is in contrast to the variable stability airplane where essentially only one flight condition at a time can be investigated.

For the last item in this category, a part of the re-entry of vehicles such as the X-15 or Dynasoar could be simulated. For example, during a portion of the re-entry a sustained normal acceleration greater than one g builds up. This could be simulated by having the airplane perform wind-up turns. At the same time the airplane would be made to have the same short period dynamics as associated with the re-entering vehicle. Thus, the dynamics research vehicle would represent the re-entry by simulating the most important forces and motions of the re-entering vehicle.

The problem could be handled in the following way. The analog and digital computers would be used to solve the re-entry vehicle equations of motion. The pilot's side stick and rudder pedal motions would serve as inputs to the computers and the pilot's instrument panel would display conditions about the re-entry vehicle. This is similar to the way in which a ground simulation of the problem would be conducted. In flight, however, the computed output quantities would be used as command inputs to the F-101 electrical input system and the airplane sensors would furnish feedback and stabilization signals.

To give you a better idea of how this equipment is used in a simulation study, a program will be outlined. The problem is the simulation of a self-adaptive controller on an advanced aircraft. The next slide shows a block diagram of how this could be accomplished.

The problem may be divided into two parts, the simulation of the advanced airplane and the simulation of the adaptive controller. The first part can be accomplished in a manner shown on the right hand side of the slide, using a technique similar to that used in variable stability studies. That is, the feedback quantities are used to modify the stability characteristics of the F-101 so that they match those of the airplane being studied. For example, in the longitudinal case the angle of attack and pitch rate signals are fed-back through the control surface to modify the stability derivatives C_{m_a} and C_{m_q} respectively, so that the desired short period dynamics i.e. frequency and damping are obtained. With the computer equipment aboard, information about the desired stability characteristics would be programmed as a function of altitude and Mach number. By furnishing Mach number and altitude through the air data computer the F-101 could be made to fly like some other airplane

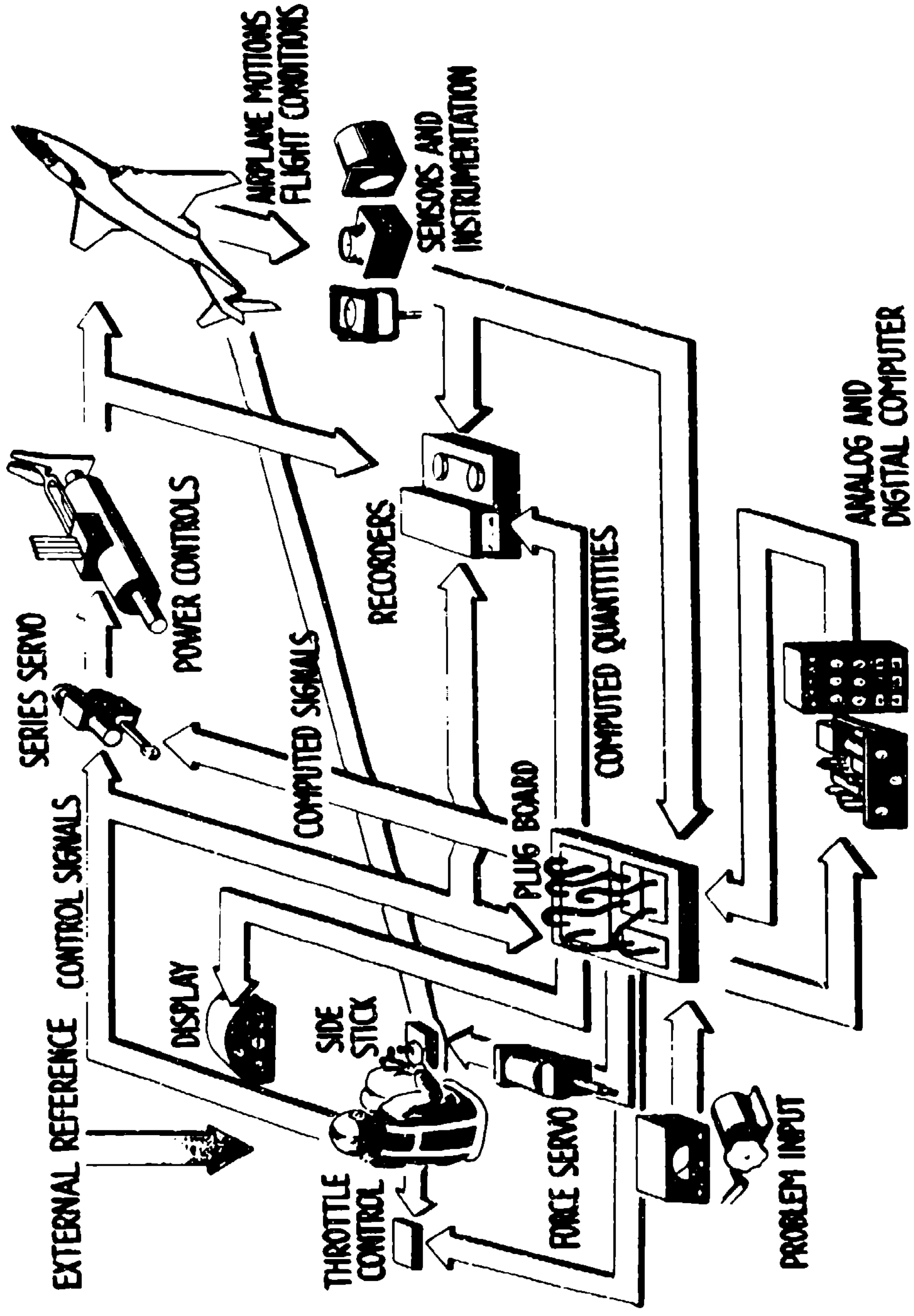
over a wide range of flight conditions. Thus the right hand side of the slide, that is the series servos, sensors, air data computer and variable stability simulator, represents the dynamics of the aircraft being studied.

The other portion of the slide shows a representation of the simulation of the adaptive control technique under study and the input commands as shown by the block marked " pilot' s input" . Any of several techniques including those mentioned previously could be simulated in a study program. The sensed signals for feedback to the adaptive controller can be taken directly from the airplane sensors in most cases.

Thus flight problems related to adaptive controllers and pilot opinions about them can be obtained directly without resorting to hardware build up of each system.

In conclusion, the dynamics research vehicle should be a useful facility for system and airplane simulation studies. Because of its versatility it should be usable for many research programs. It should help to reduce system lead time since it provides the ability to flight test the proposed technique before the system equipment has been built. As to the status of the vehicle, all major pieces of equipment except the analog computer are to be delivered for installation within the next few months. The analog computer is scheduled for July 1 delivery. It is expected that the vehicle will be operational about the end of the year.

EQUIPMENT INSTALLED IN RESEARCH AIRPLANE



NASA L-1267-1 SJOBERG, FOUDRIAT 1/13/59

ANALOG COMPUTING EQUIPMENT

COMPONENT	QUANTITY
OPERATIONAL AMPLIFIERS	100
RELAY AMPLIFIERS	8
SERVO MULTIPLIERS AND FUNCTION GENERATORS	6
SERVO RESOLVERS	4
HIGH SPEED MULTIPLIERS	4
DIODE FUNCTION GENERATORS	4
DIODE LIMITERS	10
COEFFICIENT POTENTIOMETERS	80
PRECISION REFERENCE POWER SUPPLY	1

NASA L-1267-2 SJOBERG, FOUDRIAT 1/13/59

FEATURES OF THE ANALOG COMPUTER EQUIPMENT

SIGNIFICANT NONLINEAR FUNCTION
GENERATING EQUIPMENT

PATCHBOARD SYSTEM

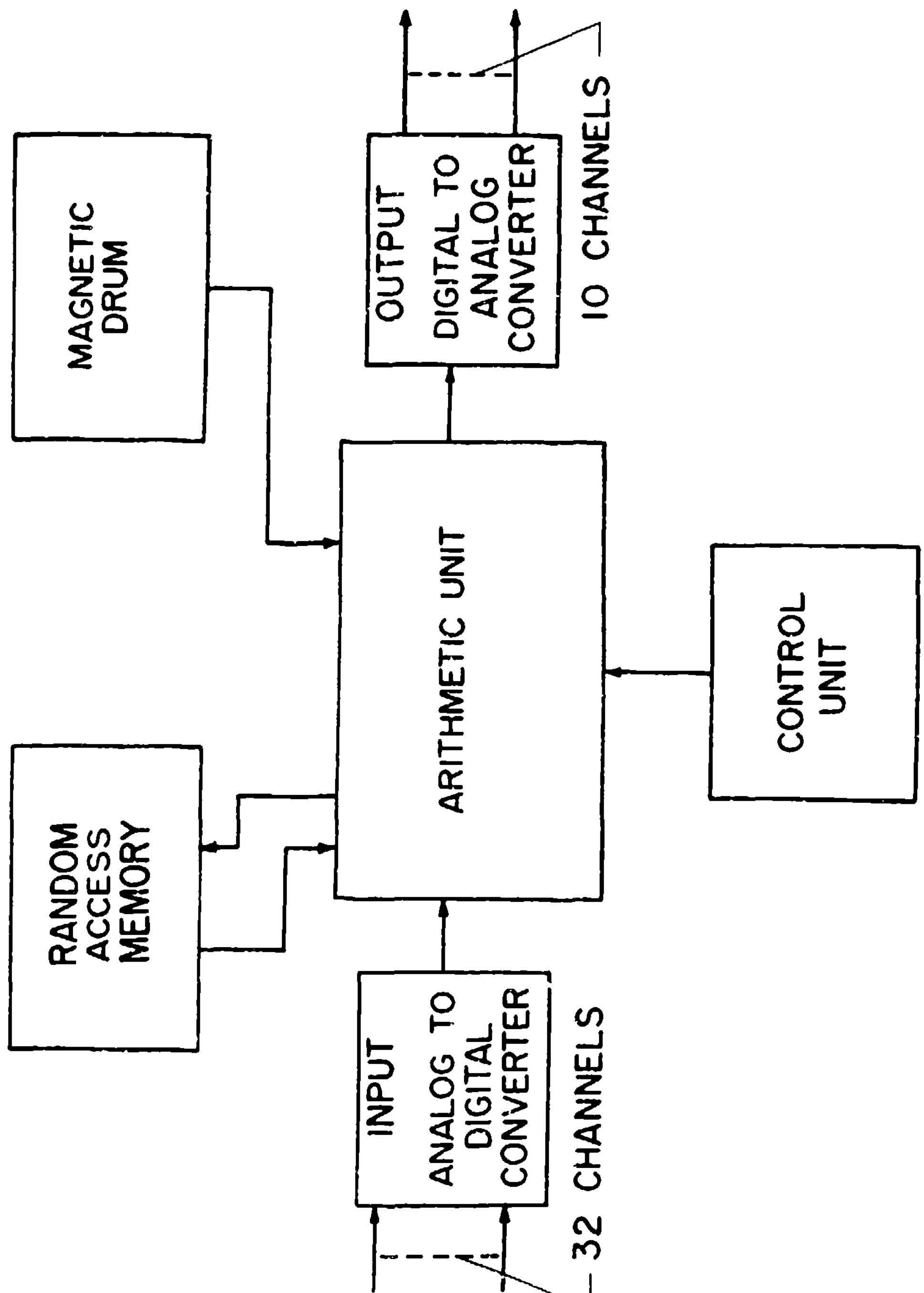
TIME VARYING PARAMETER GENERATING EQUIPMENT

MULTI-RECORDING SYSTEM

OVERLOAD EVENTS RECORDING SYSTEM

NASA L-1267-3 SJOBERG, FOUDRIAT 1/13/59

BLOCK DIAGRAM OF DIGITAL COMPUTER



NASA L-1267-4 SJOBERG, FOUDRIAT 1/13/59

ELECTRICAL INPUT CONTROL SYSTEM CHARACTERISTICS

ELECTRO-HYDRAULIC

SERIES INSTALLATION

FULL AUTHORITY

DYNAMIC PERFORMANCE :

PHASE SHIFT	$< 3°$ AT 1 CPS
	$< 45°$ AT 10 CPS
AMPLITUDE RATIO	$< 1.3 \times$ STATIC AMPLITUDE RATIO

DUALED SYSTEMS

LIMIT SENSORS

SIDE LOCATED PILOTS CONTROLLER

TORQUE SERVO FOR PILOT FEEL FORCES

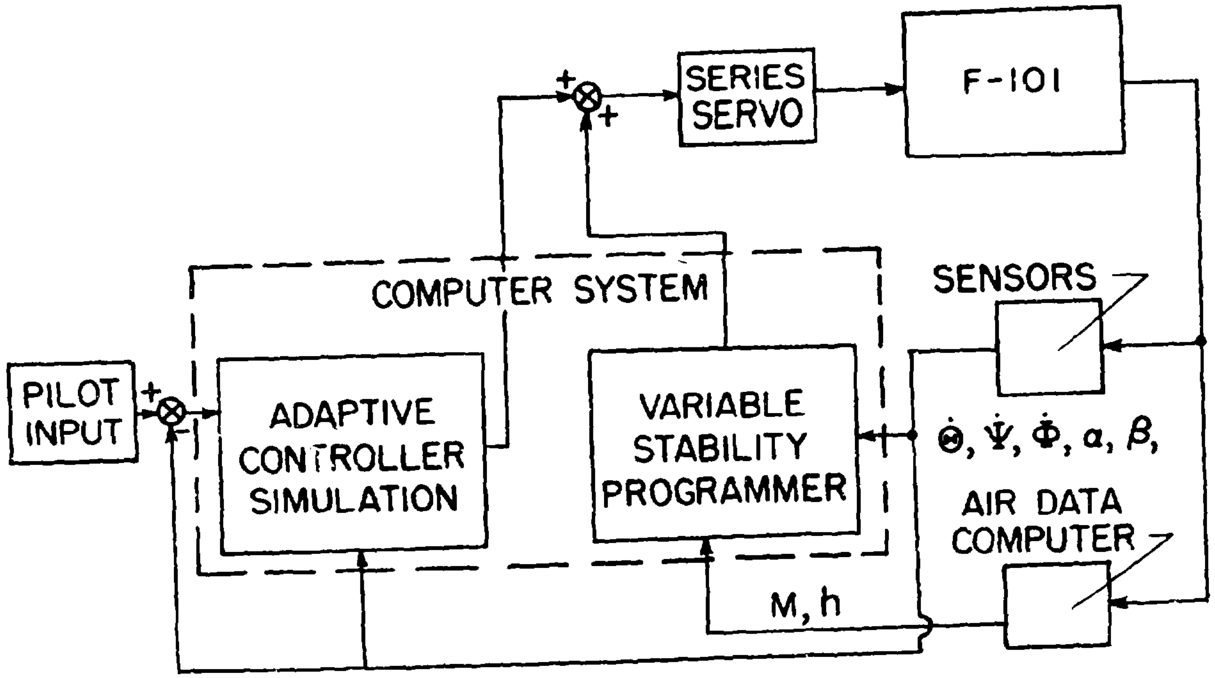
NASA L-1267-5 SJOBERG, FOUDRIAT 1/13/59

CAPABILITIES OF DYNAMICS RESEARCH VEHICLE

STUDIES OF SYSTEMS	SIMULATION OF VEHICLES
CONTROL SYSTEMS	FUTURE AIRCRAFT
ADAPTIVE CONTROLLERS	B-70, F-108
NAVIGATION SYSTEMS	HANDLING QUALITIES
DIGITAL AUTOPILOTS	FLIGHT CONTROLS
AUTOMATIC WEAPONS DELIVERY	WEAPONS SYSTEMS
AUTOMATIC LANDINGS	RE-ENTRY VEHICLES
UNUSAL AERODYNAMICS	X-15, DYNASOAR
PILOTING PROBLEMS	RE-ENTRY PULLOUT
	LANDING

NASA L-1267-6 SJOBERG, FOUDRIAT 1/13/59

ADAPTIVE CONTROLLER STUDY WITH RESEARCH VEHICLE



NASA L-1267-7 SJOBERG, FOUDRIAT 1/13/59

ADAPTIVE CONTROL SYSTEM PROGRAM

William C. Triplett
National Aeronautics and Space Administration
Ames Research Center
Moffett Field, California

At the Ames Research Center we have been conducting a theoretical study of adaptive control systems. This study has emphasized basic concepts and methods of analysis. One system which showed considerable promise is the type which uses the principle of high forward loop gain in connection with an on-off controller to obtain a response that is relatively insensitive to variations in aircraft parameters. In the strict sense of the word this system is not truly adaptive; however, it does have two distinct advantages - it is simple in concept and can be analyzed by conventional methods. On the other hand the action of the relay or " on-off" switch does result in a limit cycle oscillation which may be objectionable in some applications.

During this program, linear methods of analysis based on the root locus concept were used. Also a simple means of predicting the frequency and amplitude of the limit cycle oscillation was developed. Mr. McLean will discuss this program in his paper and also will show the results of an example application involving a normal acceleration system in a hypothetical air-to-air missile.

Our future plans are best illustrated by this slide. The chart shows, at the top, the theoretical work which is essentially complete and which will be discussed by Mr. McLean. The next two boxes show two parallel extensions of this basic study now in progress. These involve applications to conventional aircraft and also to re-entry vehicles.

First with regard to the conventional airplane application - the airplane chosen is the F-102 and preliminary simulation studies are now underway. This airplane was chosen because of its high performance AFCS which can be adapted for this program with minimum number of changes to hardware or instrumentation. Since the essential modifications involve only circuitry, flight tests of this system can be started with a minimum amount of development. It is planned to apply this adaptive principle to both the pitch and roll channels. In the pitch channel a normal acceleration command system will be used for the initial tests. This mode of control is normally employed in the F-102 for the automatic attack mode and is also the type of system previously studied in the missile example.

It is hoped that flight tests will not only demonstrate the feasibility of this particular adaptive system approach but will also provide a positive

check on the effects of the limit-cycle or chatter frequency. In this regard there are several questions to be answered. First, can the chatter frequency and amplitude be predicted adequately; what are tolerable limits on chatter amplitude; and is there objectionable coupling with airplane structural modes? Furthermore, these tests should give some idea of how well the adaptive system can cope with rapid changes in aerodynamic parameters such as encountered through the transonic region and in low-speed, high angle of attack flight.

The first part of the flight program involves automatic control with command inputs supplied by knob adjustment. The next step, as presently planned, would involve direct pilot control by means of a side-arm or information stick controller. These tests would be mainly pilot opinion studies, and again one of the main areas of interest would be the effects of chatter.

Now turning to the right side of the chart we have a project involving the control of re-entry type vehicles. At present a study is being made to define the control requirements of such vehicles which will traverse the entire flight regime from orbital speed and altitude to landing. This regime will involve reaction controls for attitude control at high altitudes, aerodynamic control at low altitude, and perhaps a mixed mode of operation during an intermediate portion of the flight. This program involves simulation studies of a representative configuration with conventional and also with adaptive type autopilots in order to determine the relative merits of the two concepts.

Later, more complete studies will be made of systems which are controlled directly by the human pilot. For these tests a two-degree-of-freedom motion simulator will be used to assess the effects of aircraft rolling and pitching motions on the pilot's ability to control the vehicle. This again will be a pilot opinion type of study.

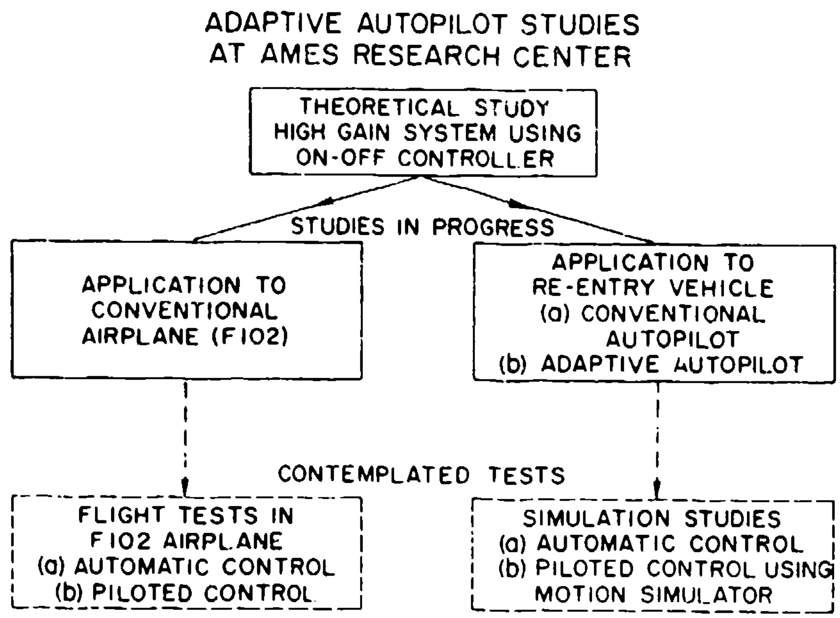

ADAPTIVE AUTOPILOT STUDIES
AT AMES RESEARCH CENTER
THEORETICAL STUDY
HIGH GAIN SYSTEM USING
ON-OFF CONTROLLER
STUDIES IN PROGRESS
APPLICATION TO
CONVENTIONAL
AIRPLANE (F102)
APPLICATION TO
RE-ENTRY VEHICLE
(a) CONVENTIONAL
AUTOPILOT
(b) ADAPTIVE AUTOPILOT
CONTEMPLATED TESTS
FLIGHT TESTS IN
F102 AIRPLANE
(a) AUTOMATIC CONTROL
(b) PILOTED CONTROL
SIMULATION STUDIES
(a) AUTOMATIC CONTROL
(b) PILOTED CONTROL USING
MOTION SIMULATOR

Fig 1

ON AN ADAPTIVE AUTOPILOT USING A NONLINEAR FEEDBACK CONTROL SYSTEM

John D. McLean and Stanley F. Schmidt
National Aeronautics and Space Administration
Ames Research Center
Moffett Field, California

As Mr. Triplett has mentioned, we at Ames Research Center have been studying adaptive autopilots to try to understand the fundamental principles by which they can be derived and determine methods for analysis of their behavior. In this paper I will present the results of our studies to date. I shall first show how conventional theories which are documented in many texts on servomechanisms can be used to design a system. These theories show that an on-off feedback control system can give the desired characteristics except that a chatter frequency must exist in the system. The theory will then be applied to a normal acceleration type autopilot and methods will be presented for predicting the chatter frequencies and amplitudes. Some results and problem areas with possible solutions will then be presented.

The theory by which one can make an adaptive autopilot is best understood with reference to the block diagram shown on the first slide. The aircraft is represented by G(s) which has variable characteristics, N(s) is a network or filter, H(s) is a feedback transfer function produced either by instruments or a network, and K is a gain constant. The ideal invariant transfer function which we would like the system to have, that is our model, is represented by M(s). The arrangement in the upper diagram is based on the concept of applying the same input to the aircraft system and the model and using the error between desired and actual outputs as a corrective feedback signal. We have not concerned ourselves with what the characteristics of the model should be but rather with how to make the system behave in the desired fashion. The principles involved can best be understood by transforming the system to the single loop equivalent in the lower diagram. Now we have a typical closed loop autopilot system preceded by a network. If H(s) is invariant with flight conditions, it is clear that in order to produce a transfer function from R to C which is independent of changes in G(s), the closed loop portion of the system must also be independent of G(s). Therefore, the problem of designing an adaptive autopilot is reduced to one of making the closed loop system response invariant over the flight envelope. Since this is the case, it is simpler to turn our attention strictly to the closed loop portion, and the next slide is more suitable for this purpose.

There are two possible approaches to keeping the transfer function of this closed loop system invariant with changes in G(s). One possibility is to devise means for measuring the characteristics of G(s). These measurements can then be used to adjust the network in a fashion that will compensate for changes in

G(s). The other approach depends upon the fact that closed loop systems inherently tend to be insensitive to changes in the forward loop parameters. The reason for this insensitivity can be shown by considering the closed loop transfer function below the diagram. As K becomes very large the transfer function approaches the reciprocal of H(s) and hence becomes invariant. The high gain alone is not sufficient to insure a desirable system because the system must also be stable. It will now be shown how linear methods of analysis can be used to insure stability.

We have chosen as our example system the normal acceleration autopilot shown in the next slide. The transfer function representing the aircraft has a natural frequency ω_a, a damping ratio ζ_a, and an aerodynamic gain K_a. This representation is reasonably valid for tail controlled aircraft. The transfer functions of the measuring instruments used to obtain the feedback have a second order denominator of natural frequency ω_i and damping ratio ζ_i and a second order numerator of natural frequency ω_o and damping ratio ζ_o. The limiter accounts for the rate limit of the servo which is assumed to be a pure integration. This integration is desirable to provide the system with a steady state gain of unity. The system is not practically realizable because the higher order dynamics of the servo have been neglected, but it is sufficiently complex for the development of theory which will apply to the realizable case.

Now let us consider the problem from the standpoint of stability as we try to make the gain high. We will use the root locus for this purpose. The pole at the origin represents the servo and the high frequency complex poles are those of the instruments. The zeros are formed by the numerator of the instrumentation transfer function. The aircraft poles are subject to motion dependent on flight conditions and might be anywhere within the shaded area. Irrespective of the location of the aircraft poles within the shaded area, the roots will move toward the zeros as the gain is increased. It can be seen that the problem associated with the high gain is the fact that loci from the instrumentation poles will cross the imaginary axis. Linear analysis indicates the system will become unstable if the gain is made high.

If the crossover gain is high enough so that the aircraft poles are near the zeros for all flight conditions the problem is reduced to one of keeping the gain of the system at the desired level. One scheme of insuring this is to measure the damping of the high order mode and adjust the gain K to keep this damping constant. A second alternative, which we have chosen to use, takes advantage of the limiting in the system. We simply make the gain K very high and due to the limiting action the system oscillates with a limit cycle or chatter of frequency ω_c at an amplitude determined by the frequency

response of the linear elements following the limiter. You will recognize that this is the principle used in the pitch rate control system developed at Minneapolis-Honeywell. The methods presented in this paper for estimating chatter frequency and amplitude are just as applicable to such a pitch rate autopilot as to the normal acceleration example discussed here.

The fact that limiting will restrict the oscillation to a limit cycle can be predicted by treating the limiter or on-off controller as a gain which is dependent on input signal level. This concept is useful in determining qualitative performance since it allows one to use linear methods of analysis. For this analysis we need not regard these roots as being fixed but rather they move back along the root loci dependent upon the amount of limiting. For example, if a large step were applied to the system, it can be seen that saturation would immediately occur and the response of the system would be governed by the open loop poles.

For this system, then, the output will have an approximately linear relationship to low frequency inputs and there will be a chatter superimposed on this low frequency response. If the chatter can be made small enough, this approach would seemingly produce an adaptive autopilot.

Next, I will show a means of predicting the chatter frequency by an approximate method which is less time consuming than conventional linear techniques. In most practical systems ω_c will be much larger than either the natural frequency of the aircraft or that of the zeros. A pole zero plot of such a system is shown in the next slide. Note that the phase shifts produced at ω_c by the zeros and aircraft poles are nearly equal and we may assume for purposes of determining ω_c that these poles and zeros cancel each other. Thus, in the practical system the chatter frequency is determined primarily by the dynamic characteristics of the servo and instruments. Also we can see that the higher the chatter frequency, hence the more desirable the system, the more valid this approximation becomes.

If the zeros and aircraft poles are eliminated there remain a pole at the origin and several higher order poles. The pole at the origin produces a phase shift of 90° at every point on the imaginary axis so that at ω_c the combined phase shifts of the higher order poles is also 90°. We can use geometric relations to find the tangent of this combined phase angle and then ω_c can be found by equating the denominator to zero. For the sixth order example shown, this means that if vectors from the two complex instrument poles and the one real pole are drawn to ω_c, then the sum of the angles of these vectors with the real axis must equal 90°. The double angle tangent

formula can be used to find the tangent of the combined phase angle. The results of several such calculations are shown in the table. The ω's and the ζ's, and P_1 represent dynamics of the instruments or higher frequency characteristics of the servo. The seventh order case results in a quadratic equation in ω_c^2, but we know that 90° phase shift occurs at the lower frequency (270° being the higher one). Realistic values of ζ_1 and ζ_2 in this case give a maximum ω_c of about $0.6\sqrt{\omega_1\omega_2}$. Considering only an even number of higher order poles it can be shown that ω_c will always be a multiple of the geometric mean of their frequencies and, of course, it must be lower than the lowest frequency present. Once the chatter frequency is known the amplitude can be quite readily predicted by describing function techniques which will be shown later.

These methods of prediction were applied to the fifth order normal acceleration system taken as an example earlier and the results were checked by analog computer simulation. The aerodynamics used were those of a hypothetical missile chosen to be representative of high performance air-to-air missiles. The variations in parameters over the flight envelope were about 7 to 1 in natural frequency, 80 to 1 in aerodynamic gain and the damping ratio varied from 0 to 0.3. Three flight conditions were chosen, the two extremes of the flight envelope and an intermediate case. The gain constant was incorporated into the limiter and a very high open loop gain was obtained by removing the feedback from the limited computer amplifier. Different chatter frequencies were produced by changing the instrumentation denominator.

The frequency predicted by the approximate method is plotted as a function of the exact frequency computed by the root locus method on the next slide. Note that with these approximate formulas we can get a reasonably close estimate of the exact chatter frequency. For the example chosen the predicted frequency was always lower than the exact one.

The chatter amplitude was predicted using the frequencies measured on the computer. The method of prediction and the comparison of predicted amplitudes with the measured amplitudes are shown on the next slide. The prediction method is to consider the output of the limiter as a square wave of amplitude equal to the limit level. The input to the aircraft and servo can be approximated by the fundamental component of this square wave which is a sine wave of amplitude $4/\pi$ times the limit level. The chatter amplitude C_c is simply this amplitude times the magnitude of $G(j\omega_c)$.

Since the amplitude prediction is more accurate than the frequency prediction the best design procedure would appear to be first to determine the

chatter frequency which would give a tolerable amplitude and then design for that frequency. It should be noted here that actual hardware will have phase shifts due to nonlinearities that cannot be readily accounted for in this analysis. Therefore, it would be advisable to make the design chatter frequency somewhat higher than one which would give an acceptable amplitude.

We have a means for predicting chatter frequency and amplitude rapidly and the question now arises as to whether the low frequency characteristics of the practical system will be as desired. For the answer to this question we again turn to the analog computer and the results are presented in the next slide. The system used is the one taken as an example earlier except that instrument damping is greater than critical, and the instrument frequency was chosen to give a fairly low chatter frequency. The root loci are shown below the corresponding step responses and these responses are for the two extremes of the flight envelope. The one on the left is for an aircraft natural frequency of 3.6 radians/sec, a gain of 0.11 and zero damping. The instrument frequency is about 16 times that of the aircraft and results in the chatter frequency being high compared to the aircraft natural frequency. Thus, the chatter amplitude is well filtered and the step response corresponds quite closely to the zero positions, that is, to the desired response. The response on the right is for an aircraft natural frequency of 25 radians/sec, a gain of 8.8 and 0.3 damping ratio. The instrument frequency here is only a little more than twice that of the aircraft and the chatter frequency is less than three times the aircraft frequency. This low ratio of chatter frequency to aircraft frequency coupled with the high gain of the aircraft results in a very high chatter amplitude. In addition, one can see that instead of following the desired response, shown by the dashed line, the step response is very heavily damped. This means that the roots are quite far back on the loci with the dominant one probably near the origin. There are then two important questions to be answered. These are: First, for those examples where the chatter frequency cannot be made high compared to the aircraft frequency, what can be done to reduce the amplitude to an acceptable value? Second, what technique could be used to predict the actual pole locations for small signal inputs in the presence of chatter?

Considering the question of how to reduce the chatter amplitude, two methods are shown on the next slide. The upper system measures the chatter amplitude by means of a high pass network, rectifier, and filter. The output of the filter adjusts the limit level through a dead zone. If the chatter amplitude exceeds the threshold of the dead zone, the limit level is reduced. This results in a proportional decrease in chatter amplitude. The lower system consists of a lead network before the limiter and a network with the reciprocal transfer function following the limiter. The chatter frequency is not altered since the phase shifts of these networks cancel. Since the amplitude at the output of the limiter is constant, attenuation is provided by the lag network. The step responses for the high chatter amplitude case

from the previous slide are shown to the right of each system and the desired response is again shown in dashed lines. Both systems are effective in reducing the amplitude and no attempt has been made to determine which is better.

The second question, that is, why the step response is so heavily damped, is best answered with reference to the next slide. As was mentioned a high gain limiter, or on-off controller, acts for sinusoidal signals as a variable gain which is dependent on the ratio of the input magnitude to the limit level. The graph on the left is a normalized curve illustrating how the gain varies as a function of the ratio of input amplitude to limit level. The solid curve is for a single low frequency input to the limiter. Note that since the gain is essentially infinite for very low inputs the curve goes to unity. It is shown in Tsien's "Engineering Cybernetics" that the addition of a high frequency sinusoidal dither signal causes the low frequency average characteristic to behave as indicated by the dotted line. The characteristics of this dotted line are determined by the dither amplitude which for this graph is 0.1 the limit level. Thus, the addition of dither to a limiter gives, for low frequency inputs, an equivalent gain reduction which can be computed if the dither amplitude is known. For our example, the dither was assumed to be the chatter frequency where the amplitude at the input to the limiter could be computed using linear techniques. By using the equivalent low gain obtained to find the pole positions on the root loci the low frequency characteristic of the step response could be predicted. The results of this prediction are shown in the step response on the right and very good correspondence can be noted.

In summary, I would like to review the following points. First, it has been shown that one way of designing an adaptive autopilot is by the principle of using high gain. There are many ways of insuring the gain be high, and we exploited only one of these ways, the use of a high gain limiter. Second, we have shown that for a high gain saturated control system suitable for adaptive autopilot application, a limit cycle or chatter must exist. Techniques have been shown for analyzing such a system by which one can predict the chatter frequency, chatter amplitude, and the low frequency response in the presence of chatter. Third, the application chosen was for a normal acceleration type autopilot; however, there is no reason why the same type system, that is a high gain saturated control system, could not be used in any of the other modes. And last, the chatter frequency has been shown to be dependent primarily on instrument and servo characteristics. In order for this system to function satisfactorily, the chatter frequency must be quite high compared to the aircraft natural frequency. The example shows it is desirable to have the ratio of instrument to aircraft frequency in excess of 10 in order that the chatter frequency be high. This suggests that if conventional servos and instruments are to be used, a reduction in static margin of the aircraft which reduces its natural frequency would be helpful.

REFERENCES

1. Chestnut, Harold and Mayer, Robert W., " Servomechanisms and Regulating System Design" , Vol I, General Electric Co., 1951. John Wiley and Sons, Inc., and Chapman and Hall, Ltd., London.

2. Truxal, John G., " Automatic Feedback Control System Synthesis, McGraw-Hill Book Co., Inc., 1955.

3. " A Study to Determine an Automatic Flight Control Configuration to Provide a Stability Augmentation Capability for a High-Performance Supersonic Aircraft" , WADC Technical Report 57-349 (a series of four progress reports covering work done from March 1, 1957 through February 28, 1958).

4. Tsien, H. S., " Engineering Cybernetics" , McGraw-Hill Book Co., Inc., 1954.

ADAPTIVE SYSTEM WITH MODEL

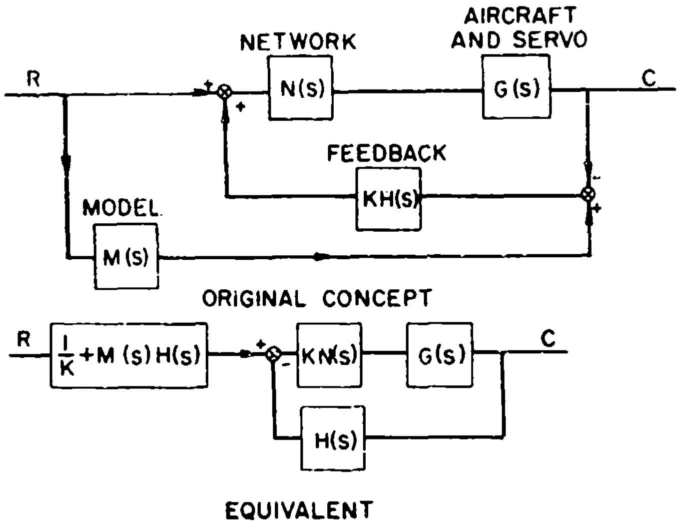
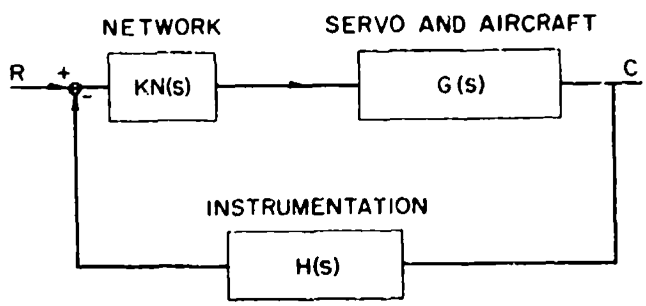


Fig 1

BLOCK DIAGRAM OF TYPICAL AUTOPILOT



$$\frac{C}{R} = \frac{KN(s)G(s)}{1 + KN(s)G(s)H(s)} \approx \frac{1}{H(s)}$$

Fig 2

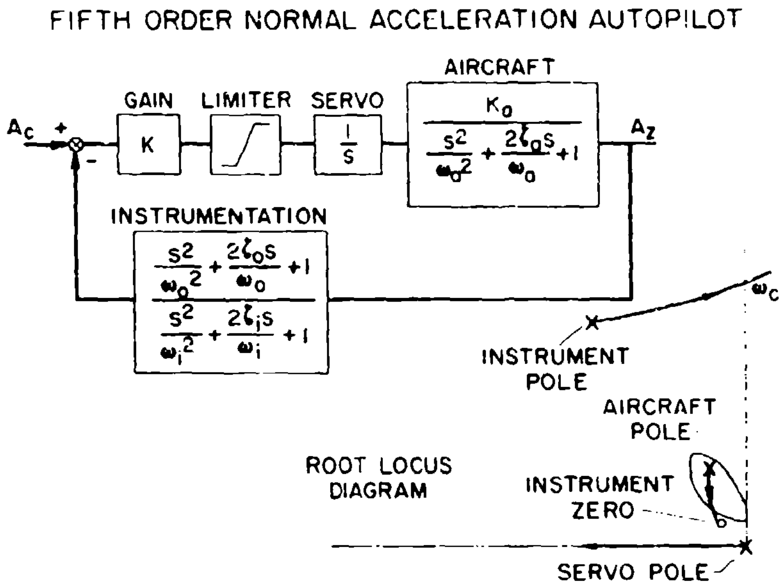
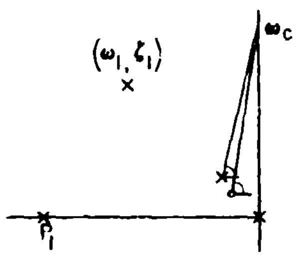


Fig 3

APPROXIMATE METHOD FOR DETERMINING CHATTER FREQUENCY



POLE-ZERO PLOT FOR SIXTH ORDER EXAMPLE

SYSTEM ORDER		CHATTER FREQUENCY FOR TWO ZEROS
4	$\omega_c =$	∞
5	$\omega_c =$	ω_1
6	$\omega_c =$	$\sqrt{\frac{P_1 \omega_1^2}{P_1 + 2\zeta_1\omega_1}}$
7	$\omega_c =$	$\sqrt{\frac{\omega_1 \omega_2 (b - \sqrt{b^2-4})}{2}}$

$$b = \left(\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1} + 4\zeta_1\zeta_2\right)$$

Fig 4

FREQUENCY PREDICTION FOR FIFTH ORDER SYSTEM

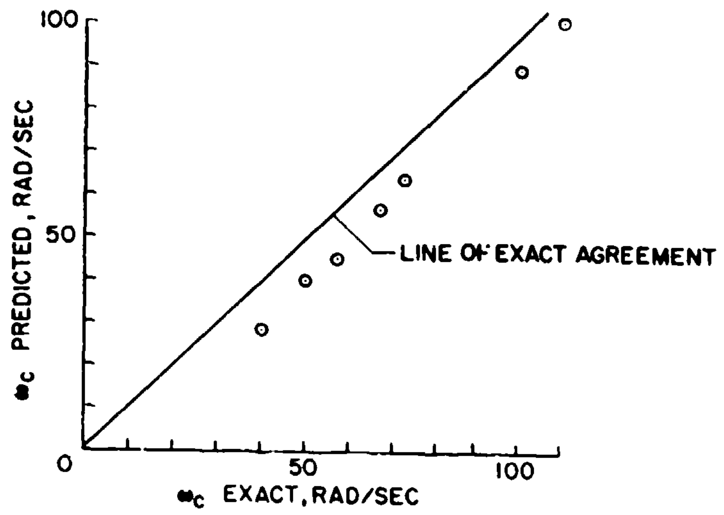


Fig 5

AMPLITUDE PREDICTION FOR FIFTH ORDER SYSTEM

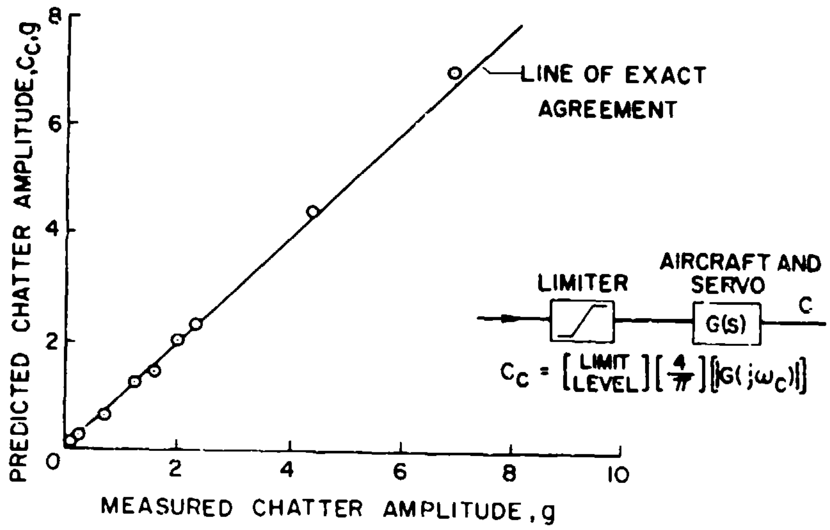


Fig 6

FIFTH ORDER SYSTEM FOR TWO FLIGHT CONDITIONS

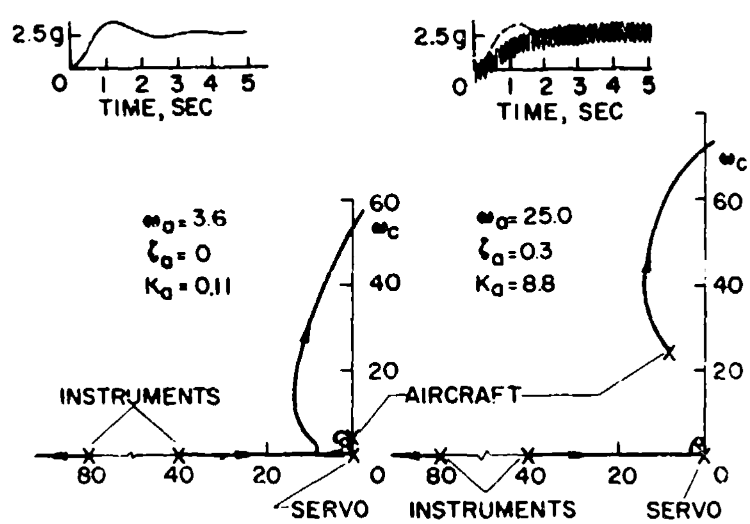


Fig 7

TWO METHODS FOR REDUCING CHATTER AMPLITUDE

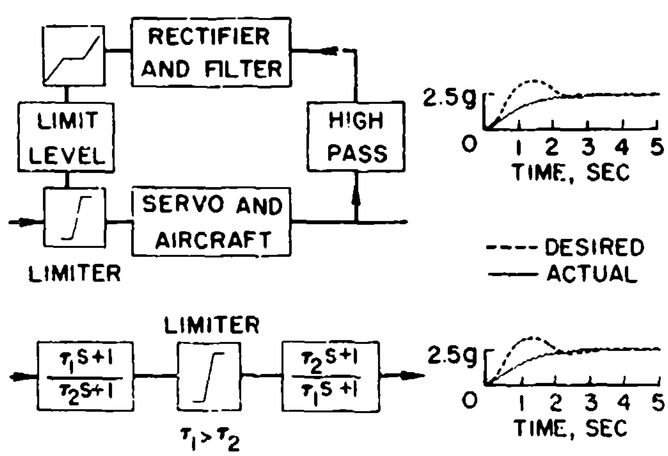


Fig 8

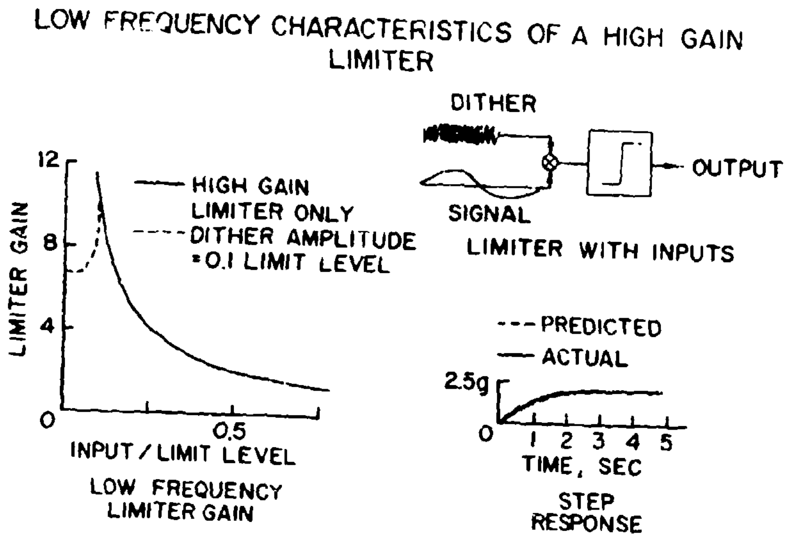

LOW FREQUENCY CHARACTERISTICS OF A HIGH GAIN LIMITER
DITHER
OUTPUT
SIGNAL
LIMITER WITH INPUTS
12
8
4
0
LIMITER GAIN
HIGH GAIN LIMITER ONLY
DITHER AMPLITUDE =0.1 LIMIT LEVEL
0.5
INPUT / LIMIT LEVEL
LOW FREQUENCY LIMITER GAIN
PREDICTED
ACTUAL
2.5g
0
1 2 3 4 5
TIME, SEC
STEP RESPONSE

Fig 9

SUPERVISORY CONTROL SYSTEM

by

Robert L. Cosgriff and Robert B. Lackey
Antenna Laboratory
Department of Electrical Engineering
The Ohio State University
Columbus 10, Ohio

INTRODUCTION

Today there are several types of adaptive control systems, systems which change as the environment of the system changes. These self adjusting systems can be divided into several categories and each category has its particular applications, its advantages and disadvantages. The various types of adaptive control systems can be classified in terms of versatility, complexity and speed of response. The major types of adaptive controls are listed and their characteristics tabulated in the table which follows. The degree of complexity is indicated by numbers from one to ten with unity indicating a simple system and 10 a complex system. Versatility is also indicated by numbers from zero to ten with zero indicating no versatility. The time necessary for the system to adapt once a change in environment is made is indicated as $n\tau$ where τ is the average time constant of the controlled system. These tabulated values are not absolute in nature but merely give an indication of system properties.

Type 1(a) system is by far the most versatile and in time it will adjust the system to a true optimum condition even if instability arises during the adjustment. Type 1(b) systems are similar to type 1(a) although not so versatile because of multiple criteria. Instability may result because of conflicting criteria.

Type 2 systems are all characterized by having a test input signal. However, the speed of response and complexity vary greatly. Type 2(c) systems will be described in a later section. Type 3 systems are the least versatile but are the simplest in construction. This type of system must be very carefully designed.

The remainder of this paper will be devoted to type 2(c) systems, one of several systems investigated in some detail at the Ohio State University Antenna Laboratory. (See references 1, 2, 3, 4, and 5 for results of other investigations.)

GENERAL DESCRIPTION OF SUPERVISORY CONTROL SYSTEM*

If values of a limited number of parameters of a linear or quasi-linear

* For other types of supervisory control systems see reference 5.

TABLE

Type	System Description	Versatility	One variable Adjusted: Complexity	One variable Adjusted: Time of Response	n variables Adjusted: Complexity	n variables Adjusted: Time of Response
1(a)	Optimizing (single criterion)	10	3	50τ	4	$50n\tau$
1(b)	Optimizing (multiple criterion)	4	6	10τ	$8n^{1.5}$	5τ
2(a)	Correlation (random)	3	10	20τ	$10n^{1.5}$	20τ
2(b)	Correlation (transient)	3	5	4τ	$5n$	4τ
2(c)	Correlation (sinusoidal)	3	2	τ	$2n$	$\frac{n+1}{2}\tau$
3	Quasi-linear design	2	1	τ	n	τ
4	Programmed	2	3	$\tau/4$	$3n^2$	$\tau/4$

system are to be controlled, a supervisory control system can be devised which measures and then adjusts the parameters of interest. The simplest method that can be used to accomplish this purpose consists of inserting a sinusoidal test signal into the system being adjusted and then sensing the output signal from this system. By proper use of phase sensitive detectors the parameter of interest can usually be detected. By comparing this parameter with a standard reference an error signal can be produced which will actuate a control motor which in turn will adjust or correct the parameter of interest. Conceptually, this type of supervisory control system differs from the conventional control system only in that a parameter of the controlled system, rather than a variable is adjusted.

EXAMPLE OF SUPERVISORY CONTROL SYSTEM

In this section a simple example of a supervisory control system is considered and test results given. See Fig. 1. Here the block labeled G(p) is considered a hypothetical aircraft where δ is elevator deflection and $\dot{\gamma}$ is the rate of change of the angle between the velocity vector and an inertial reference. The parameters A, f and k are all dependent upon the environment and velocity of the aircraft; however insofar as this paper is concerned only f will be controlled and A and k will be assumed to be constant. (The fact that these assumptions are made should not be construed as meaning that the variables A and k cannot be controlled by means of techniques described but rather that they are made to simplify the description of techniques that can be employed.) If the servo has a much faster response than the air frame the auxiliary feedback path which is to be adjusted and represented by kp will cause the effective transfer function relating x and $\dot{\gamma}$ to be modified from

$$\frac{AG_S}{p^2 + fp + k} = \frac{\dot{\gamma}}{x}$$

to

$$\frac{\dot{\gamma}}{x} = \frac{AG_S}{p^2 + (f + k\,G_S A)\,p + k}$$

Here G_S is the static gain of the servo.

Adding the test signal to the input of the servo system will cause $\dot{\gamma}$ to have a sinusoidal signal component of the form

$$\frac{AG_S \quad C}{\left[(k - w_o^2)^2 + w_o^2\,(f + k\,G_S A)^2\right]^{1/2}} \cos(w_0 t + \theta)$$

which will have a quadrature component given by

$$\frac{AG_S\, C\ (f + k\, G_S A)\ w_0 \sin w_0 t}{(k - w_0^2)^2 + (f + k\, G_S A)^2\ w_0^2} .$$

If w_0^2 is equal to k, this latter expression simplifies to

$$\frac{AG_S\, C}{(f + k\, G_S A)\ w_0} \sin w_0 t .$$

This particular component of $\dot{\gamma}$ can be separated from $\dot{\gamma}$ by means of a phase sensitive detector shown in Fig. 1. Subtracting the output of this detector from the reference signal yields an error signal which actuates the motor and thereby adjusts k. At the steady state condition

$$\frac{AG_S C}{(f + k\, G_S A)\ w_0} = k_0$$

The foregoing system has been simulated by means of an analogue computer and the response is indicated in Fig. 2. Rapid changes were made in f adjusting it from one constant value to another and the variable $f + k\, G_S A$ plotted. Note that $f + k\, G_S A$ returns to a constant value in about one period of the test signal, namely in time t_d given by

$$t_d = \frac{2\pi}{w_0}$$

GENERAL CONSIDERATIONS

In design of high speed supervisory control systems of the type described, many practical considerations must be taken into account. First, if the output of the detector is amplified and directly excites the control motor spurious unstable modes may be encountered in that k will have a periodic component and the overall system reduces to a linear system with time varying coefficients. Such systems can have unexpected unstable modes of operation.

The choice of w_0 is of importance for two reasons. First, the upper limit for the speed of response is generally proportional to w_0. Thus for high speed operation w_0 must be large.

Second, if w_0 is chosen near the resonant frequency $\sqrt{k}$, noise, both internal and aerodynamic, may cause k to have a noise component larger than desired. For this reason it is generally desirable for w_0 to be somewhat larger than $\sqrt{k}$.

If all three parameters, A, f and k are to be controlled at least two test signals must be employed. In general the number of test signals required is equal to one half the number of variables to be adjusted.

Finally, one is interested in the complexity of the equipment required for the supervisory control unit. Generally this equipment is far less complex than is expected particularly if other components, normally installed can be used for sensory purposes.

1. Cosgriff, R. L., " A Study of Nonlinear Servomechanisms" , Ohio State University Antenna Laboratory reoort AF 18(600)-88, E. O. 112-61 SR-6f4, Chapter 4, 1953.

2. Cosgriff, R. L., " Stabilization of Nonlinear Feedback Control Systems" , Proc. IRE, Vol. 41, pp. 382-385, March 1953.

3. Cosgriff, R. L., Nonlinear Control Systems, McGraw-Hill Book Co., Inc., New York, 1958, Chapter 4.

4. Cosgriff, R. L., Emerling, R. A., " Optimizing Control Systems" , Applications and Industry (AIEE publication) No. 35, March 1958, pp. 13-16.

5. Cosgriff, R. L., Nonlinear Control Systems, ibid, Chapter II.

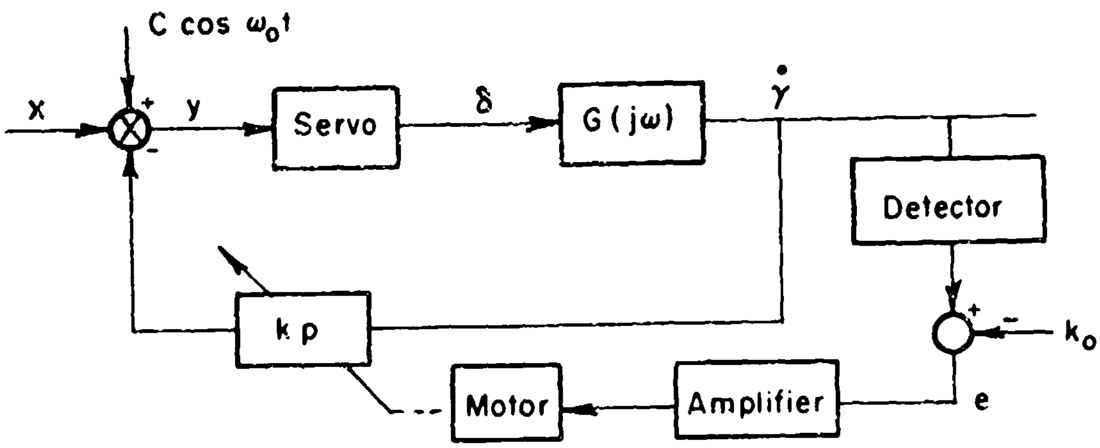


Fig. 1. Supervisory Control System for Hypothetical Aircraft.

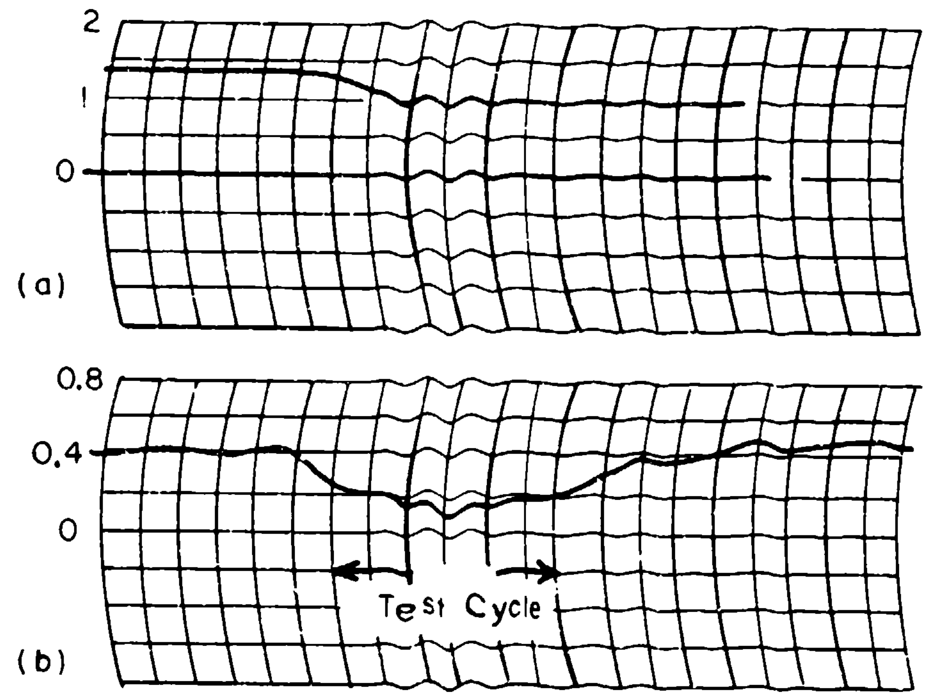


Fig. 2. Typical Response.
(a) Natural damping factor
(b) Total damping factor.

THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS SYMPOSIUM

SESSION II

Dr. C. S. Draper, Chairman
Massachusetts Institute of Technology

OPENING REMARKS BY CAPT. R. R. RATH

During the break, it was pointed out to me that there has been a bit of an oversight. It seems that up to now there has been no definition of what an adaptive control system is. That may not have been an oversight. This is a highly controversial issue. I think I might be safe in giving a definition because we are not going to allow any questions or discussion at this time. I like the definition that we at the Flight Control Laboratory have been following as a guide when discussing an adaptive system. We consider a system adaptive when it maintains a desired performance throughout the entire flight regime of a vehicle in a closed loop fashion with no air data scheduling of control parameters and with a minimum of advance information about the vehicle characteristics. This covers a broad area, but if a system does this, it operates the way we want it to and, therefore, we feel that we can consider it to be adaptive. This subject will be open for discussion during the open forum. The chairman of the second session will be Doctor C. S. Draper, who needs no further introduction.

Dr. C. S. Draper
Head, Department of Aeronautics
Head, Instrumentation Laboratories
Massachusetts Institute of Technology

It is an honor and a pleasure for me to be given the privilege of sitting up here all by myself on the platform and supposedly being the chairman of this session from which we are going to have so much dynamic information. I suspect that this was done so that I would not talk too much and I am going to try to restrain myself. As I sit here looking at all the attendees, I find that I have known a large part of this group for many years. I am sort of forced to think about new techniques, not in adaptive control systems, but in the methods of getting things done. We start out and get thinking people in on the job before it is really well defined. I can't compete with General Davis' stories, but I would like to illustrate my point by a little story that I think is very pertinent.

The story deals with a contractor that had a beautiful wife and a son about five or six years old. The contractor told his son that he was about to have a little sister or a little brother, and the little sister or brother would be along sometime after the first of the year. The boy said, "Gee, wouldn't it be nice if we had it here for Christmas?" His father told him, "Well, I'm afraid this is a little hard to do. I don't think we can do this". So the son said: "Well, if you are building a building and want to do it a little faster you put about a hundred more men on the job, why don't you do the same thing here?"

Actually, we are now doing research and development by this procedure. It used to be that a few people, or a few organizations, worked on a new development until the idea was pretty well jelled. When something significant came out a lot of other people then went on to make the equipment or do whatever was required. Now we don't wait for the impetus to be born, we actually get people in on the deal when the conception is nothing more than a probability. As far as I know this is a new technique in the history of the world. I don't believe we ever did it much in this country, the Germans didn't do it very much, and whether the Russians do it or not I don't know. It would be interesting to find out.

I am going to take a few minutes here and try to make a few definitions that perhaps will give a little meaning to the overall picture before we get involved in mathematics. Later on you can argue with me if you want to, but I would like to define flight control as the process of maintaining the vehicle involved within a range of attitudes and motions that provide safe operation, and permit the effective utilization of guidance. Guidance is the process of indicating deviations of the actual vehicle path from the desired path and generating a correction command signal for the flight control system. In

other words, guidance stems from subjectory type of things. We are interested primarily in what the center of gravity is doing and not concerned about the attitude of the vehicle. The flight control system merely enables you to realize the guidance correction that the guidance system has found to be necessary.

To sketch in the background of this thing, as I see it, flight control is a very, very old art. It goes all the way back to the days of the gliders. Both flight control and guidance in those gliders were provided by the human that was being lifted along with the vehicle. The man looking out sensed the deviation from the path that he wanted and he translated those deviations in the motions of his own body so that he provided both guidance and flight control. The eyes were the sensing media and the pilot's body was an effective medium for realizing flight control. This system got into a lot of trouble because they thought the way to control the glider was to make it very stable and, in effect, have the same conditions that you have in a ship. The ship sort of rides on the waves and you turn a rudder to steer it. The trouble, of course, with these stable vehicles were that they were so stable that the human being couldn't do what he wanted to do. This didn't produce very high class flights, and led to catastrophic results before they got into bad air conditions.

The Wright Brothers have been given credit for a lot of things and most of these are things that I think they didn't do. What they did do was to provide a machine which, in itself, was unstable, but when you put the man in it to provide the sensing, the judgment for developing guidance commands, and the ability to translate those commands into motions for control purposes, the combination resulted in satisfactory flight stability. By making the vehicle itself unstable, but controllable, and putting the man in the loop the Wright brothers came out with an overall result that was a pretty good flying machine for those days. As a matter of fact, the pattern that they had established is the one that is very largely used today.

The replacement of the man and his sensing by an instrument started in 1918 with the gyroscopic turning gear and continued with the Guggenheim competition. The Guggenheim developments led Doolittle and Brown to the blind flying business where more of man's senses were replaced by gyros; however, the man continued to generate guidance and do a considerable part of the flight control. Autopilots that have replaced more of man's functions came along in the early 30's. I remember seeing the Sperry Gyroscope Company's first autopilot. They made three of them and, if I remember correctly, they had a terrible time selling them. Nobody was interested in them. Whiley Post took one of them and he flew around the world and you know what happened after that.

Developments have followed in this field for a matter of twenty years or so until we have the all maneuver autopilot today. Flight control systems

of today are a far cry from those first few autopilots. Now the flight control systems are adapted to receive guidance commands either from radio or radar, or from inertial systems carried within the vehicles. The system that provides guidance is not identical with the system that provides flight control.

I would like to make a couple of remarks regarding adaptive control systems and I would like to define what I call optimalizing control systems. Optimalizing control systems are exactly what the word says. I use the word " optimal" instead of " optimize" because optimize in the dictionary refers to a human type that is always looking too far in the optimistic direction. These systems replace the human function of refining performance as it has been used in many, many systems. For example, if you are conducting a test and you want to find out what fuel mixture ratio gives you the best power, you fix the mixture ratio and you look at the scale beam on the dynamometer. You then change the mixture control and if the scale beam goes up you change it a little bit more in the same direction. Some time you will come to a point where changing the mixture ratio in the same direction causes the dynamometer beam to go down. In this case you reverse the direction of your mixture control adjustment. You do the same thing if you are trying to get the maximum economy out of flying an airplane. You check how many gallons you have used per hour with a certain mixture ratio and then you try again. This is the basic philosophy of the optimalizing systems and it seems to me that they are adaptive systems. A paper " optimalizing control" was written by Dr. Lee L. Lanning and myself in 1950. It was presented at an ASME meeting in San Francisco in 1950 and resulted in an ASME transaction paper in 1951.

The idea of an adaptive system was first used in anti-aircraft fire control systems in 1939, based on the idea that if you had a target at long range you automatically adjusted the parameters of your system so that it gave you a smooth refined solution. If the range became shorter then you made the system faster but the solution became less accurate. When you started out to get a solution in the aircraft fire control business you deliberately changed the parameters of the system so that you would get a fast solution in the shortest possible time. If the target started to maneuver, you in effect took a reading of the answer that came out and adjusted the parameters of the system in such a fashion that you end up with the best results. As I see it, this is the philosophy used to measure the performance. For the given setting of the target you are going to try to optimalize you keep track of what happens in performance, and you make a change, and you see whether the results are better or worse than you had. If they are better, you do one thing and if they are worse, you do another. You are, in fact, using a sampling closed route method of adjusting the system.

This is not exactly what Captain Rath has said, but I think it has the same general idea. I have already talked too much and I thank you gentlemen for your attention.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY PRESENTATION

Mr. H. Philip Whitaker

An adaptive system is one that adapts itself to a changing environment, a changing character of input signals, or a changing system or component characteristic in such a manner that a desired performance will be maintained.

The development research program that has been completed by the M.I.T. Instrumentation Laboratory has investigated the characteristics of one class of adaptive control systems and their application to aircraft, missile, and spacecraft control systems. In later presentations, Dr. Li will consider the overall classification of the various kinds of adaptive systems. The type of system which I would like to discuss at this time is that which Dr. Li would call a dynamic performance adaptive system with closed-loop adjustment of system parameters. Such a system controls the dynamic performance of a control system with respect to a reference system, or specification, by adjustment of the system parameters either through a closed-loop process of nulling a performance index or through a self-optimalizing* process of seeking an optimum operating point. This type of system eliminates the uncertainties and design compromises that accompany the commonly used, open-loop, gain programs and the need for accurate estimation of the airplane performance functions prior to system design.

We have further suggested the name " model-reference adaptive system" for the type of system under consideration. A model-reference system is characterized by the fact that the dynamic specifications for a desired system output are embodied in a unit which is called the model-reference for the system, and which forms part of the equipment installation. The command signal input to the control system is also fed to the model. The difference between the output signal of the model and the corresponding output quantity of the system is then the response error. The design objective of the adaptive portion of this type of system is to minimize this response error under all operational conditions of the system.

*Definition of Optimum Response - It is obvious that criteria are needed for specifying an " optimum" response. In this regard the following definitions are proposed.

Optimum control of the aircraft is said to result if the control system provides close control of the transient and steady-state responses of the aircraft so as to utilize the capabilities of the aircraft most effectively in fulfilling the specifications of its flight mission. These specifications of course vary with the type of aircraft and are different for separate portions of the flight mission of any one aircraft (for example, the landing and the

terminal interception phases). They may or may not require the maximum capabilities of the aircraft, but in almost all cases they will specify such characteristics as response time, damping, dynamic and static errors, and control of interference effects. This definition of optimum lays greatest emphasis upon meeting the mission specifications of the vehicle.

Figure 1 is a functional block diagram of a model-reference system. The characteristic feature of the model-reference type of system is the model, which is the physical embodiment of the design specifications of the system. A change in the model is exactly equivalent to a change in the system specifications. The first consideration, then, is to decide upon the system specifications, which of course must be compatible with the performance capabilities of the aircraft. When this is done, the model performance will also be compatible with the aircraft capabilities. Further, if different specifications exist for each mode of system operation, these differences can be incorporated into the model. The output of the model is the desired response of the system and is physically available as a reference signal. The command input to the system is also sent as an input to the model, and thus all of the information required for making the system adapt is obtained from the normal operational inputs to the system. If the performance function of the system were exactly the same as that of the model, the outputs of the system would be identical to those of the model. The system would thus meet its performance specifications, and its response would meet our definition of optimum. In general, the performance functions will not be identical, and the difference between the system response and the model response can then be a measure of response error which can in turn be used to generate error functions which are measurable criteria of system performance. These error functions (or quantities) then serve to generate command signals to change either the controllable parameters of the control loops or the characteristics of the input signal so that the desired response of the flight control system will result.

Note also that the adaptive control equipment is making adjustments to a control system whose feedback control loops are closed independently of the adaptive controls rather than being closed through the adaptive equipment. If the latter equipment fails, the control system still remains as a closed-loop system.

The adaptive control features can be added to any existing control system without major alteration of the control signal paths. In a multi-loop system the adaptive features can often readjust the remaining parameters to result in satisfactory flight characteristics even though complete failure of the adaptive controls for one parameter occurs. (We discovered this when it happened to us in flight.)

For these reasons it appeared that greater flexibility could be obtained with a model-reference system. When this was considered together with the relative ease with which the required equipment could be built and installed

in the aircraft currently bailed to the Instrumentation Laboratory, it was decided to investigate this type of adaptive system.

To evaluate the techniques developed during this program, they were applied to an all-maneuvering flight control system developed earlier at M.I.T. and flight tested in an F-94A airplane. They are not restricted to use only with this system, however. In the complete system, seven parameters were varied. Three of these were in the pitch sub-system, one in the rudder coordination sub-system, and three in the yaw sub-system. Only limited flight test time was available for the program, and as a result it was not possible to flight test the complete system at one time. On two flights it was possible, however, to have six of the seven parameters controlled automatically by the adaptive system.

There is insufficient time available this morning to cover the details of the entire system. They are presented in the final report of this program, and this report is available here today. To illustrate the design procedures, let us consider the yaw sub-system in detail, and if time permits briefly outline the main features of the pitch and rudder coordination sub-systems. All of the slides to be presented are taken from figures found in the final report, and you can study them in more detail by referring to the report later.

The design procedure consists of the following steps:

1. Design of a model to meet the system psecifications
2. Selection of the control system loop configuration
3. Determination of which parameters should be varied and how they affect the system response
4. Determination of error criteria which will adjust the parameters
5. And finally, analysis and simulation to determine the convergence times and dynamic operating performance of the system.

Since no mission specifications had been set up for the system of this development program, an arbitrary dynamic performance specification was chosen. This was expressed as the requirement that the system generate an aircraft yaw angular velocity in response to a command input, and that for a step function input a response time of approximately 3 seconds with no overshoot would result. The loop configuration in this case was chosen to be that of figure 2.

This figure presents a functional block diagram of the adaptive yaw system. This is an orientational control system which stabilizes the aircraft

to the yaw reference orientation established by the yaw integrating gyro. This yaw system produces yaw angular velocity with respect to inertial space proportional to a yaw command signal input. The yaw angular velocity is generated by rolling the aircraft to establish a roll angle while minimizing aerodynamic sideslip. Two degrees of freedom are thus involved, and the yaw system is accordingly more complicated than the pitch system. It has been found that the rudder coordination system required to control sideslip can be analyzed separately from the outer loops which control roll angle and yaw angular velocity. The rudder coordination system was also adaptive and is described in the report. In the discussion that follows, it will be assumed that the rudder system is operating correctly, and if time permits we will return to it later.

For the present application, the dominant modes of the system can be represented by a third order performance function exhibiting one real pole and a pair of complex conjugate poles. Therefore, a third order model-reference was chosen, and in order to meet the system specifications the real pole exhibited a characteristic time of 1.4 seconds, and the second order poles were characterized by an undamped natural frequency equal to 1.65 radians/second and a damping ratio of 0.8.

An additional design feature which increases the flexibility available to the designer was also investigated during this program. It was recognized that meeting the same system specifications at low dynamic pressure flight conditions would require a very high open-loop sensitivity for the yaw orientational control loop. Such gains may be undesirable from fail-safe considerations. Further, since parallel control servos were used in the test airplane, past experience had indicated that the high loop sensitivities would result in control stick deflections due to random turbulence to which the pilots would object. If the loop sensitivity were arbitrarily limited at some maximum value, however, the error criterion used to control that sensitivity could no longer be satisfied at those flight conditions that require higher values. The remaining control loops would readjust to produce the best system possible under the circumstances, but the system would cease to operate about its optimum point, and all the error quantity signal levels could be expected to increase.

There are several solutions to this problem. The one that was investigated here was chosen for its apparent reasonableness and simplicity. It was recognized that one might indeed desire a different system specification at the lower dynamic pressure conditions. In particular it may be desirable to have a slower responding system when the aircraft is near its stalling speed. This can be accomplished very simply by making the first order term in the model performance function vary once the limited value of the orientation loop is reached. In operation, whenever high sensitivities were called for, the model slowed down until the maximum available sensitivity was sufficient to permit the error criterion to be satisfied. In this sense, the model itself was adaptive, and the sampling criteria which were derived from the model were also adaptive.

With the performance specifications established in the design of the model, there remained the task of selecting error criteria and assigning the various loop parameter controls to minimize them. In our approach to date, various functions of the response error have served to define error quantities. The criteria of performance are the specified optimum values of the error quantities usually taken to be either minimum or null values. In general, the error quantities are examined over some interval of time called the sampling time. In this approach, the sampling time has been controlled by the input and output signals of the system model. The sampling begins with the initiation of a normal operating input to the system and is terminated at a time that is controlled by the magnitude of the output of the model in relation to the input. This enables one to tie the length of the sampling interval to such quantities as the rise time or the solution time of the dynamic model.

There are three parameters which can be varied to control the dynamic response of the system. These are the open-loop sensitivities of the three control loops. For convenience the three parameters chosen express these open-loop sensitivities in terms of the ratios of the aileron displacement to the three output quantities yaw angle, roll angle, and roll rate since these are the measurable ground calibration quantities. The three parameters are represented by the notation: P_{yoc}, the yaw orientational control loop parameter; P_{rs}, the roll stabilization loop parameter; and P_{rd}, the roll damping loop parameter.

In selecting an error quantity for the orientational loop parameter, it was observed that this sensitivity directly affects the magnitude of the torque applied to the airplane in response to an input command. It is thus effective in controlling the initial portion of the response, or the rise time of the system. Therefore, the error quantity chosen to control P_{yoc} was the integral of the error sampled over the rise time of the model, arbitrarily taken to be the time at which the model output reached 70% of the input. The error criterion was that the integral be zero.

Of the three variables, the roll stabilization parameter exerted the greatest effect upon the system stability. An error quantity that afforded a simple mechanization was desired, and a nulling rather than a minimizing quantity was preferred to reduce convergence time. Thus the error criterion for this parameter specified that the integral of the error sampled over the response time of the model was to be zero.

The integral of the absolute value of the error was chosen as the error quantity for the roll rate damping loop, and the design criterion was that this integral be a minimum. Even when the error criteria for the previous two loops had been satisfied, oscillations could result due to insufficient damping. The absolute value operation adds an increment to the error quantity for each half cycle of an oscillation, and thus the integral becomes large when oscillations are present.

The next step in the design procedure is the analysis and simulation of the system to determine its static and dynamic performance characteristics.

Figure 3 presents some of the general features of useful error quantities. Before examining these error quantities it is appropriate to consider some general features of the properties of useful error quantities discussed in Chapter 2. If one first considers the case of two variable parameters, the extension to more variables is straightforward. If the parameters are P_1 and P_2 and the corresponding error quantities are $(EQ)_1$ and $(EQ)_2$, one can plot families of curves of $(EQ)_1$ versus P_2 for constant values of P_1. A typical presentation for a nulling criterion would be similar to that of Figure 3a. In the vicinity of any operating point defined by the set (P_1, P_2), the change in $(EQ)_1$ can be written

$$d(EQ)_1 = \frac{\partial (EQ)_1}{\partial P_1} dP_1 + \frac{\partial (EQ)_1}{\partial P_2} dP_2 \qquad (1)$$

where the derivatives are evaluated at the point P_1, P_2. The second partial can be evaluated by cross-plotting the data for the desired value of P_1. Similarly, we could evaluate the change in $(EQ)_2$ as

$$d(EQ)_2 = \frac{\partial (EQ)_2}{\partial P_2} dP_2 + \frac{\partial (EQ)_2}{\partial P_1} dP_1 \qquad (2)$$

If one then selects a value of P_2, the value of P_1 which $(EQ)_1$ would select is given by the intersection point on the $(EQ)_1 = 0$ axis, and these can be plotted as in Figure 3b. If the process is repeated for the second error quantity data, another curve is obtained as in Figure 3c. If the two curves intersect at only one point, both error criteria are satisfied only for the values of P_1 and P_2 corresponding to the point of intersection.

Ideally, the second terms on the right-hand sides of Equations (1) and (2) would be zero, in which case the error quantities would be functions only of their associated parameters. If this were true the corresponding data for Figure 3a would show only one curve rather than a family of curves, and the Figure 3c would appear as shown in Figure 3d. On the other hand the error quantities become useless if there is no intersection in the range of usable values of P_1 and P_2 as in Figure 3e.

As you will remember the error quantities used to set P_{yoc} and P_{rs}

were two different samples of the integral of the error. The data corresponding to part c of figure 3 is shown in figure 4.

The error quantity selected for the roll damping loop was the integral of the absolute value of the error, and the type of minimum this quantity exhibits is shown in figure 5 for the case for which $(EQ)_{yoc} = (EQ)_{rs} = 0$.

As might be surmised the use of three error criteria result in some redundant action. Actually the final operating point is defined by the minimum of the integral of the absolute value of the error and could probably be attained through some method of time sharing the error quantity among the three parameters. The other two criteria, however, perform the very important function of greatly reducing the convergence time of the system as will be shown subsequently.

One can obtain a geometrical representation of the optimalizing action of the system by looking at a three-dimensional space defined by three orthogonal axes the coordinates of which are the values of P_{yoc}, P_{rs}, and the error quantity for the third parameter. All possible combinations of these three quantities define a surface resembling a bowl-like shell, the bottom of which is the minimum value of the integral of the absolute value of the error. There is a similar shell for each value of the roll damping loop parameter as shown in figure 6.

The intersections of the shell and planes parallel to the $P_{yoc} - P_{rs}$ plane define contours of constant value of the integral as shown in figure 7. Two dashed lines are plotted on the figure. One of these corresponds to points at which the error criterion for P_{yoc} is satisfied, and the other for points at which the error quantity for P_{rs} is satisfied. These curves actually are projections of two curves on the surface of the shell. These two curves intersect at the minimum value of $(EQ)_{rd}$ due to the previous choice for P_{rd}.

To show the effect upon the system damping, the poles of the dominant second order mode were obtained for the data of figure 7. These are shown in figure 8. It is desirable that convergence take place in such a manner that regions of instability are avoided. Figure 9 shows that if any two of the error criteria can be satisfied, the system will be stable.

A movie was shown showing the simulation of the system on a high-speed, suppressed, time-scale analogue computer. The type of test that was simulated was one in which the system parameters had been deliberately set at initial values far from their optimum settings. The response of the system was shown as the parameters were changed by the adaptive process.

Flight test results of the system are presented in figures 10, 11, 12, and 13. These show the results of flight tests in which the initial conditions chosen for the loop parameters represent errors that are far greater than those which would be encountered in practice. In particular, figure 11 shows that the system will recover from an unstable initial condition using only one sample of error information.

In summary, we have presented the results of an adaptive control system research program which has investigated the characteristics of the model-reference type of adaptive system. Briefly, these characteristics are as follows:

1. The system provides closed-loop control of the system parameters so that a specified dynamic performance will result.

2. The system specifications are embodied in a model-reference which forms a portion of the equipment installation.

3. The design techniques can be applied to any existing system without major alteration of the system configuration.

4. All information required for performing the adaptive operations is obtained from the model and system responses under normal operating inputs.

5. Sampling is controlled entirely by the interrelationship between the input and output of the model.

6. Convergence times of the order of 10 seconds of sampling time for large errors in parameter initial conditions has been experienced in flight tests. The use of nulling criteria for some of the parameters greatly reduces convergence time.

We do not claim that the full potentiality of these systems has been reached. Rather, we think a door has been opened, and what lies beyond warrants a further research effort.

Finally, it should be acknowledged that the impetus for the model-reference system evolved from some of the early work performed by Capt R. R. Rath of Wright Air Development Center who deserves much of the credit for keeping interest in the entire area of adaptive control systems alive.

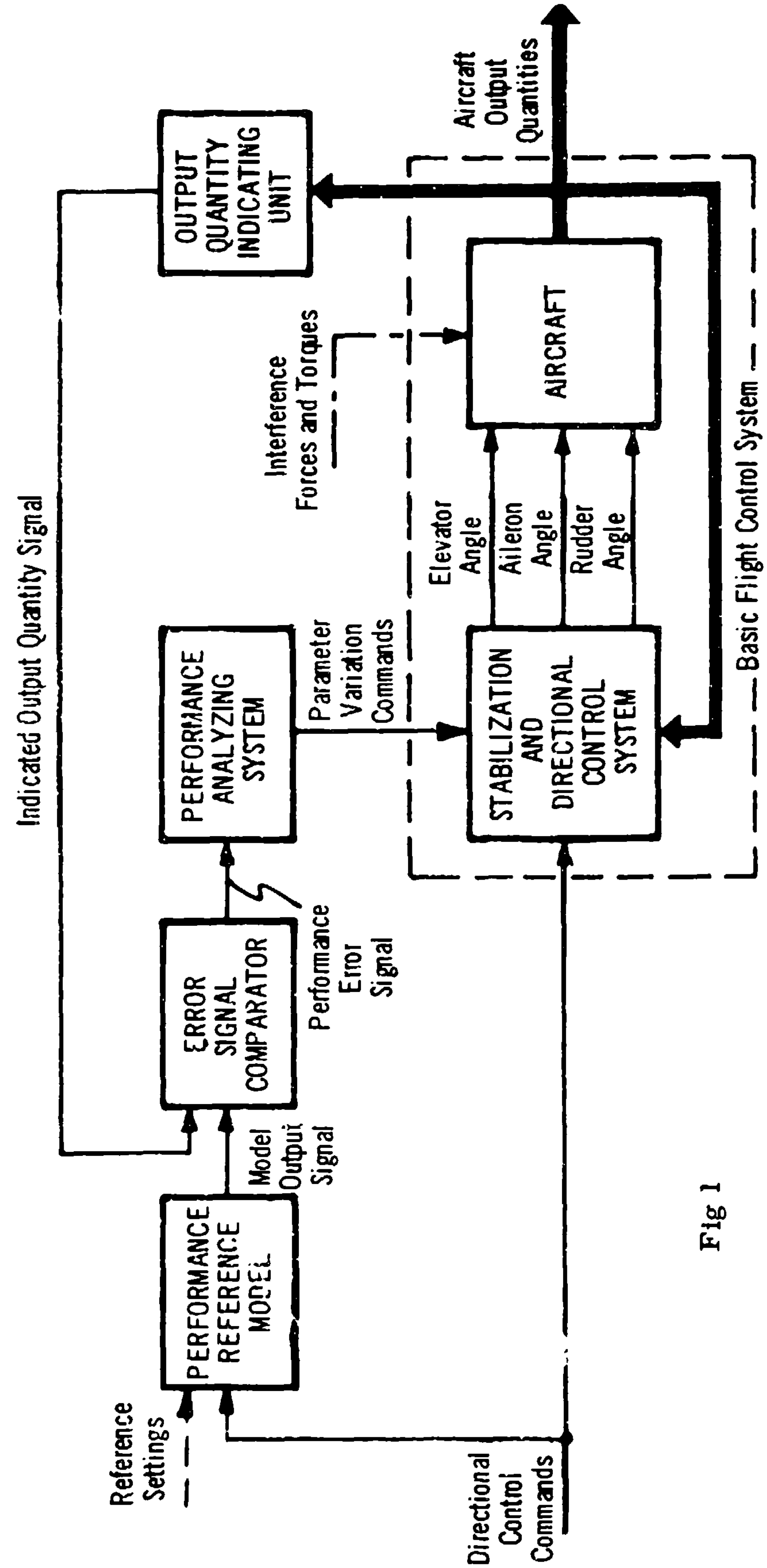

PERFORMANCE REFERENCE MODEL
ERROR SIGNAL COMPARATOR
PERFORMANCE ANALYZING SYSTEM
OUTPUT QUANTITY INDICATING UNIT
STABILIZATION AND DIRECTIONAL CONTROL SYSTEM
AIRCRAFT
Reference Settings
Directional Control Commands
Model Output Signal
Performance Error Signal
Parameter Variation Commands
Indicated Output Quantity Signal
Interference Forces and Torques
Elevator Angle
Aileron Angle
Rudder Angle
Aircraft Output Quantities
Basic Flight Control System

Fig 1

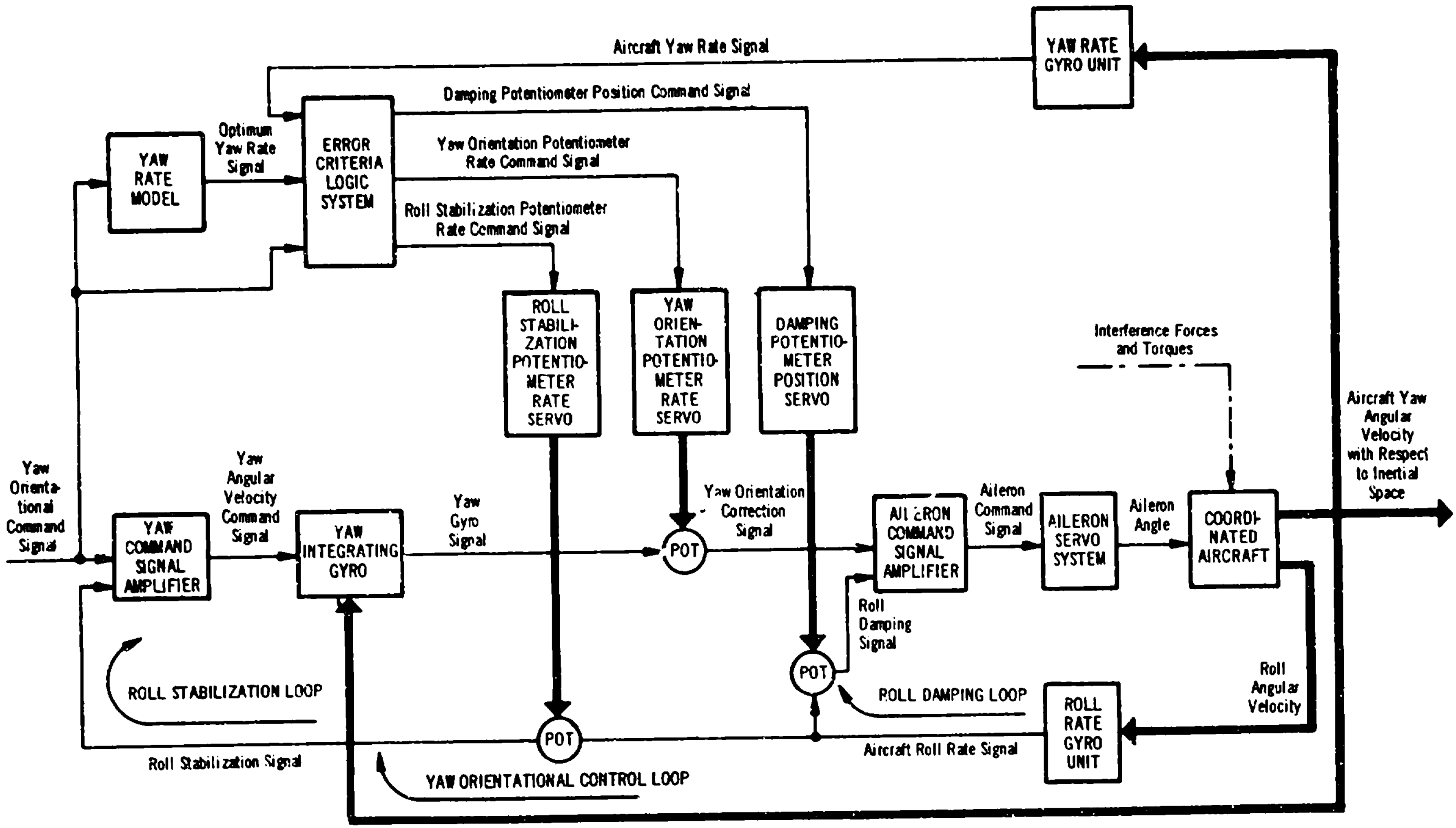
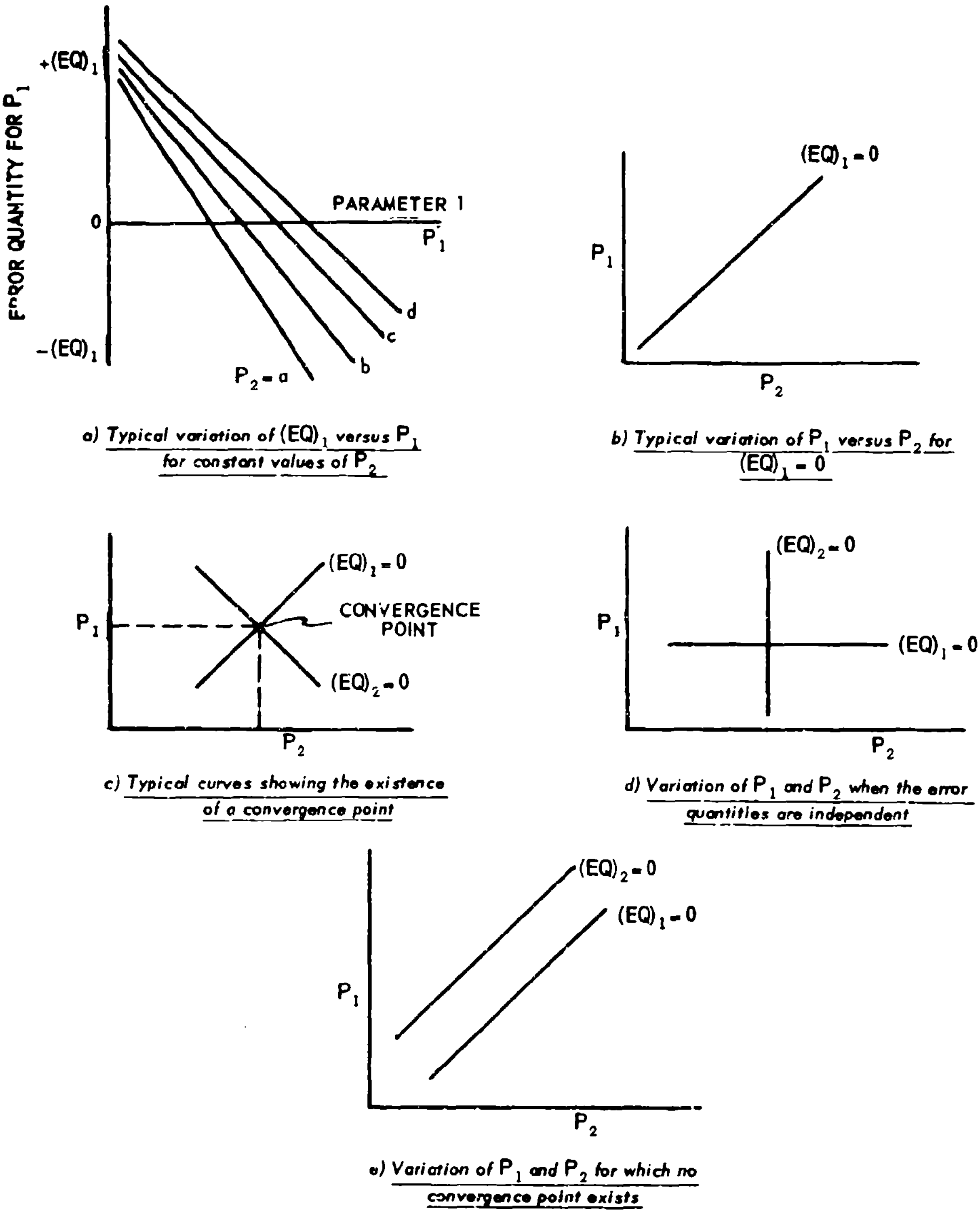


Fig 2 Functional block diagram of the adaptive yaw orientational control system.



a) Typical variation of $(EQ)_1$ versus P_1 for constant values of P_2

b) Typical variation of P_1 versus P_2 for $(EQ)_1 = 0$

c) Typical curves showing the existence of a convergence point

d) Variation of P_1 and P_2 when the error quantities are independent

e) Variation of P_1 and P_2 for which no convergence point exists

Fig. 3 Properties of error quantities.

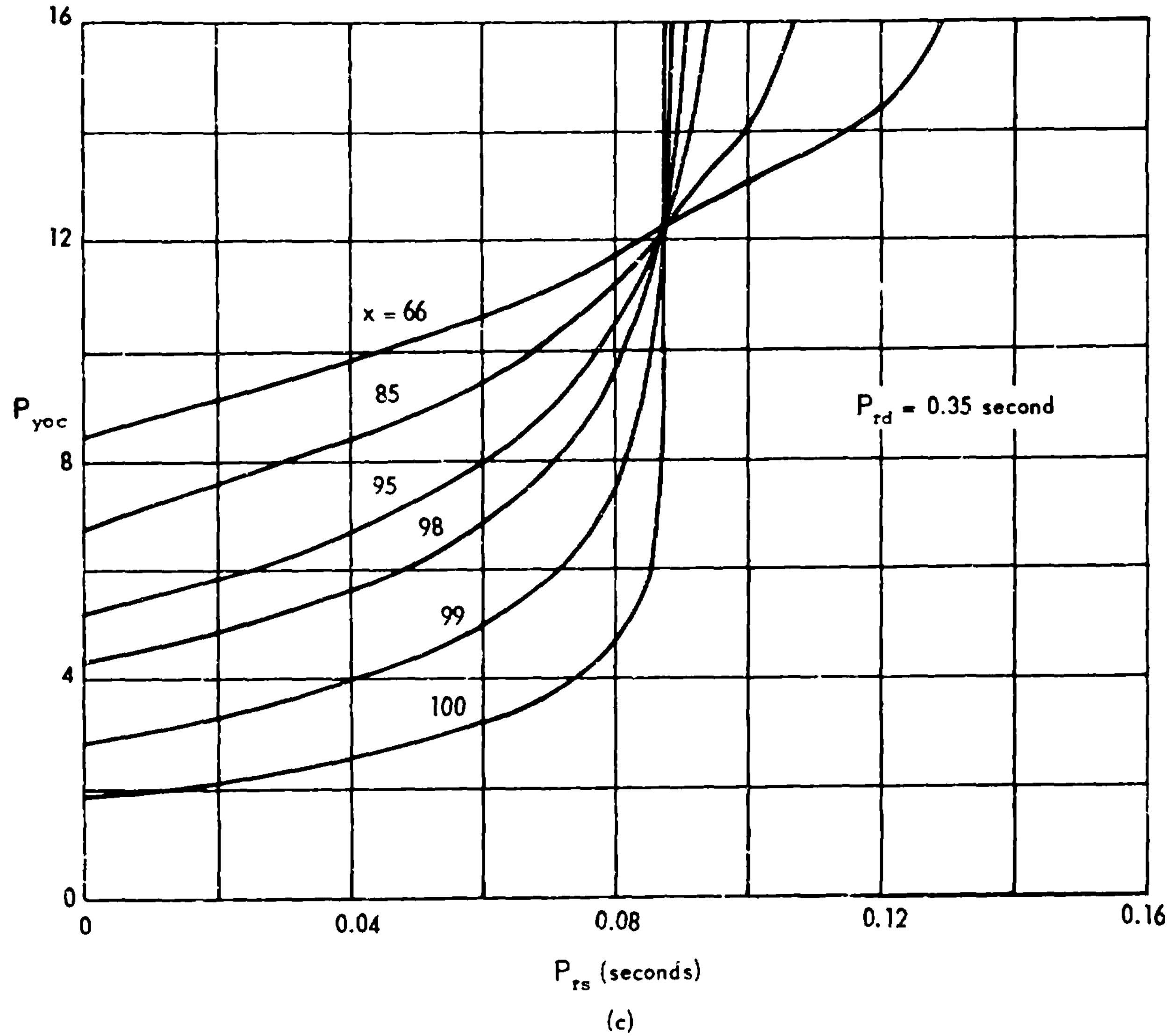


Fig 4 Combinations of the values of the system parameters for which $\int_0^{t_{s(x)}} (E) W_{(M-A)Z_A} \, dt = 0$ for several values of the sampling interval for the system of Fig. 2

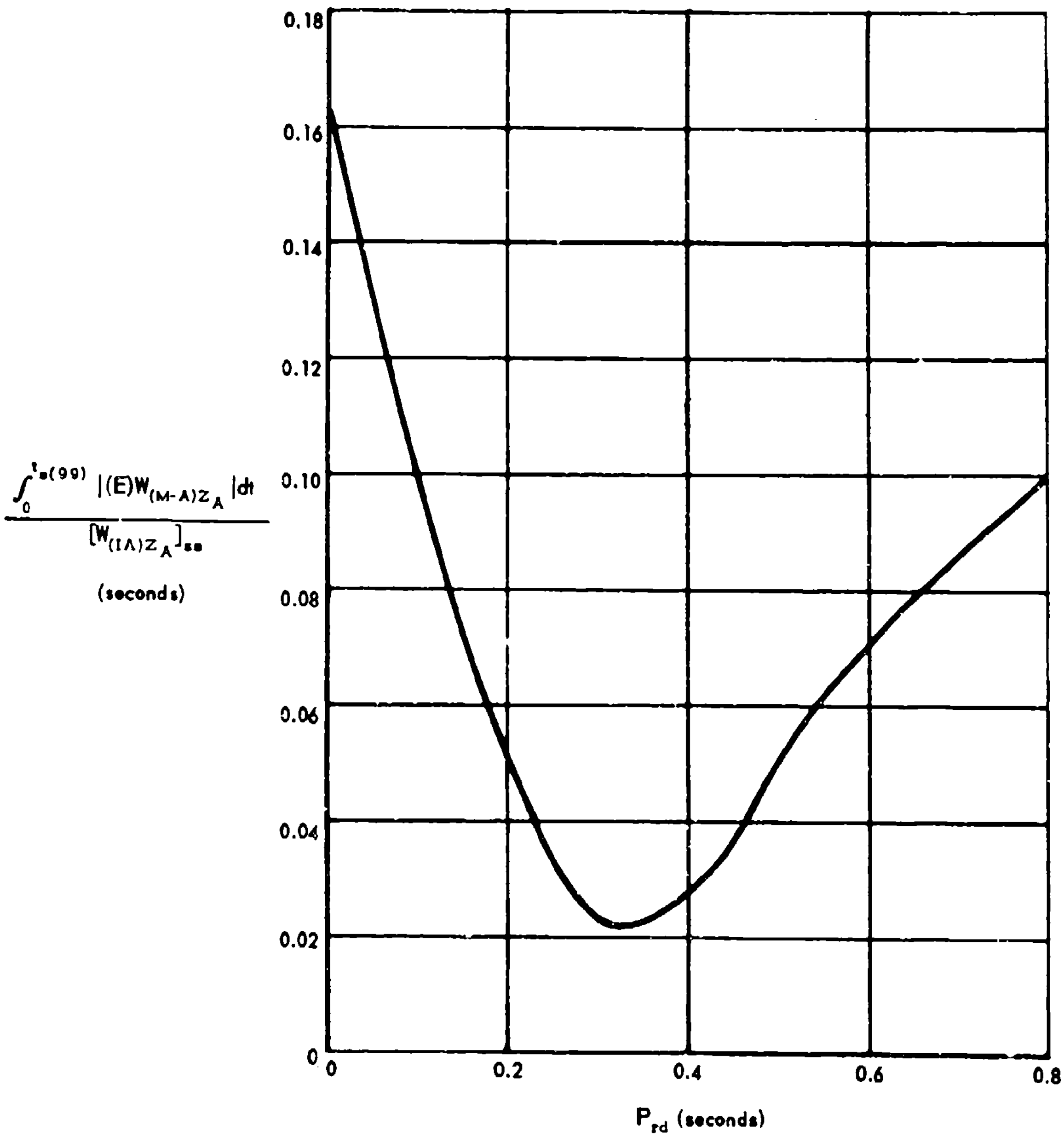


Fig 5 The variation in the normalized value of $(EQ)_{rd}$ as a function of P_{rd} for various types of system operation for the system of Fig. 2

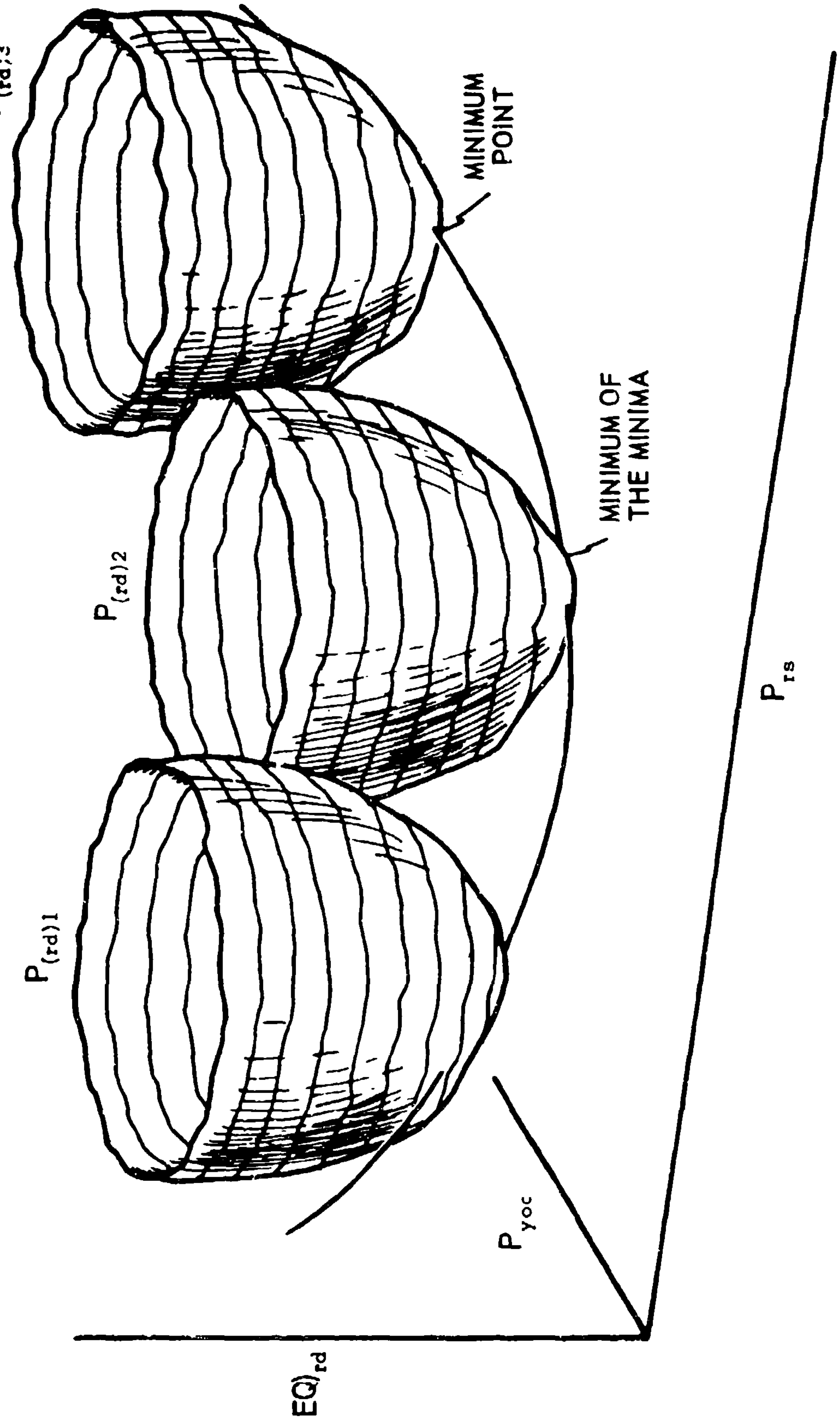


Fig 6 Geometrical representation of the optimalizing action of the yaw system.

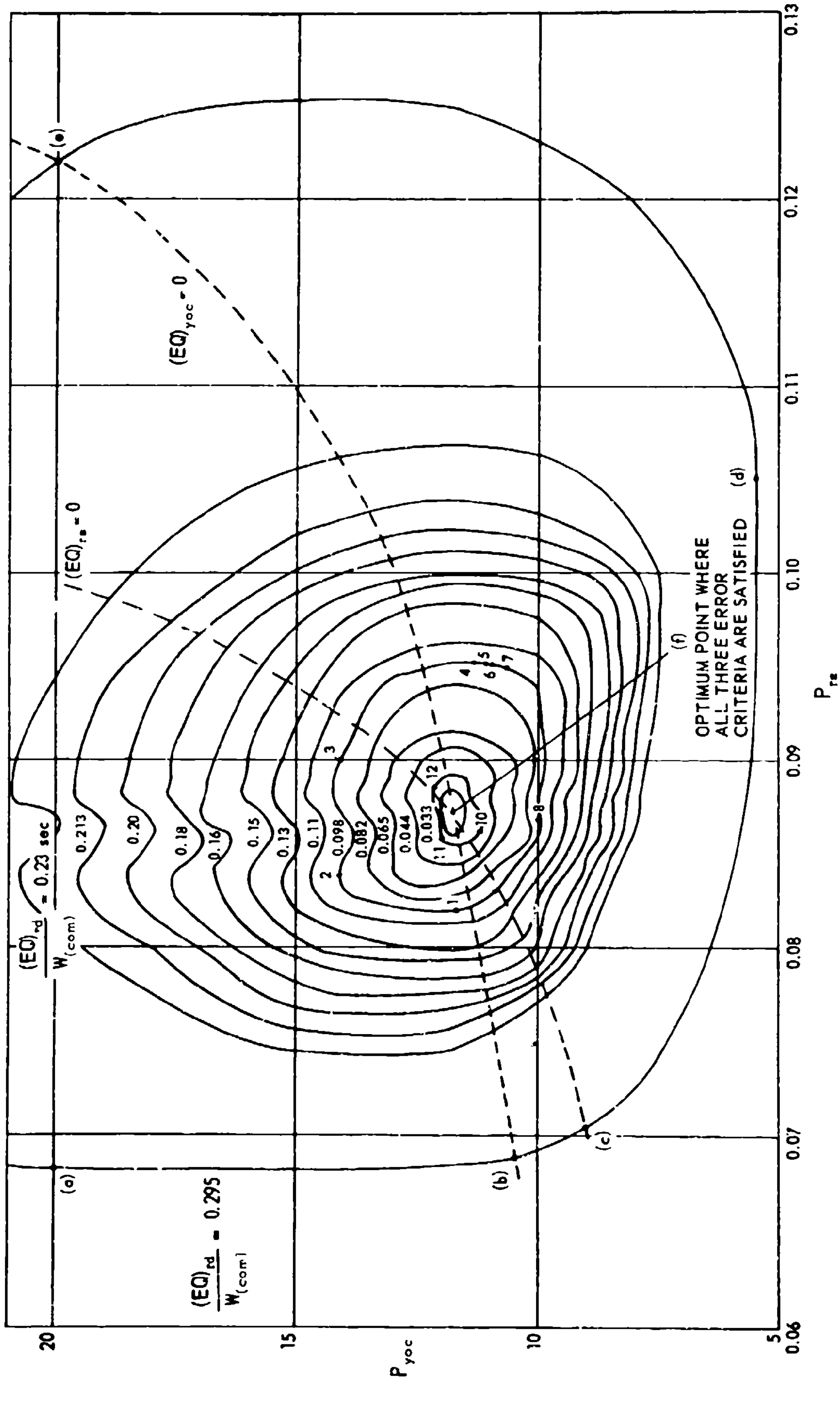


Fig 7 Contours of constant $(EQ)_{rd}/W_{(com)}$ for P_{yoc} as a function of P_{re} with $P_{rd} = 0.35$.

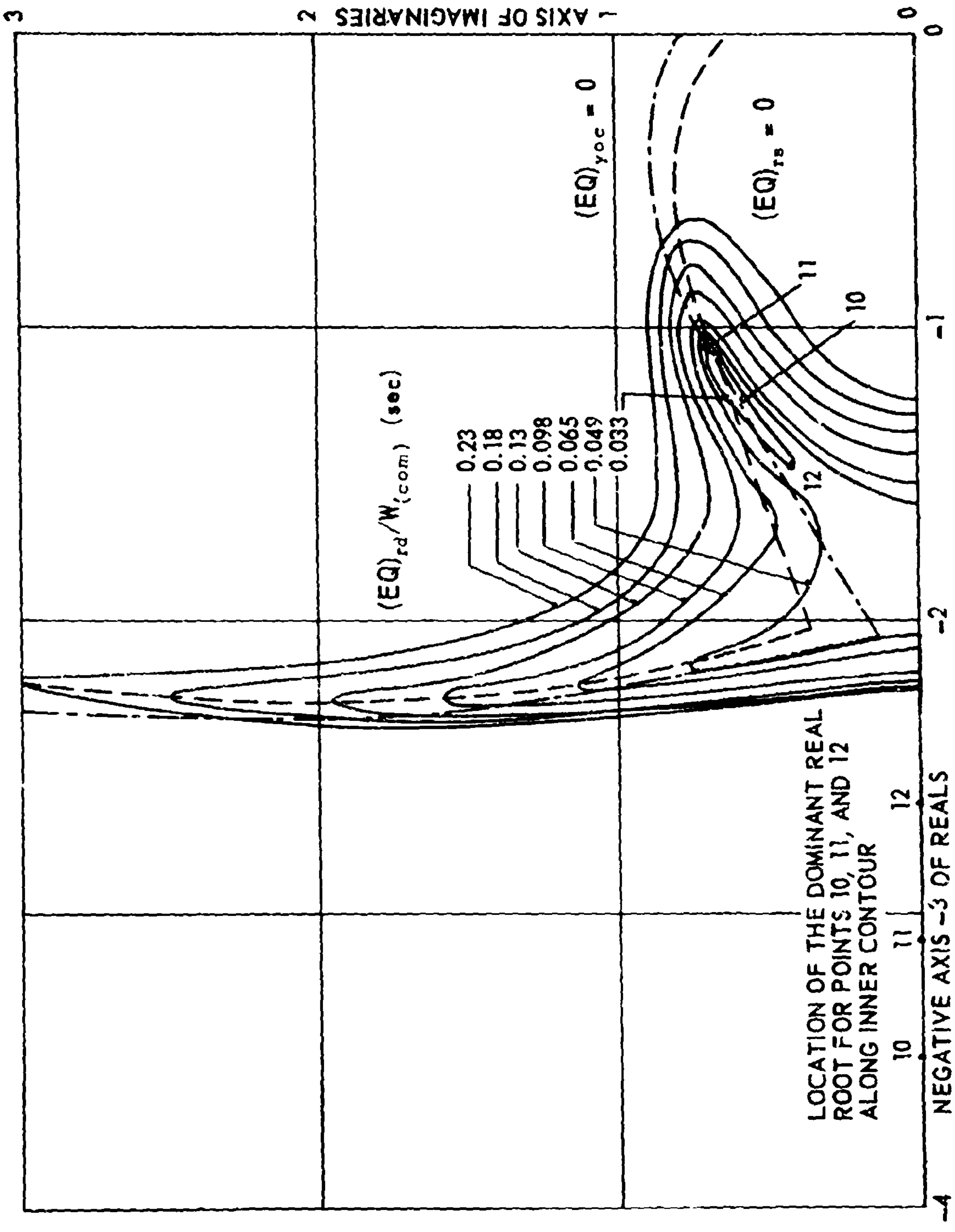


Fig 8 Complex plane location of the dominant second order poles of the system of Fig. 2 for the parameter values corresponding to the contour plot of Fig. 7

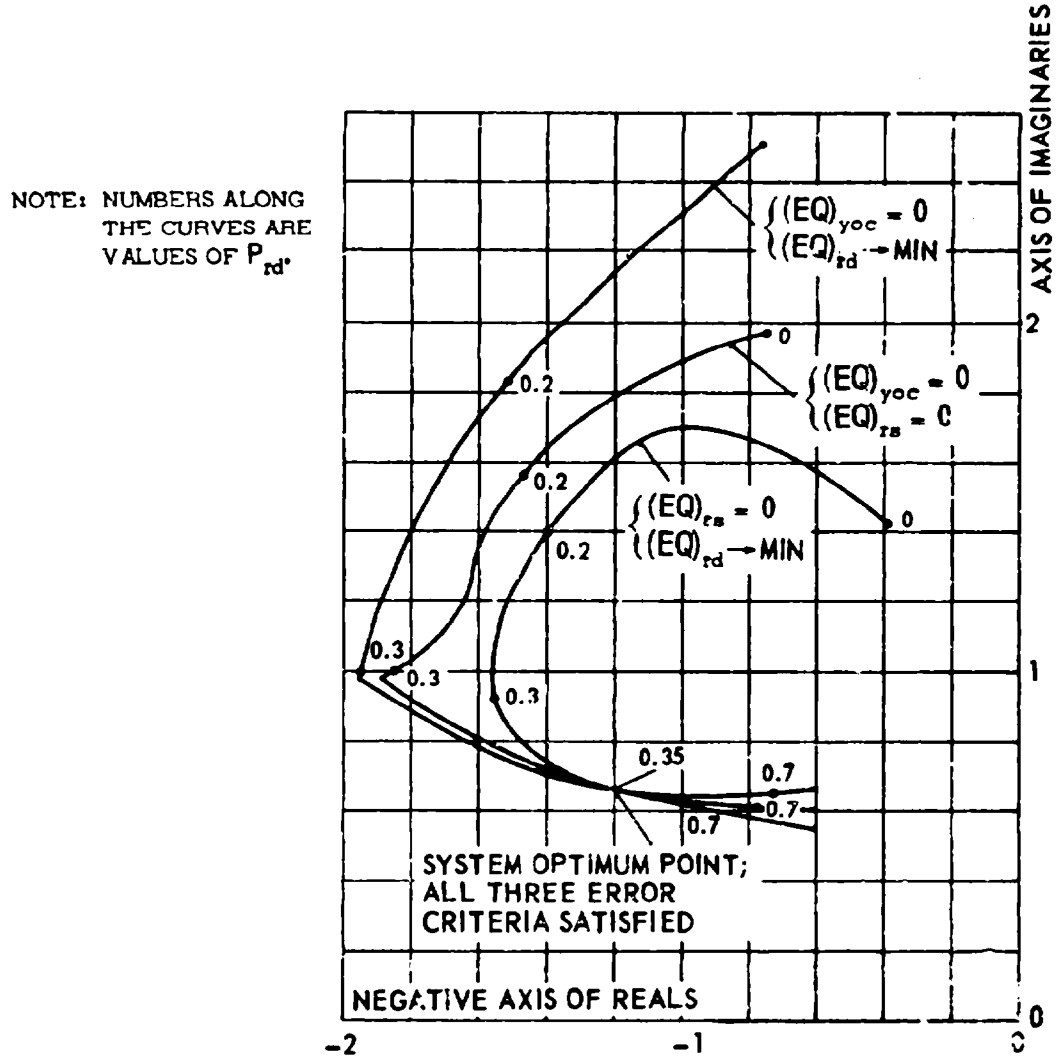


Fig 9 Complex plane loci of the poles of the dominant second order mode of the system of Fig. 2 for values of the loop parameters which satisfy the error criteria.

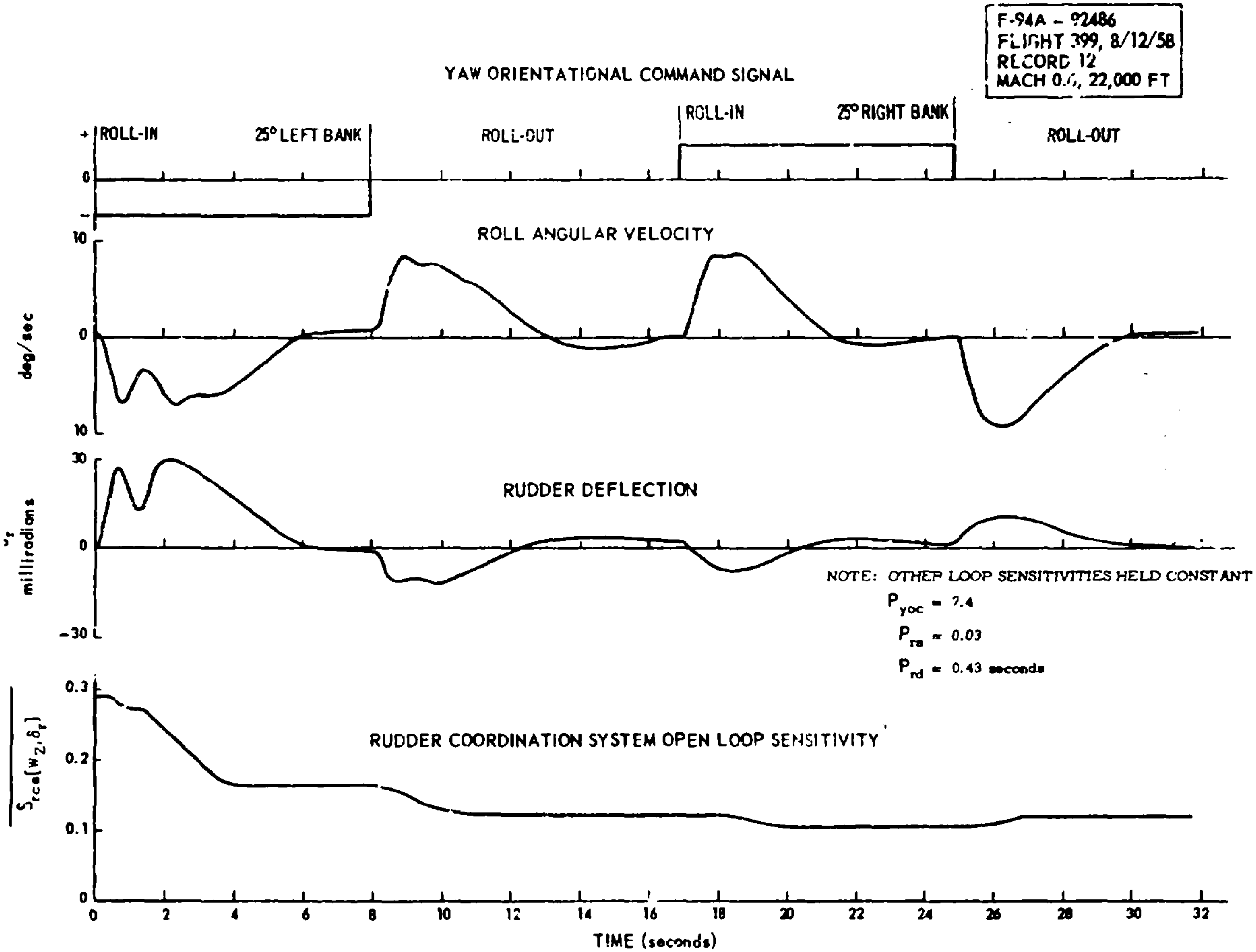


Fig. 10. Flight test results of the yaw optimalization system: effect of the operation of the automatic adjustment of the rudder coordination system loop sensitivity.

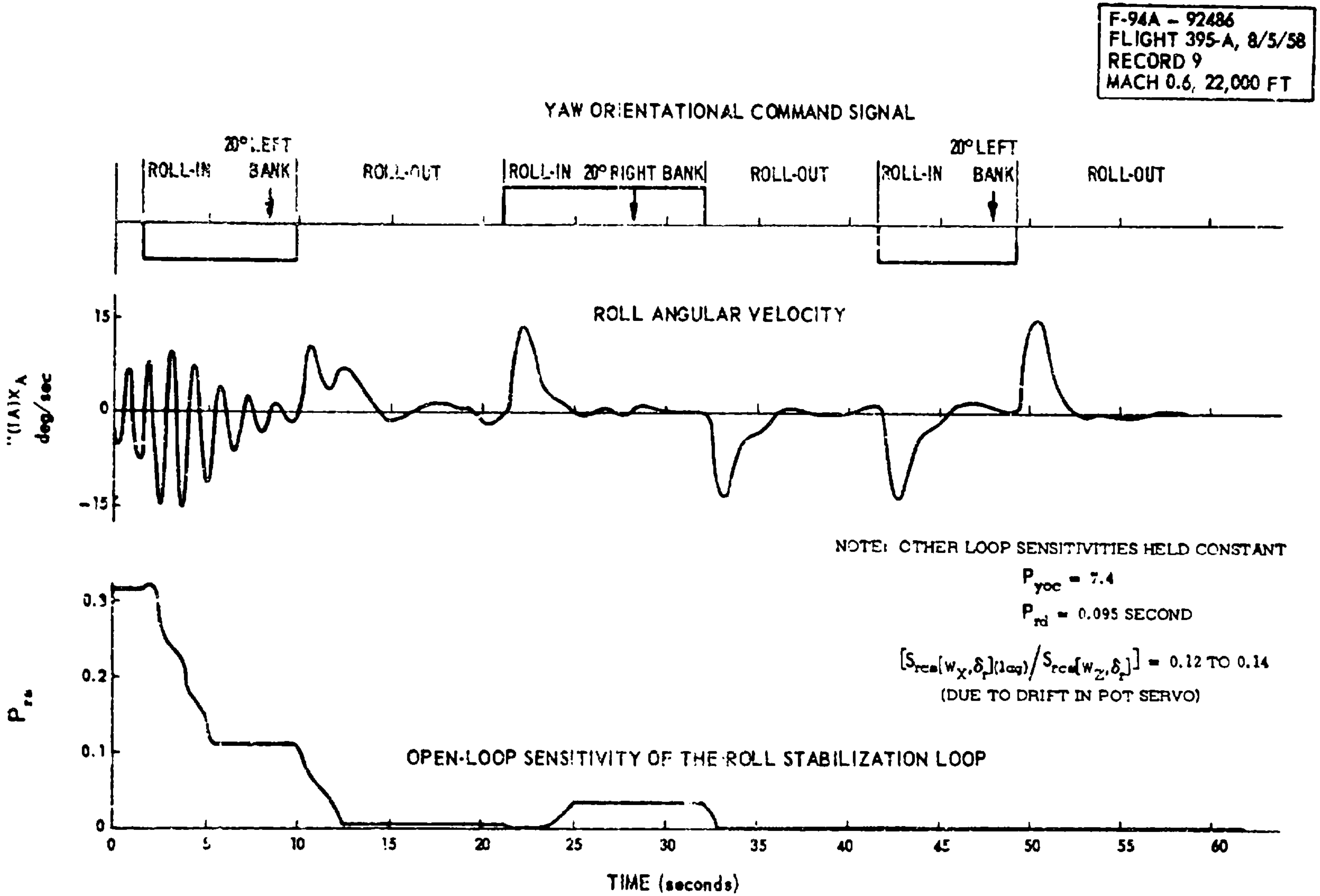


Fig 11 Flight test results of the yaw optimalization system: effect of the operation of the automatic adjustment of the roll stabilization loop sensitivity.

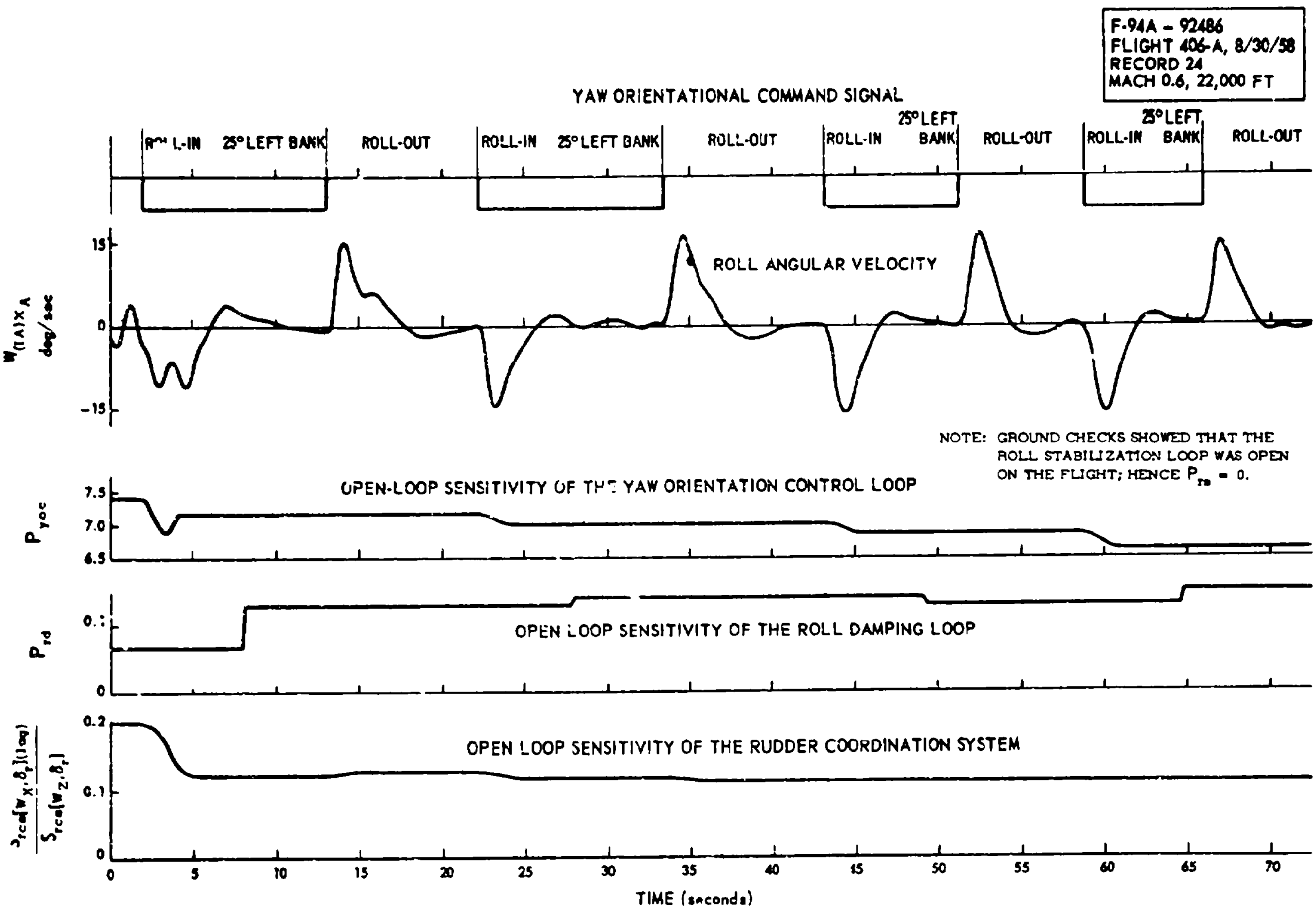


Fig 12 Flight test results of the yaw adaptive system: automatic adjustment of all loop sensitivities.

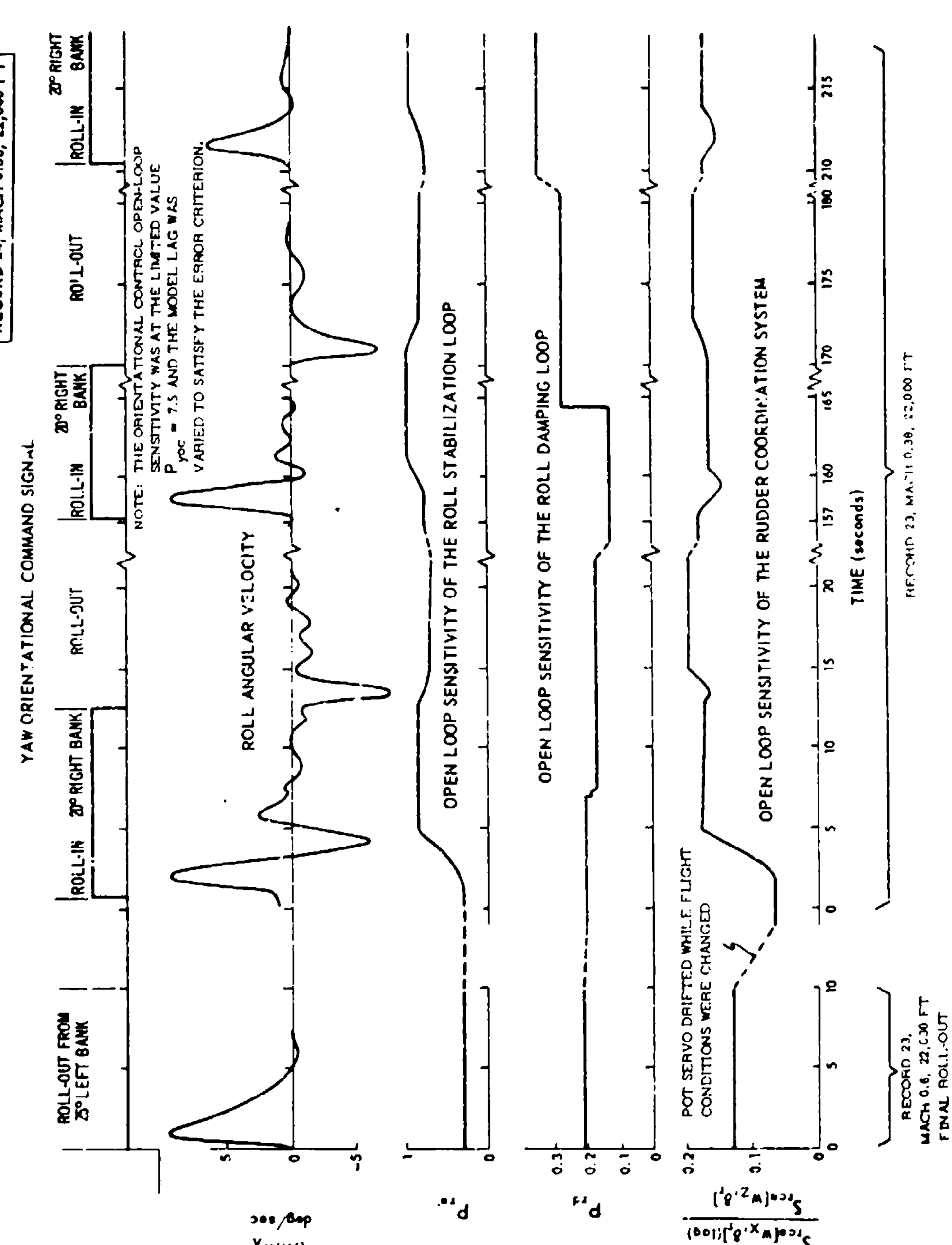


Fig 13 Flight test results of the yaw optimalization system: automatic adjustment of all loop sensitivities.

THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS SYMPOSIUM

SESSION III

Dr. C. S. Draper, Chairman
Massachusetts Institute of Technology

Dr. C. S. Draper
Head, Department of Aeronautics
Head, Instrumentation Laboratories
Massachusetts Institute of Technology

In order to add a little something either as confusion or clarification to this symposium, I will take a few minutes, while there are still people coming in, to try to illustrate what I think of adaptive controls and optimalizing controls, so far as definitions are concerned.

The question has been raised, why should you work with a thing that you call an adaptive control, and what do we mean by optimalizing control and is there any difference? I can give a couple of examples that are merely illustrative but perhaps will illustrate one or two of the concepts involved. Now the idea that became an optimalizing control was really an adaptive control in the beginning. This was a control that changed the parameters on a system operating under conditions that could not be predicted. The simple situation was that of an anti-aircraft system. Hitting the target would always be easy if the pilot was cooperative and continued to fly a smooth course. Then there was no need of changing the parameters of the system. However, if he did not cooperate, and I am sure he never intended to cooperate, but started on a group of basic maneuvers in which the rate of change of range and angular velocity both began to show considerable change, the adaptive system was then made to shift the parameters of the fire control system to make it faster in order to keep up with this unpredictable performance that the target was putting on. You couldn't hope to hit him but you could hope to stay up with him in such a fashion that if you smeared around your shots where he was, he would be in considerable danger if he made any mistakes. This was a method of adapting a control from the feedback not from the output. The rate of change of the inputs were used to change the condition rather than the parameters of the system; the optimalizing of the system on the other hand definitely was intended to utilize some type of an operating arrangement that did have an inflexion point in the performance curve. For example, the case of fuel mixture ratio. In that instance the system could have been predicted and you could have put in a programming control. If you didn't know what the program was, then changing the mixture ratio and seeing what happened, and comparing that with what had happened before you made the change, would allow you to optimalize the system. In this case you could have done it with programming, but in order to do it with programming you would have to determine the characteristics of the system. The whole philosophy was merely that of using the system itself as a measuring device for telling you what the performance was under a given condition and determining how that performance changed when you changed one of its parameters. These are a few ideas that are a little different. I think they will add a little more confusion perhaps to an already confused situation.

Adaptive Flight Control System

Mr. S. S. Osder, Sperry Gyroscope Company

INTRODUCTION

This paper presents a summary of research and development accomplished on a program to evaluate the feasibility of an adaptive automatic flight control system under Contract AF 33(616)-5075. The work was performed from May 1957 to May 1958, for the United States Air Force, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio by the Aeronautical Equipment Division of the Sperry Gyroscope Company, Division of Sperry Rand Corporation, Great Neck, New York. The system investigated employs so-called optimum response models in the control system forward loop and uses linear and non-linear techniques to force the controlled vehicle to execute maneuvers with minimum error from the response dictated by the model. The stability problems associated with these high gain control techniques are alleviated by means of a Performance Computer which continuously detects stability boundaries and adjusts automatic flight control system gains to within a desired margin of these boundaries. The required adjustments are determined by extracting the essential information from the vehicle-autopilot closed loop impulse response. Periodic low amplitude excitation impulses below the human pilot's detectable threshold are applied to the system with the Performance Computer establishing the required adjustments on the basis of monitored response. Adaptive stabilization and maneuvering configurations for a variety of vehicles were obtained in analog computer studies which included actual physical equipment mock-ups. The Performance Computer was also used to provide automatic cross-over between reaction jet and aerodynamic controls during exit and re-entry maneuvers. A complete description of the Sperry adaptive flight control system program for the Air Force is reported in the following documents prepared for WADC on Contract AF 33(616)-5075.

1. First Interim Technical Report - Feasibility Study-Automatic Optimizing Stabilization System, S. S. Osder; Sperry Report No. 3265-3683, September 1957.

2. Second Interim Technical Report - Feasibility Study-Automatic Optimizing Stabilization System, S. S. Osder, I. N. Hutchinson; Sperry Report No. 3265-3702, November 1957.

3. Third Interim Technical Report - Feasibility Study-Automatic Optimizing Stabilization System, S. S. Osder, I. N. Hutchinson; Sperry Report No. 3265-3733, February 1958 (Confidential)

4. Final Technical Report - Feasibility Study-Automatic Optimizing

Stabilization System, S. S. Osder, I. N. Hutchinson; Part I and II Contract No. AF 33(616)-5075, Sperry Report No. 3265-3746, WADC TR 58-243, ASTIA Document No. AD 155576, June 1958.

DISCUSSION

The system investigated employs three basic adaptive techniques aimed at providing optimum performance throughout the complete range of rapidly varying flight conditions. These techniques may be classified as:

1. Passive adaptation by virtue of linear feedbacks around an optimum response model.

2. Alteration of controller structure as a function of measured performance by means of non-linear modification of system input signals.

3. Measurement of the critical part of the closed loop impulse response and adjustment of controller parameters as a function of this measurement.

Before describing the manner in which these techniques are employed, it would be valuable to elaborate somewhat on the philosophy behind the classifications.

a. Passive Adaptation by Linear Feedback - Consider first the passive adaptation which is an inherent property of every system with negative feedback. This is easily demonstrated by the control system block diagram illustrated by figure 1. The dynamic element being controlled, H_A, may be an airplane transfer function which relates the aircraft's attitude with the input moments represented by X_d and X_c. The moment X_d is an external disturbance such as may be caused by a gust induced variation in lift. If the control loop represented by the controller H_1 is not closed, the aircraft's response to the disturbance will be given by

$$\left[\frac{\Theta_o}{X_d}\right]_{\substack{\text{Open}\\ \text{Loop}}} = H_A \qquad (1)$$

where H_A is defined by the combination of aerodynamic and inertial forces acting on the aircraft.

Now, if we close the loop containing the controller H_1, the output response to the same disturbance, X_d becomes

$$\left[\frac{\Theta_o}{X_d}\right]_{\substack{\text{Closed}\\ \text{Loop}}} = \frac{H_A}{1 + H_1 H_A} \tag{2}$$

If $H_1 H_A > 1.0$

$$\left[\frac{\Theta_o}{X_d}\right]_{\substack{\text{Closed}\\ \text{Loop}}} \approx \frac{1}{H_1} \tag{3}$$

which is the result to be expected in any application of negative feedback. But, the implications of equation (3) are very intriguing when adaptive performance is being sought. This equation states that the controlled aircraft's response to the disturbance will be independent of the aircraft. Thus the aircraft can fly through any altitude and Mach number regime, or even transform itself from a heavy low speed transport to a hypersonic vehicle, but the response will remain constant, determined only by the controller transfer function. This is obviously as adaptive as a control system can get, and no complex array of computing equipment is required. However, as practical designers of automatic flight control systems will readily testify, so remarkable a controller as the H_1 of equation (3) cannot in general be achieved with physically realizable equipment. Practical limitations are imposed by the finite attainable servo bandwidths and the effects of control system non-linearities. Nevertheless, in many instances, it is possible to attain a fairly good approximation of the desired H_1. This compromise value of H_1 can be employed to greatest advantage when the autopilot design incorporates an optimum response model and the so-called conditional feedback technique. In effect, this technique permits us to use information contained in the input signal to maximum advantage in minimizing dynamic errors from the desired response.

b. Optimum Response Model - The optimum response model technique can be used to make the system's response to commands independent of the system's response to disturbances. Figure 2 is a block diagram which illustrates a flight control loop with conditional feedback around an optimum response model. The essential difference between figures 1 and 2 is the utilization of a model which will shape the command to represent desired performance. This is compared to actual performance, resulting in generation of an error, ϵ which is further operated on by the conditional feedback block.

The arrangement of a block diagram to show a conditional feedback loop based on a performance model is an illustration of a manipulation of system block diagrams, aimed at showing adaptation capability of virtue of linear feedback. It is emphasized that the use of the model concept

does not by itself represent any fundamental advance in servomechanism art. It is, in reality, no more than a rearrangement of the conventional servomechanism block diagram. To illustrate, the diagram of figure 2 can be easily redrawn into its single loop equivalent as indicated in figure 3. All that has happened is that a pre-filter $[1 + g(s)]$ has been inserted into the command path.

Representing the system block diagram to incorporate an optimum response model emphasizes a very important design concept which points the way toward the attainment of an adaptive capability. This concept involves a design which permits the separation of command and disturbance response. Such a separation permits the over damping of the aircraft autopilot closed loop disturbance response without compromising the response to commands. This principle can be demonstrated by the following simplified illustration. Consider first the representation of the control block diagram incorporating a model as shown on figure 2. The conditional feedback in this figure is the error signal which represents the difference between the actual and the desired response to the command, Θ_i. This block diagram can of course be redrawn to show the model as a pre-filter on the command as illustrated in figure 3.

Equation 3 described the response to a disturbance, X_d, showing that it approaches dependence upon only the controller transfer function, H_1. However, from figure 3 it is seen that the response to the command Θ_i is defined by

$$\frac{\Theta_o}{\Theta_i} = \frac{g'(s) \quad H_1(s) \quad H_A(s)}{1 + H_1(s) \quad H_A(s)} \tag{4}$$

Again if $H_1(s) \; H_A(s) > 1$

$$\frac{\Theta_o}{\Theta_i} \approx g'(s) \tag{5}$$

which states that the response to the command Θ_i is defined only by the pre-filter (or model) transfer function. The first step, therefore, toward designing an automatic flight control system which displays an inherent adaptive capability is to define the optimum response model. This turns out to be a rather simple task since the optimum response model for an attitude command is a first order lag while the optimum response model for an attitude rate command such as a pilot's stick force input is an integrator preceded by a first order filter.

c. Non-Linear Modification of Controller Structure - The next problem which must be overcome in our quest for adaptive performance relates to our ability to maintqin H_1 (s) H_A (s) greater than 1.0 so that equation (5) can be obtained from equation (4). Studies of aircraft dynamic variations encountered throughout a typical flight regime indicate that in some instances a single set of autopilot gains can provide nearly constant airplane-autopilot dynamics over a complete subsonic and supersonic range of flight conditions. However, in most cases a single set of gains represents too severe a compromise in performance so that the objective of keeping H_1 (s) H_A (s) greater than 1.0 for most of the flight regime cannot be met. The critical factor in the aircraft dynamic change which necessitates a reduced autopilot aircraft combination gain is the approach of the aircraft natural frequency to the bandwidth of the control system (including sensors and actuators). The adaptive system described in this paper will automatically detect the requirement that autopilot gain must be reduced at these critical flight conditions. However, the "non-linear" modification of controller structure effectively compensates for the reduced linear system gains by making the system respond as though its gains were several times higher than the linear gain parameters would imply. Thus, the use of a non-linear controller permits the criterion of H_1 (s) H_A (s) $>$ 1.0 to be maintained even when linear stability considerations force the gain of H_1 (s) to be lowered below the minimum value required for a linear adaptive configuration.

d. Impulse Response Measurement - It remains, then, to discuss the method by which the autopilot gain is raised or lowered to reflect the maximum gain restrictions imposed by the controller bandwidth limitations at the various flight conditions. The technique employed is based on the extraction of required gain adjustment data from the aircraft -autopilot closed loop impulse response. Many methods for obtaining the impulse response of an unknown dynamic element have appeared in the recent servo-mechanisms literature. These methods, however, involve fairly complex equipment since they employ such techniques as sampling and storage of a transient's past history and correlation function computations. Complexity of this nature would be unavoidable if we attempted in-flight measurement of a completely unknown dynamic element. This, however, is not true in the case of an airplane transfer function. Since we know the basic form of the aircraft's equation of motion, we also know the general form of the transfer function H_A (s) discussed previously. That is, we can always approach the problem with some qualitative knowledge of the poles and zeros of H_A (s) even when we possess no information relating to the specific coefficients of the aircraft's equations of motion. From this vantage point of knowledge rather than ignorance of the controlled element, we can extract the required specific details without resorting to complex computations. In the following disucssion it will be shown that a relatively simple measurement can provide the necessary inputs to the gain adjusting circuitry. Moreover, it will be shown that this

measurement always remains independent of the automatic flight control loop in that the required equipment may be inserted or removed without disturbing the basic autopilot. Thus, the gain adjusting system which is referred to hereafter as the Performance Computer need be incorporated only in those applications where the first two adaptive techniques cannot adequately cope with the variations in vehicle dynamics.

c. Performance Computer - Before describing the operation of the Performance Computer, it is important to review the nature of the stability problem which this computer must control. This can be determined with the help of figure 4 which is applicable to the pitch control system. The system stability can be determined from the open loop transfer function $H_1 H_S H$. Consider H_1 to be a simple rate plus displacement autopilot given by the following transfer function.

$$H_1(s)\, H_S(s) = K \left[1 + K_R\, s\right]\; H_S(s)$$

where K is the static gain in degrees of surface per degree of attitude error, K_R is the ratio of rate to displacement gain and H_S represents the dynamics of the servo actuator transfer function. $H_A(s)$, the rigid airplane transfer function may be described for the case of pitch control as

$$H_A(s) = \frac{\Theta}{-\delta_E} = K_A \frac{\left(\frac{s}{\omega_1} + 1\right)\left(\frac{s}{\omega_2} + 1\right)}{\left(\frac{s^2}{\omega_A^2} + \frac{2\zeta_A s}{\omega_A} + 1\right)\left(\frac{s^2}{\omega_B^2} + \frac{2\zeta_B s}{\omega_B} + 1\right)} \qquad (6)$$

where K_A is the airplane gain, a quantity dependent upon a large number of aerodynamic factors, but in the short period control frequencies it is primarily determined by surface effectiveness, longitudinal moment of inertia and static stability. It is important to note that K_A has the identical effect as autopilot gain K on the open loop transfer function $H_1 H_S H_A$. In the ideal case, the variation of K_A with flight condition is completely defined so that an open loop gain schedule can be used to change the autopilot gain and thereby permit the combination KK_A to produce the desired closed loop characteristics. The problem of the adaptive autopilot involves situations in which we cannot predict K_A.

The poles of equation (6) are defined by ω_A and ζ_A the phugoid natural frequency and damping ratio, and ω_B and ζ_B the airplane short period natural frequency and damping ratio. The zeros ω_1 and ω_2 are associated with two of the airplane response time constants. The flight path angle initial response time constant to a pitch angle change is defined by ω_1 and ω_2 defines the time constant of the flight path angle as it decays to

its steady state value after the initial response. Figure 5 shows the complete root locus of figure 4 with H_S neglected. Note that there is no stability problem indicated by figure 5 since the open loop poles traverse no region of poor stability as they move toward the open loop zeros. This root locus is another way of illustrating how the autopilot can completely dominate the dynamics of the closed loop response since it demonstrates how the autopilot zero at $-\frac{1}{K_R}$ becomes the dominant pole of the closed loop system as the autopilot gain approaches infinity. Actually, a realistic gain of K = 2 to 3 is often sufficient to move the airplane short period pole 98% of the distance on the S plane to the terminating zero. Now let us consider the effect on this root locus when we include the servo dynamics, H_S. Consider first, second and third order servo dynamics defined by:

1. 1st order servo $H_S = \dfrac{1}{\frac{s}{\omega_{s_1}} + }$

2. 2nd order servo $H_S = \dfrac{1}{\frac{s^2}{\omega_{s_2}^2} + \frac{2\zeta_s s}{\omega_{s_2}} + 1}$

3. 3rd order servo

$$H_S = \frac{1}{\left(\frac{s}{\omega_{s_1}} + 1\right)\left(\frac{s^2}{\omega_{s_2}^2} + \frac{2\zeta_s s}{\omega_{s_2}} + 1\right)}$$

The form of the root loci when these three servos are included is shown in figures 6, 7, and 8. In all three cases, a new high frequency problem area is introduced, for in the previous ideal case represented by figure 5, an increase in gain always moved roots toward a higher damping region. The gain K at which these new servo loci cross the imaginary axis corresponds to the gain margin of the stabilization system. Figure 9 shows a family of curves for the loci of second order servo roots with different servo system natural frequencies. These curves demonstrate the well known technique of increasing the system gain margin by expanding the servo bandwidth, for the value of gain required to cross the imaginary axis increases with the servo natural frequency.

The important point being made in figures 6, 7 and 8 is that the only type of instability which can appear in an overdamped pitch stabilization system is a high frequency instability related to the control system bandwidth. It is noted that the dynamic lags identified with the actuator transfer function H_S could just as well be considered part of any other dynamic lag introduced in the open loop $H_1 H_S H_A$. This would apply to the gyroscopic sensors or the autopilot computing networks. The primary concept involved in the stabilization system under study in this program is the automatic detection and restraint to within stable boundaries of these " high frequency" roots. A monitoring system which can hold these roots within an acceptable margin from the right half plane will automatically set the system gain at the maximum attainable value.

It is important to emphasize that the " high frequency" roots are the

only limitations to system gain margin. Gain margin is defined as that value of K which will cause a cross-over of the " high frequency" roots into the right half plane. This reasoning is applicable to types of pitch stabilization configurations other than those illustrated by the root loci of figures 6, 7, and 8. For example, it applies to systems such as those which employ second derivative pitch data and integrating servos as in figure 10. Moreover, a similar analysis can demonstrate that this same principle is applicable to aircraft roll and yaw attitude control systems. It is noted, however, that the attitude control systems must employ displacement references and must command aircraft moments in those axes about which the attitude displacement errors appear if we wish to use the high frequency instability as a performance criterion. Thus, roll attitude errors must be corrected by commanding yawing induced rolling moments and heading errors corrected by commanding yawing moments. In control configurations which act as dampers only (rate autopilots) the high frequency instability discussed above will appear but other types of oscillations may develop before the high frequency roots pose a stability problem. For example, a yaw damper has an optimum gain, and any increase in gain above this value can lead to a decrease in lateral stability at periods several times greater than the " stick fixed" dutch roll period.

Figure 11 is a block diagram illustrating the technique employed for bounding the critical high frequency roots within a range of acceptable damping ratios. The control system is excited by a combination of the attitude control signal and the small amplitude narrow pulses inserted at the servo actuator input. Because of the orientation of the poles and zeros of the aircraft-attitude control system combination, only the oscillatory mode associated with the high frequency servo roots will be excited by a small amplitude impulse. It is noted that if we wished to measure the complete impulse response of the closed loop airplane-autopilot combination, we could measure the airplane response rather than the actuator response. However, if we wished to detect the influence of all the poles and zeros inherent in the equation of the airplane-autopilot closed loop transfer function, we would have to use an excitation impulse many times larger than the one required to excite only the critical high frequency pole pair.

The system illustrated in figure 11 measures the response δ_E to a pulse δ and shapes this response to a series of fixed amplitude pulses. The number of these output pulses is a measure of the number of reversals of δ_E in response to the excitation impulse and therefore is proportional to the damping ratio of the " high frequency" roots. The logic circuitry establishes the criteria for autopilot gain changes as a function of an excess in output pulses during a given sampling period. The sampling period and the excitation pulse shape and amplitude are established in the excitation and sequence control. The logic circuits establish the boundaries of acceptable damping ratios of the critical " high frequency" roots. The combination of the Sequence Control, logic circuitry, pulse shaper and counter are referred

to as the Performance Computer.

The performance computer's function is to automatically seek the maximum attainable gain within the specified stability limits. By operating the autopilot at its maximum permissible level we always operate with a stabilization configuration which gives the closest approximation to the attainment of complete adaptation as defined by equations 3 and 5. The performance computer generates the periodic pulses which are used to probe the aircraft's environment. In general, the rate of pulsing can be made a crude function of the rate of change of the aerodynamic environment. The pulse rate was slowed down when conditions were static, and automatically accelerated when the aerodynamic static and dynamic pressures changed. In the case of missiles or reentry research vehicles this sophistication need not be used. Actually, no additional circuitry is required in the excitation and sequence control for this variable pulse rate function since the basic timing flip-flop is designed to increase frequency as a function of a voltage (ac or dc) which represents the rate of change of the flight environment.

Figure 12, then, is a comprehensive block diagram of the adaptive configuration which was evaluated in extensive simulator studies which utilized actual hardware such as servo amplifiers, hydraulic actuators and a breadboard Performance Computer.

f. Performance Computer Implementation - The circuitry required to implement the performance computer function was relatively simple and noncritical. In the breadboard system build on Air Force Contract AF 33(616)-5075, the logic circuits employed electro-mechanical relays to control the speed and direction of the gain adjusting motor. These relays represented a complexity and weight penalty which can easily be eliminated with semiconductor switching devices. The relays were used in the breadboard system because they provided a valuable flexibility which permitted changes to the logic system with a minimum of circuit modifications.

A logic diagram of the performance computer is shown on figure 13. The system operates by inserting impulses into the automatic pilot at controlled intervals and monitoring the control surface response. Any oscillatory motions of the actuator output will be converted into a train of pulses and counted in a decade counter. In the breadboard version, a magnetron beam switching tube was used as the counter. Part of a conventional decade counter constructed of three flip-flops can provide improved reliability for this function and thus may be used in place of the beam switching tube. It is noted that the simplest type of binary element is required for this counting function so that the more elaborate circuitry associated with high speed, transistorized flip-flops need not be used.

The number of counts detected at the decade counter will depend upon the number of surface reversals which exceed the threshold sensitivity of the

Schmitt Trigger and therefore, will be a measure of the critical root damping ratios. The beam switching tube counter shown on figure 13 was used to operate relays which dictate the direction and speed of the automatic pilot gain control motor. The circuitry was arranged to give a slow decrease in gain for a count of three, a medium speed decrease in gain for a count of four and a fast decrease for a count of five. A slow increase in gain occurs the second time a count of less than two is obtained. This logic is included in order to prevent a gain increase when a higher count is expected. When a count of two is obtained, the performance computer's criterion for correct autopilot gain is satisfied and no change occurs. It is again noted that the relays could be eliminated by employing a direct read-out from the counter circuit to actuate transistor gates which would vary the speed and direction of the gain control motor.

Figure 14 is a block diagram of the sequence and excitation control portion of the logic diagram presented in figure 13. The period of the astable multivibrator is controllable in the range 3 to 30 seconds by a low power 400 cycle control voltage. Each cycle of this astable binary circuit causes one 3 second pulse to be generated in the monostable binary. A test impulse is formed and sent to the automatic pilot at the beginning of this three second pulse and the counter and motor gates are opened to permit detection and correction of non-optimum gain. The counter and motor gates are closed at the end of the three second pulse to prevent gain changes due to random disturbances, and the counter is reset to zero to prepare for the next test period. This sequence control arrangement could be improved by the inclusion of an additional monostable binary to close the counter gate sooner than the motor gate. Since the critical frequency will generally be higher than three cycles per second, the critical root damping ratio can be determined in a half second and this modification would minimize the possibility of false high counts due to extraneous noise.

The excitation and sequence control circuits could be common for all aircraft axes. The counter and gain control logic circuit must be repeated for the pitch and roll axes. The entire performance computer function for pitch and lateral control can be packaged in a volume of 40 to 50 cubic inches and is presently envisioned as a plug-in module of a complete AFCS.

g. <u>Typical System Performance in Supersonic Aircraft</u> - Before showing results which demonstrate system performance obtained with the above configuration, some mention should be made of techniques other than the excitation impulse as a means of deriving the required information about the closed loop impulse response. At the start of the program, it was suspected that atmospheric turbulence might provide an adequate excitation to the system without requiring the addition of an impulse. This did not prove to be feasible because the random nature of this type of disturbance could not provide consistent signals to the Performance Computer. It was found that when the gust disturbances did not cause an adequate surface response for a few

sampling intervals, the stability boundaries were temporarily exceeded and divergent high frequency oscillations began to develop. These oscillations were eventually bounded and damped, but unlike the performance obtained with the periodic impulse, the instability actually reached the point of divergence. Figure 15 is a computer record which illustrates the erratic performance obtained with excitation derived from atmospheric turbulence only.

A typical demonstration of the system's capability is illustrated by the recording on figure 16. Here a supersonic airplane which experiences a severe change in dynamics is decelerated and accelerated at constant altitude so that the range from Mach=1.6 to Mach=0.4 is covered. The aircraft's pitch natural frequency varies from about 13 radians per second at the high speed condition to about 2 radians per second at Mach 0.4. In figure 16, the impulse period is seen to vary from 21 seconds to 3 seconds as the deceleration increases. During the deceleration, the Performance Computer tracks the maximum permissible gain through a range of 1.0 to 3.5 and maintains the critical root damping ratio within acceptable bounds. At the start of the acceleration phase of the maneuver, the critical root damping ratio falls temporarily below desirable levels due to non-optimum gain control motor speeds. This difficulty could easily be alleviated with the use of a gain control motor having improved speed characteristics. It is seen, however, that the performance computer recovers rapidly and develops the allowable maximum gain throughout the rest of the maneuver.

In the following recordings which illustrate maneuvering performance with an F-104A airplane, the characteristics of the non-linear control in the adaptive autopilot forward loop were dictated by specific parameters of the hydraulic actuator used. For example, this actuator's authority limits of ±5 degrees prevented an even more effective utilization of the non-linear gain controls since the actuator was effectively employed as a dual mode controller; that is proportional control for small errors and "bang-bang" control for large errors. For the authority limits available, no further improvement could be obtained by raising the gain of the non-linear channel, although in general, a considerable increase in gain above the value used could be attained before an amplitude sensitive limit cycle would occur. It is noted that several methods of non-linear error gain control were investigated before the configuration eventually used was selected. All combinations of variable gains and gain switching of the rate and displacement signal as a function of the absolute value of the error and error rate were investigated. Fortunately, the technique which gave the best results was also the simplest to implement. This technique varies the displacement gain only, and the gain increase is a simple function of the absolute value of the displacement error. Its effect is that of superimposing a "bang-bang" controller on a linear proportional control system. For the parameters used, an attitude error of greater than about 1.0 degree would command maximum surface deflection so that the aircraft is constrained to follow a commanded reference with extreme tightness even when stability consideration force a reduction in linear system gains. Figure 17 is a

plot of the surface deflection versus pitch error for the linear and non-linear system at Mach 0.6, 0.95 and 1.6. flight conditions of the F-104A. Note that the linear system gains at these different flight conditions are those which were automatically set by the Performance Computer. It is apparent from this figure that the non-linear system is extremely effective in minimizing the actual difference between the control action at the different flight conditions. Its effect is to minimize the possibility of ineffective control because of reduced autopilot gain at flight condition III. It is noted that in that particular case, the autopilot gain was reduced by the Performance Computer not because the aircraft surface effectiveness increased but primarily because the limited control system bandwidth could not cope as effectively with the increased aircraft natural frequency.

Figures 18, 19, 20, 21, and 22 are typical analog computer recordings illustrating pitch rate maneuver performance. It is noted that in these recordings, command error rates are not employed so that the effectiveness of the non-linear error gain controls can be seen more clearly. The aircraft is an F-104A at 25,000 feet and various Mach numbers. In each case, the input is a step pitch rate command. Figure 18 shows the response when the non-linear controls are not used. The command is equivalent to 2.0 incremental "g" steady state when airspeed changes are neglected. The second maneuver on this figure differs from the first in that g-limits of $\pm$ 1.0 g's are set by the g-limiter. It is noted that the g-limiter gains exceed those which would be feasible if linear signal techniques are employed and the response is sharper than can be obtained with passive cut-off methods. In this implementation, a signal proportional to the excess in g's is applied to the pitch control system only when the g-limit has been exceeded. Since extremely high gains on the g-error signal are employed, the immediate effect is a large correction in the direction to lower the pitch rate to that value corresponding to the limit g's.

It is seen from figure 18 that when the non-linear pitch error gains are not used, the instantaneous pitch error reaches as high as 3.0 degrees and there is a slight overshoot in the g-limit. On figure 19 the non-linear gain control is employed and the maximum instantaneous pitch error is limited to one degree. Also, the g-limiting is sharper. Note that the excitation impulse is always present although on figures 18 and 19 its effect cannot be seen in the pitch rate response.

Figure 20 shows the same maneuver as in figure 19 but with mild atmospheric turbulence present. It is seen that the effect of the turbulence would easily be enough to mask any manifestations of the excitation impulse as far as a human pilot would be concerned. Figures 21 and 22 are similar maneuvers at Mach 0.60 and 1.6 where the autopilot gain has been set by the performance computer to about twice and one half the value it set for the previous flight condition. In figures 21 and 22 the effects of turbulence are also illustrated. These recordings show that in mild turbulence any trace of the excitation impulse is completely masked by the gust disturbances. Also, note that the

performance computer is able to discriminate between the surface activity due to the turbulence and the activity resulting from the excitation impulse. Extensive tests on the effects of turbulence on the performance computer have shown that turbulence will cause the performance computer to lower the autopilot gain only about 1.0 to 2.0 DB. To an extent, this slightly lower gain may actually be a desirable characteristic.

h. <u>Application to Space Vehicle Exit and Re-Entry Control Problem</u> A logical extension of the maximum gain seeking automatic control system is the application to high speed vehicles which perform both inside and outside of the atmosphere. This can be appreciated if we recall that the true automatic pilot gain is a combination of the system gain and surface effectiveness. In order to maintain effective attitude control, the system gain must increase as the surface effectiveness decreases. As the vehicle departs from the atmosphere, the surface effectiveness decreases to zero and the required aerodynamic automatic pilot system gain approaches infinity Since the maximum gain seeking function of the Performance Computer will automatically try to compensate for the loss in surface effectiveness, automatic switching to reaction-jet attitude control can be obtained when the aerodynamic automatic pilot system gain reaches some specified high value. The following is a description of some of the results obtained when the Performance Computer was used to provide cross-over of aerodynamic and reaction-jet controls during exit and re-entry maneuvers of a hypothetical ballistic-glide vehicle.

The trajectory simulated on the analog computing equipment involved an initial drop at 38,000 feet at Mach 0.5, an acceleration for 80 seconds to Mach 6.0 with an eventual climb to 260,000 feet, and a ballistic descent with leveling and deceleration at about 90,000 feet. Figure 23 shows the uncontrolled vehicle response during this exit and re-entry maneuver. The vehicle was excited by a short moment pulse during the exit phase. The apparent divergence in the oscillatory amplitude as the frequency decreases is characteristic of a spring-mass oscillatory system in which the spring stiffness decreases with time.

Figure 24 shows the response to 2-degree step pitch commands during an exit maneuver with only the aerodynamic automatic pilot operating. Note the periodic impulses appearing at the control surface and the aerodynamic autopilot gain decreasing in response to the initial vehicle acceleration and then increasing as the surface effectiveness diminishes toward zero. Figure 24 shows that despite the Performance Computer's increase in the autopilot static gain to 10.0, an attitude command causes an uncontrolled tumbling of the aircraft as the air density approaches zero.

Figure 25 shows the response to 2-degree step pitch commands during an exit maneuver with only the jet reaction automatic pilot operating. This record shows that the jet reaction automatic pilot contributes little in the way of stabilization during the first 30 seconds of the maneuver, and

does not provide a very effective attitude control for another 20 seconds. Figure 26 shows the vehicle attitude stabilization response during exit and re-entry maneuvers with the combined system in operation. A reaction jet crossover switch was located on the output shaft of the Performance Computer's gain adjuster. This switch was set to cut-in reaction jet control when the automatic pilot static gain was driven to a value of 6.0. The recording on figure 26 shows that tight attitude control and good stability margins were thereby maintained at all times during the exit and re-entry. Figure 27 shows a re-entry which includes a rapid trim change equivalent to five degrees of control surface deflection in four seconds. The maximum disturbance in attitude is seen to be less than 0.25 degrees. The Performance Computer and its associated excitation impulses thus act to probe an unknown environment and establish the optimum control configuration on the basis of interaction between the aerodynamic forces and the control system response.

SUMMARY

In summary, we have described an approach to the development of an adaptive automatic flight control system which uses as a starting point the inherent adaptation obtainable from a high gain control system employing linear feedbacks around an optimum response model.

The linear system was augmented by a simple non-linear control of the error signal to give the effect of a " bang-bang" controller superimposed on a proportional control system. This non-linear augmentation expanded the inherent adaptive properties of the linear configuration. The limitations of the linear system were shown to be a function of the control system bandwidth with the upper bounds on the linear system gain dictated always by a pair of complex conjugate high frequency poles in the closed loop transfer function. A method of bounding these critical roots within a region of acceptable damping ratios by means of a fairly simple measurement of the system's response to a small excitation impulse was shown to be feasible. The feasibility of this technique was established on the basis of extensive analog computer studies in real time simulations of various control problems employing realistic physical equipment mock-ups. The actual hardware used included a multiple input hydraulic actuator with associated servo amplifier circuitry and a bread-board Performance Computer which measured the critical part of the system's closed loop impulse response. The required magnitude of the periodic excitation impulses was in general below the human pilot's detectable threshold and it was demonstrated that it should definitely be undetectable to a pilot in any of its manifestations if a slight amount of turbulence is present.

These techniques were applied with considerable success in the simulator studies to both supersonic aircraft displaying severe dynamic changes over a wide range of flight conditions and a rocket powered hypersonic vehicle in planetary atmosphere exit and re-entry maneuvers. In general, it was concluded that the straightforward linear system with the optimum response models

augmented by the simple non-linear control element could provide an attitude stabilization system displaying constant dynamics for some transonic and supersonic aircraft without requiring gain scheduling. However, in the case of vehicles which undergo extreme changes in dynamics, the Performance Computer and its associated excitation impulses could be used to automatically set the optimum automatic pilot gain throughout any range of flight conditions. An especially significant application of the Performance Computer's continuous probing of an unknown environment was demonstrated in its use as a means for providing the automatic crossover of aerodynamic surface and reaction jet controls during space vehicle exit and re-entry maneuvers.

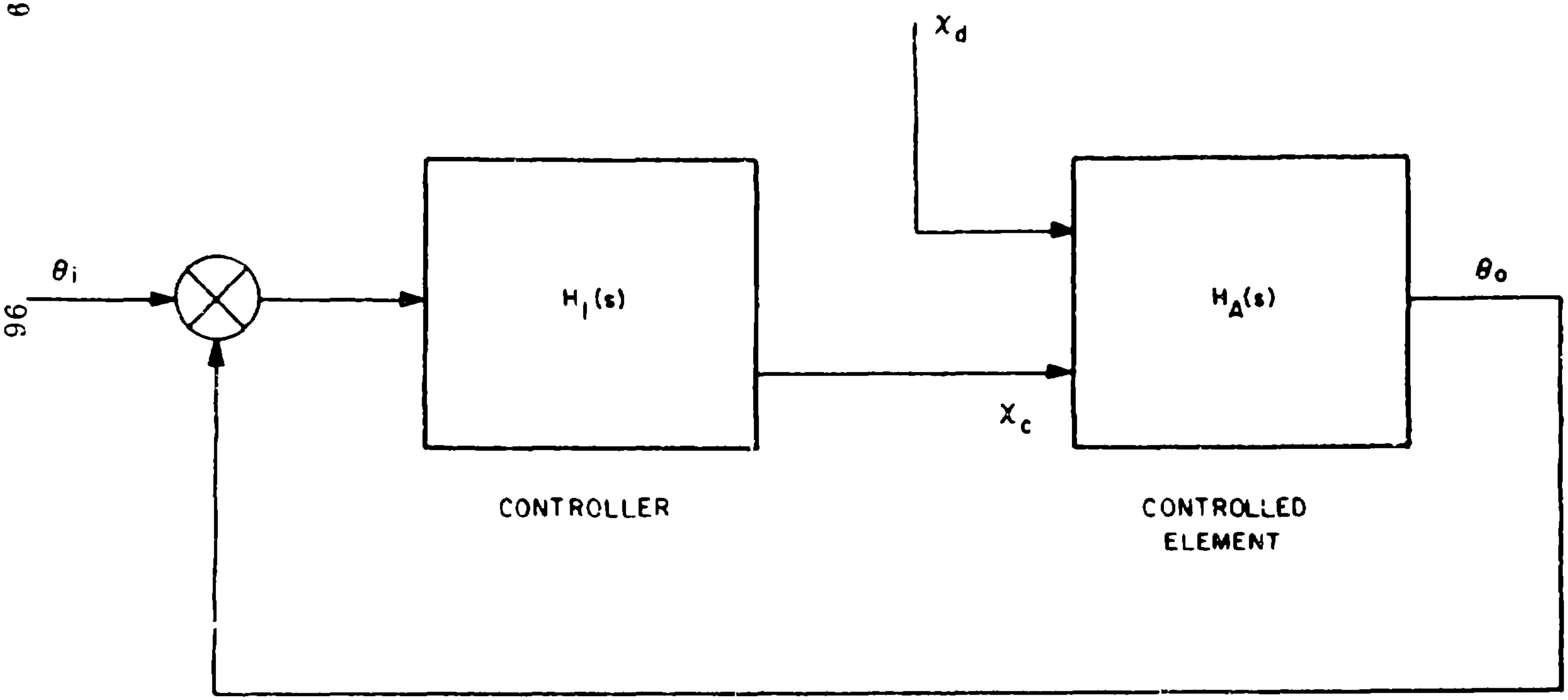


Fig 1

BASIC CONTROL BLOCK DIAGRAM

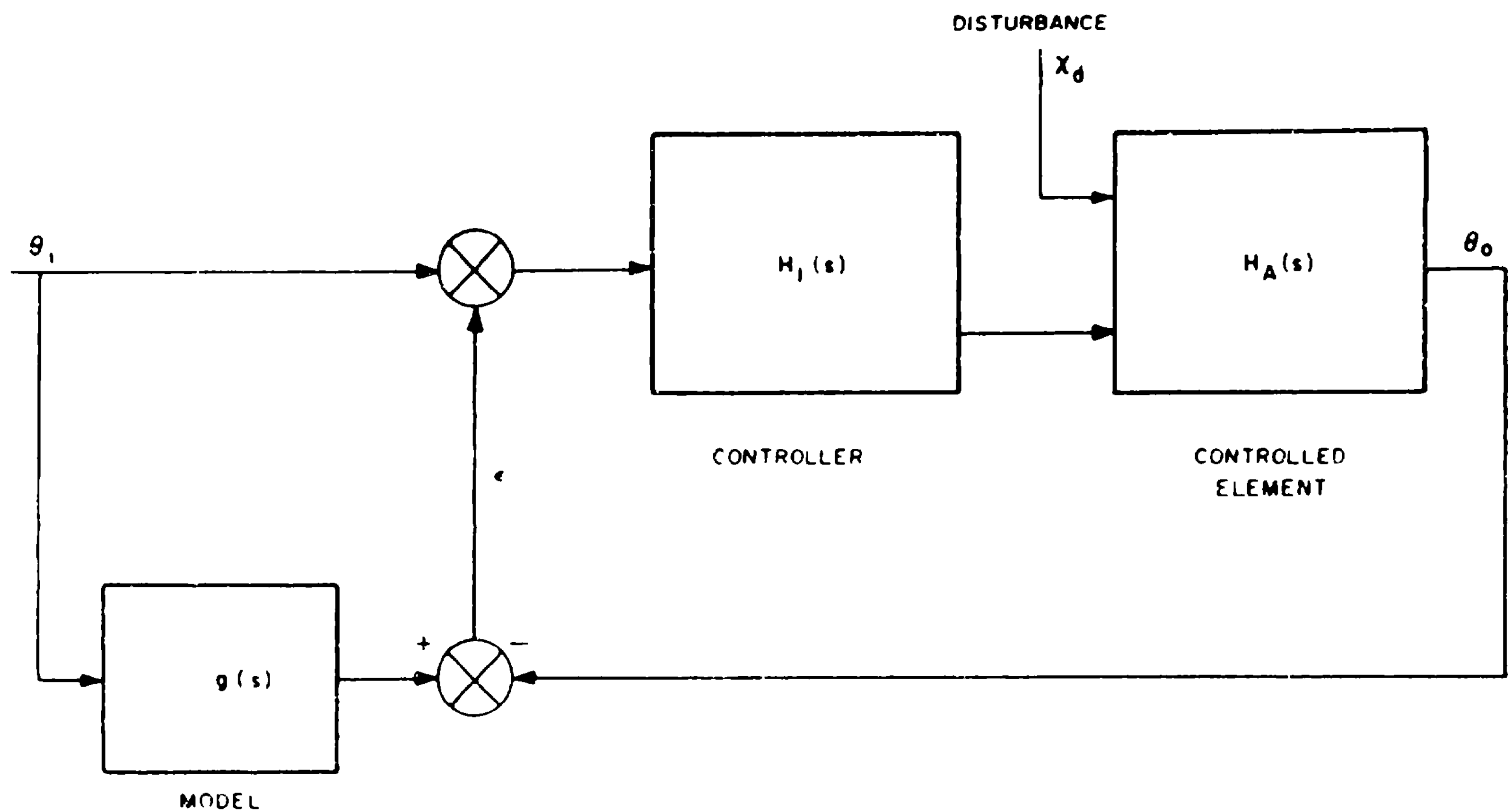
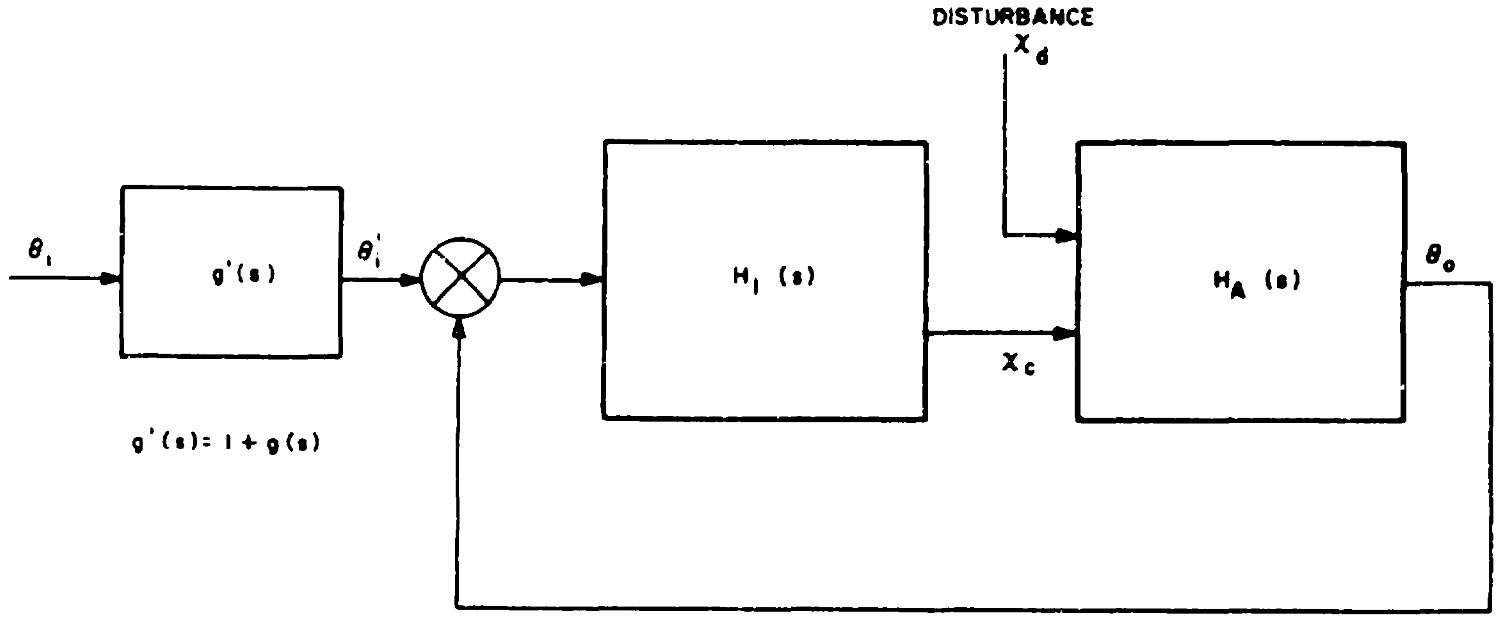


Fig 2

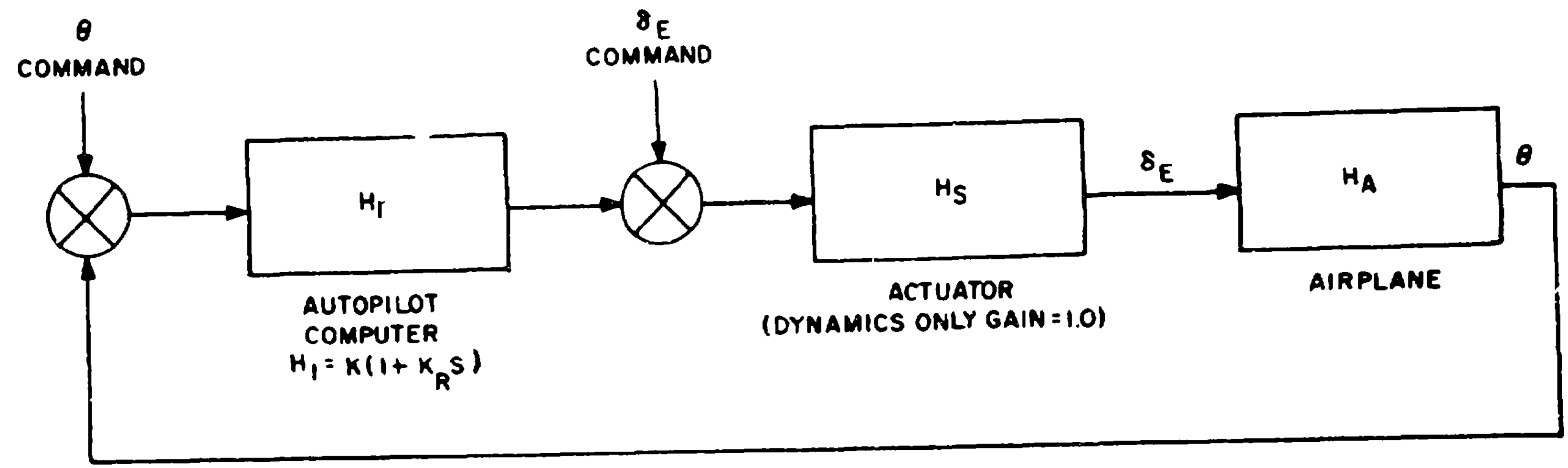
CONTROL BLOCK DIAGRAM
WITH CONDITIONAL FEEDBACK



$g'(s) = 1 + g(s)$

Fig 3

EQUIVALENCE OF
CONDITIONAL FEEDBACK CONFIGURATION
AND CONVENTIONAL SYSTEM
WITH PRE-FILTER



BASIC PITCH ATTITUDE STABILIZATION SYSTEM

Fig 4

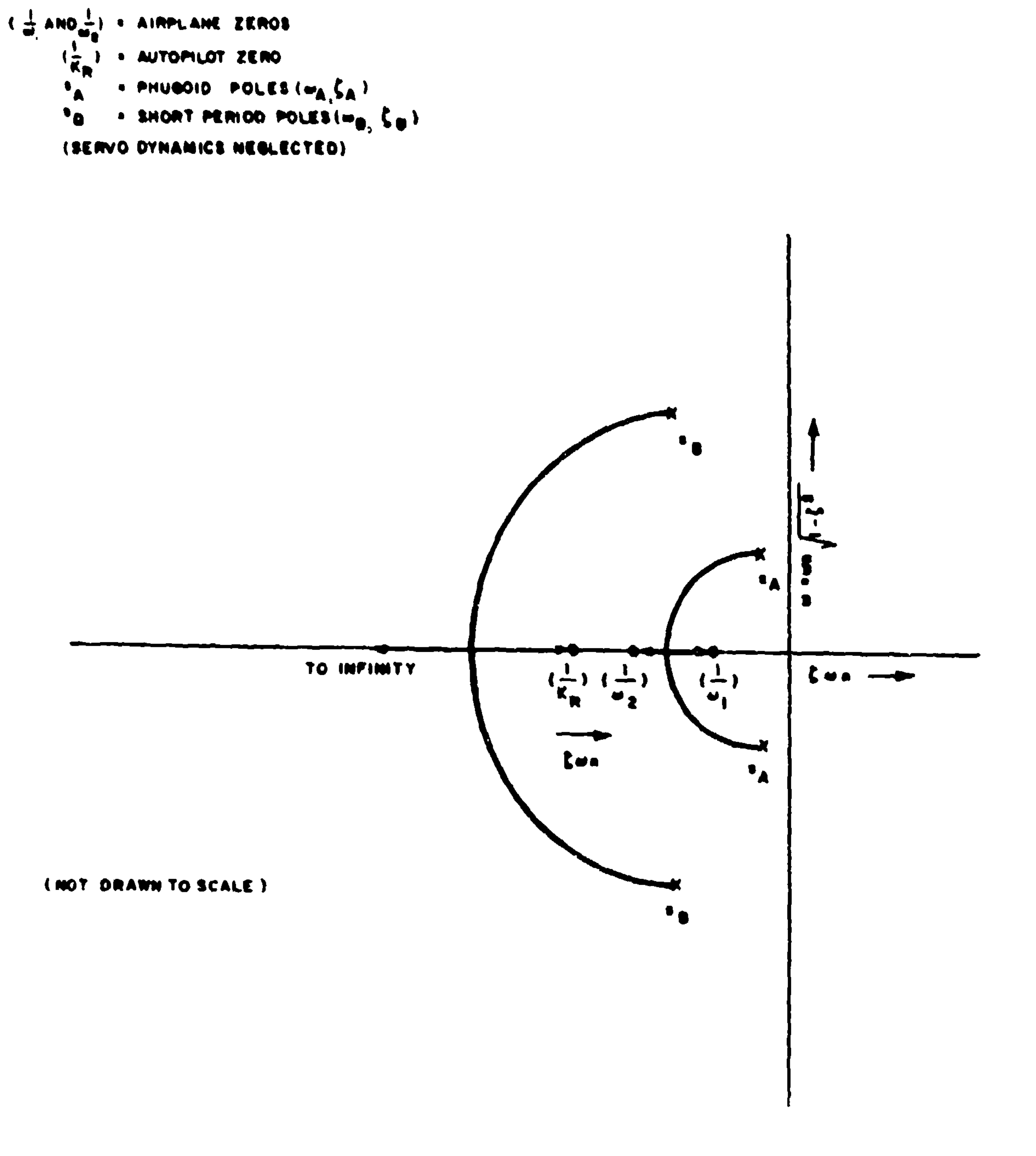


Fig 5

TYPICAL ROOT LOCUS OF
BASIC PITCH ATTITUDE STABILIZATION SYSTEM

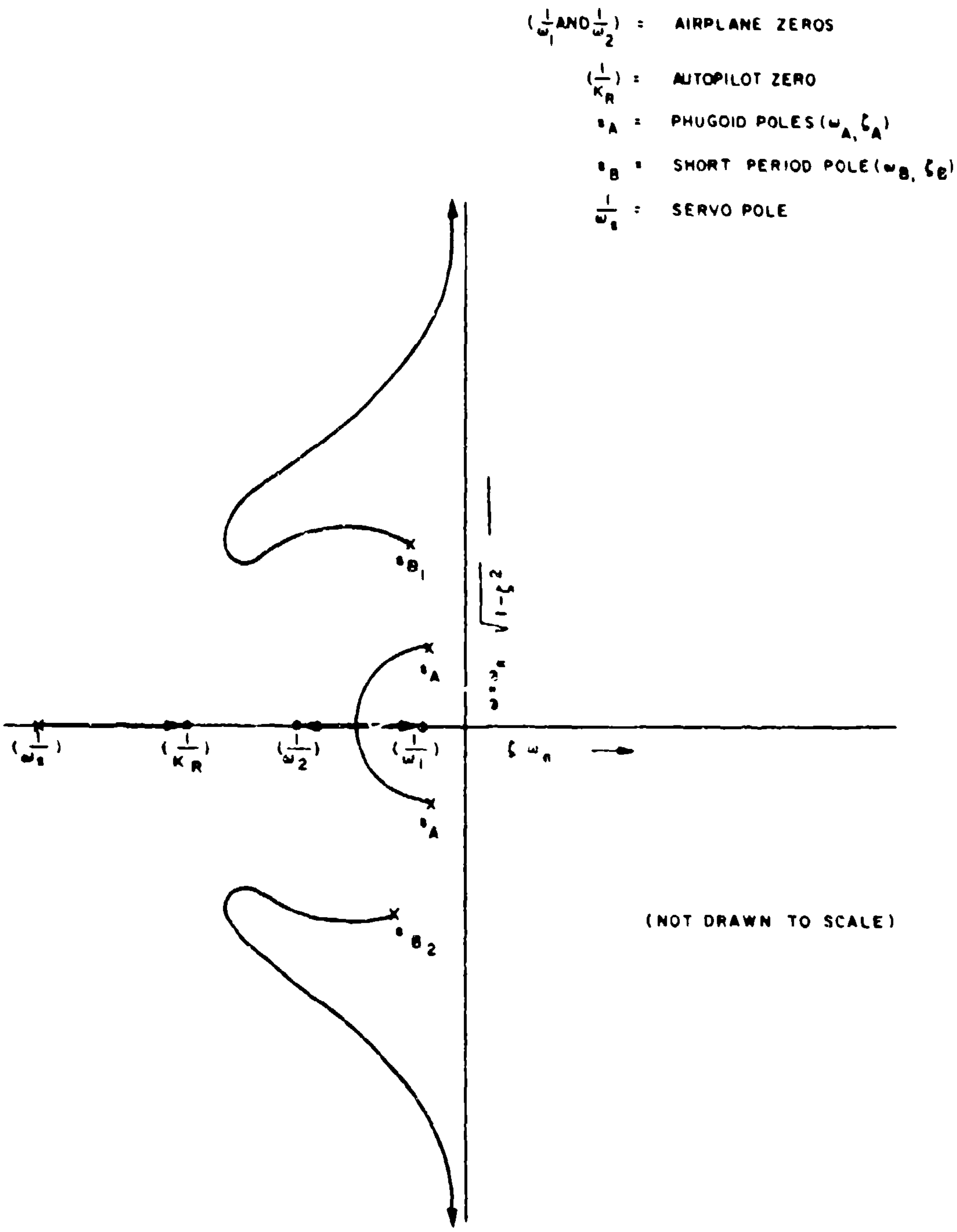


Fig 6

TYPICAL ROOT LOCUS OF PITCH
STABILIZATION SYSTEM WITH 1ST ORDER SERVO

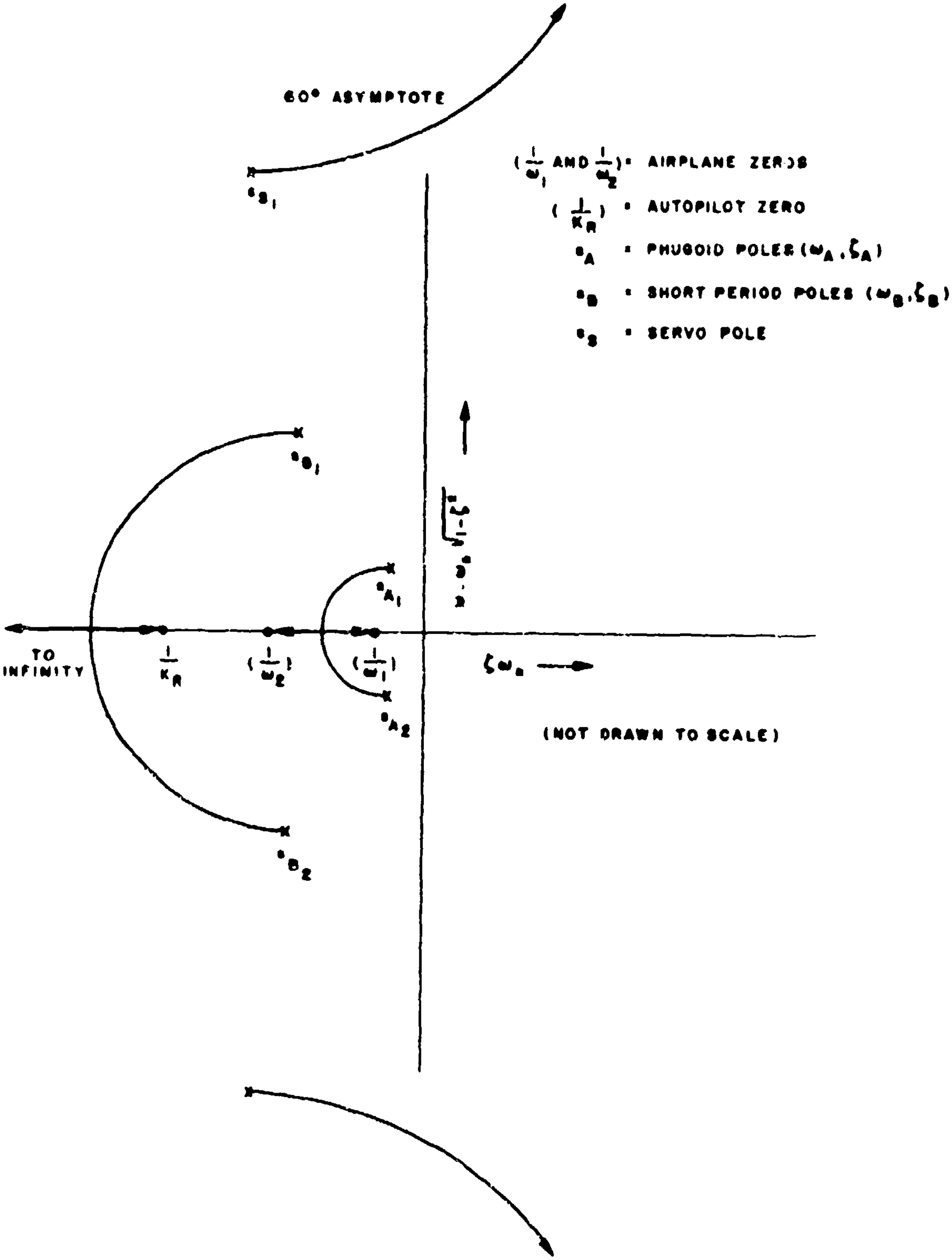


Fig 7

TYPICAL ROOT LOCUS OF PITCH STABILIZATION WITH 2ND ORDER SERVO

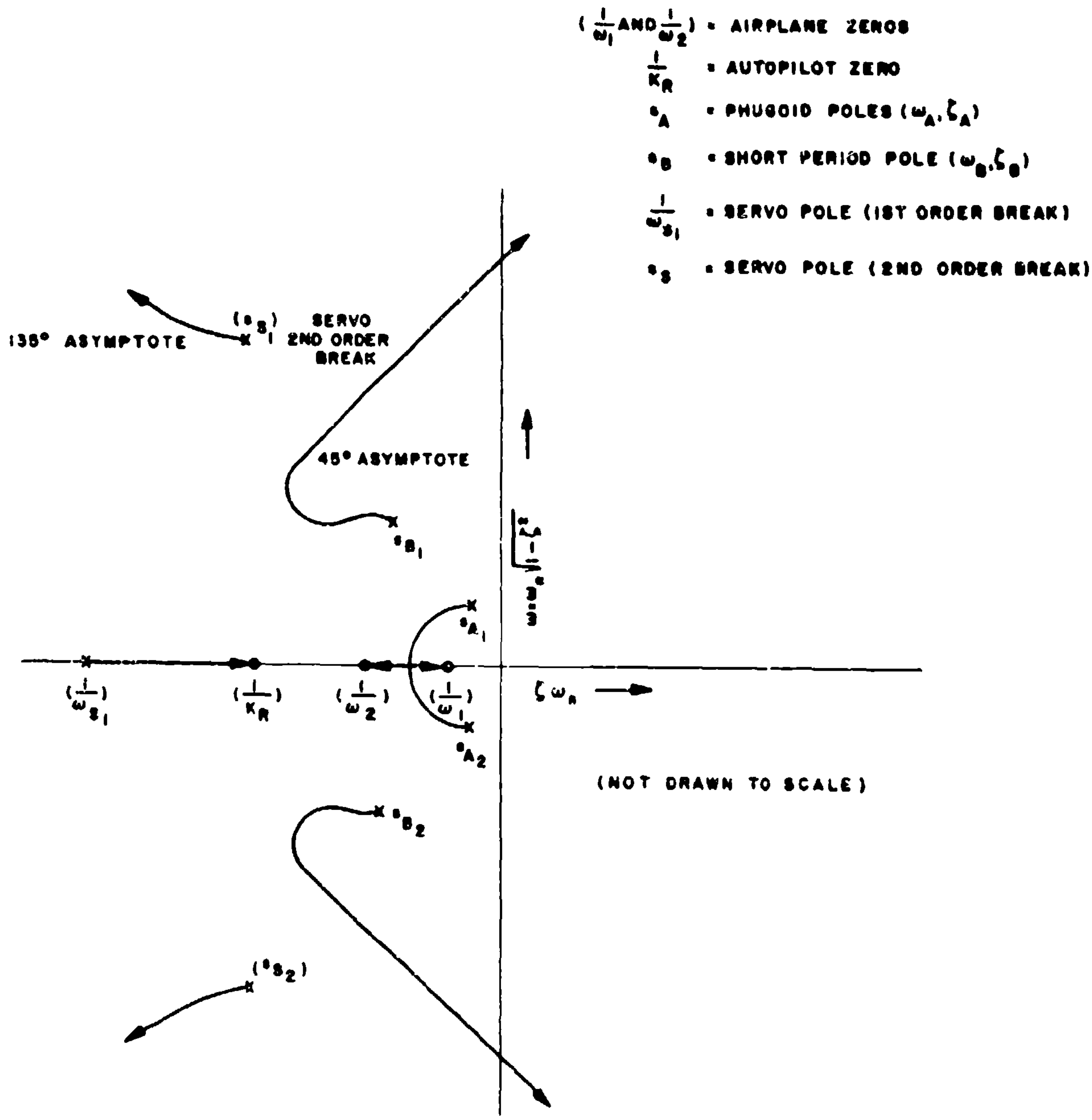


Fig 8

TYPICAL ROOT LOCUS OF PITCH STABILIZATION SYSTEM WITH 3RD ORDER SERVO

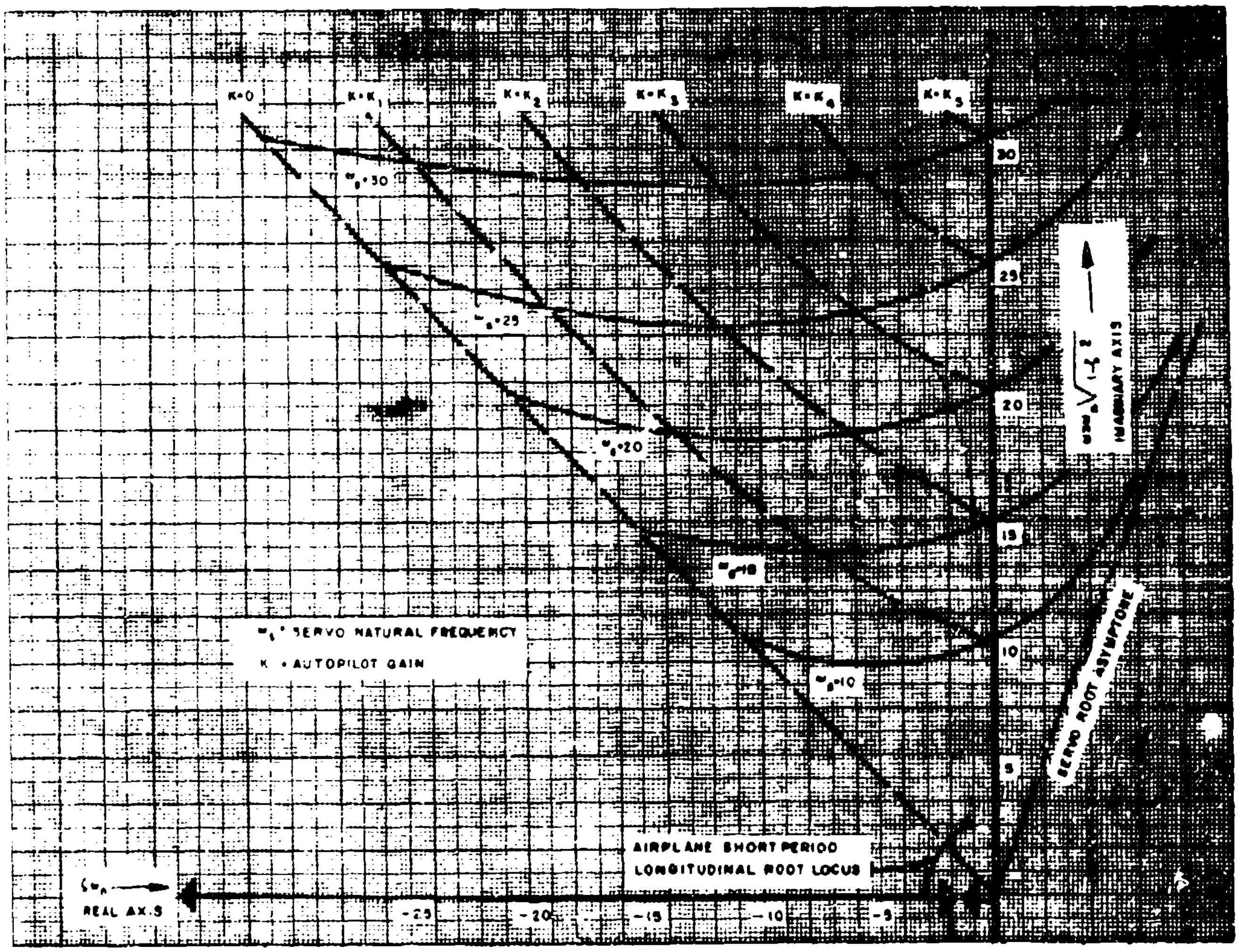


Fig 9

TYPICAL AIRPLANE-AUTOMATIC PILOT LONGITUDINAL ROOT LOCUS WITH 2ND.-ORDER SERVO

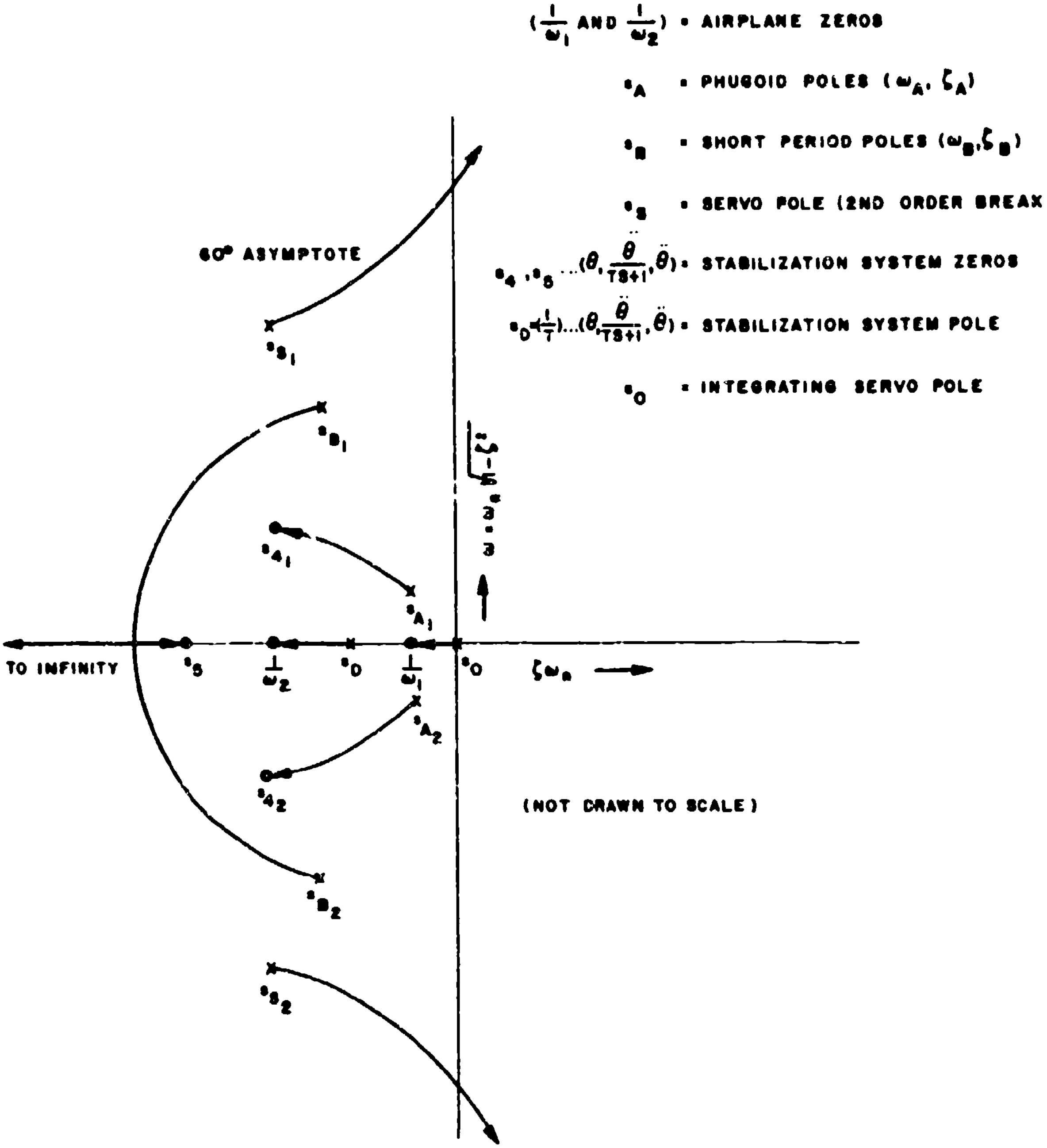
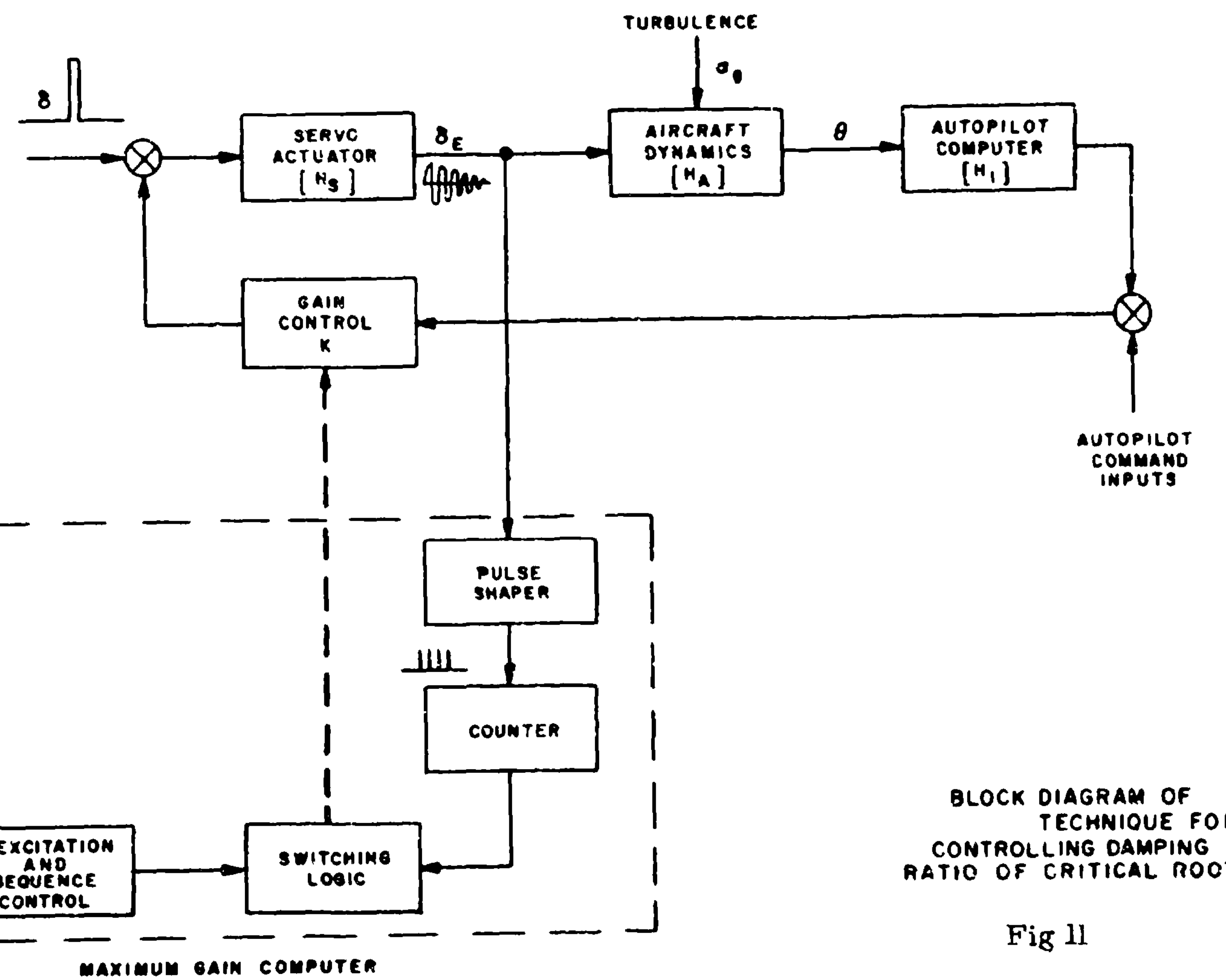


Fig 10

TYPICAL ROOT LOCUS OF PITCH STABILIZATION SYSTEM WITH 3RD ORDER INTEGRATING SERVO



BLOCK DIAGRAM OF TECHNIQUE FOR CONTROLLING DAMPING RATIO OF CRITICAL ROOTS

Fig 11

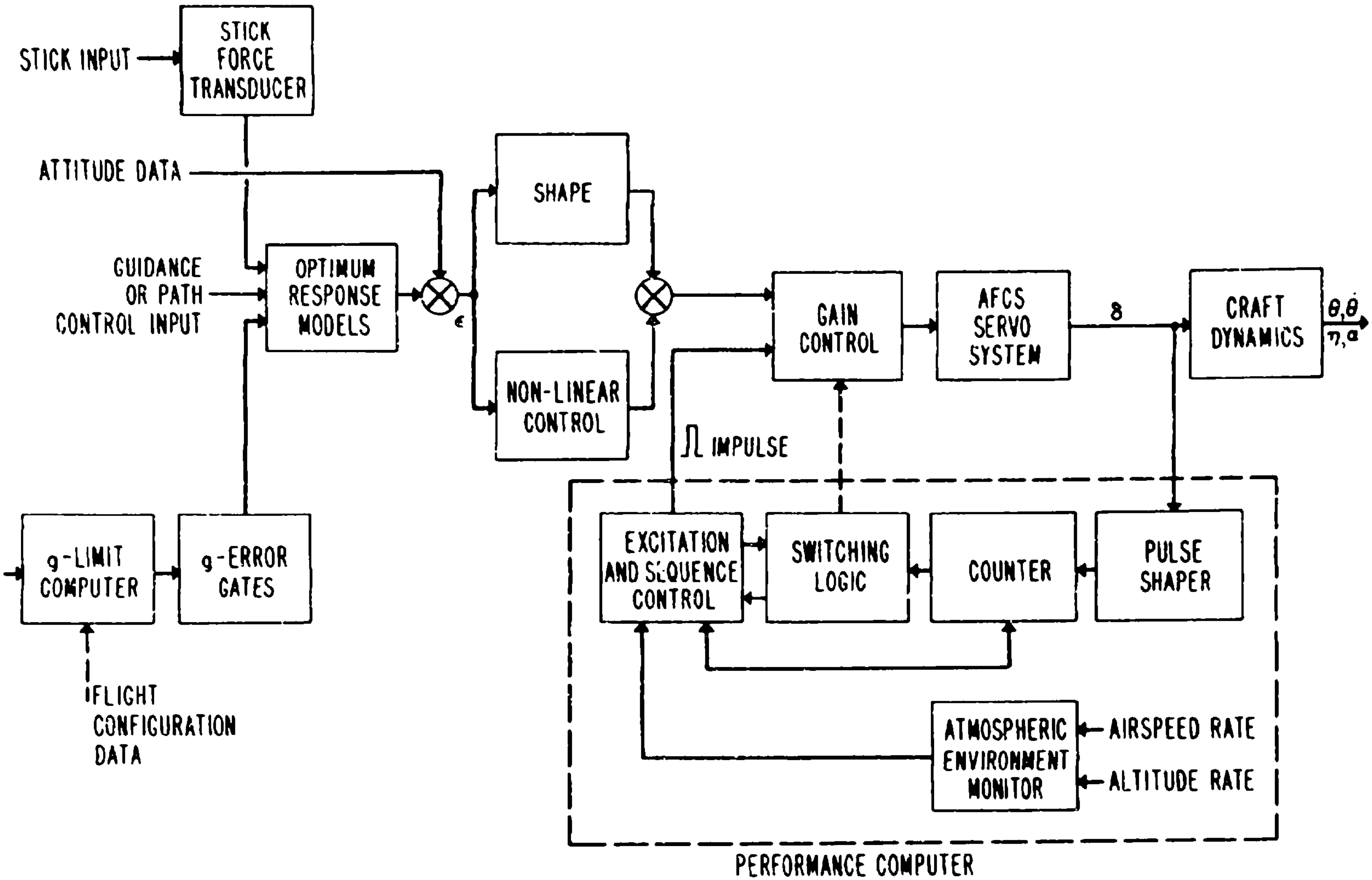


Fig 12 BLOCK DIAGRAM
ADAPTIVE STABILIZATION SYSTEM

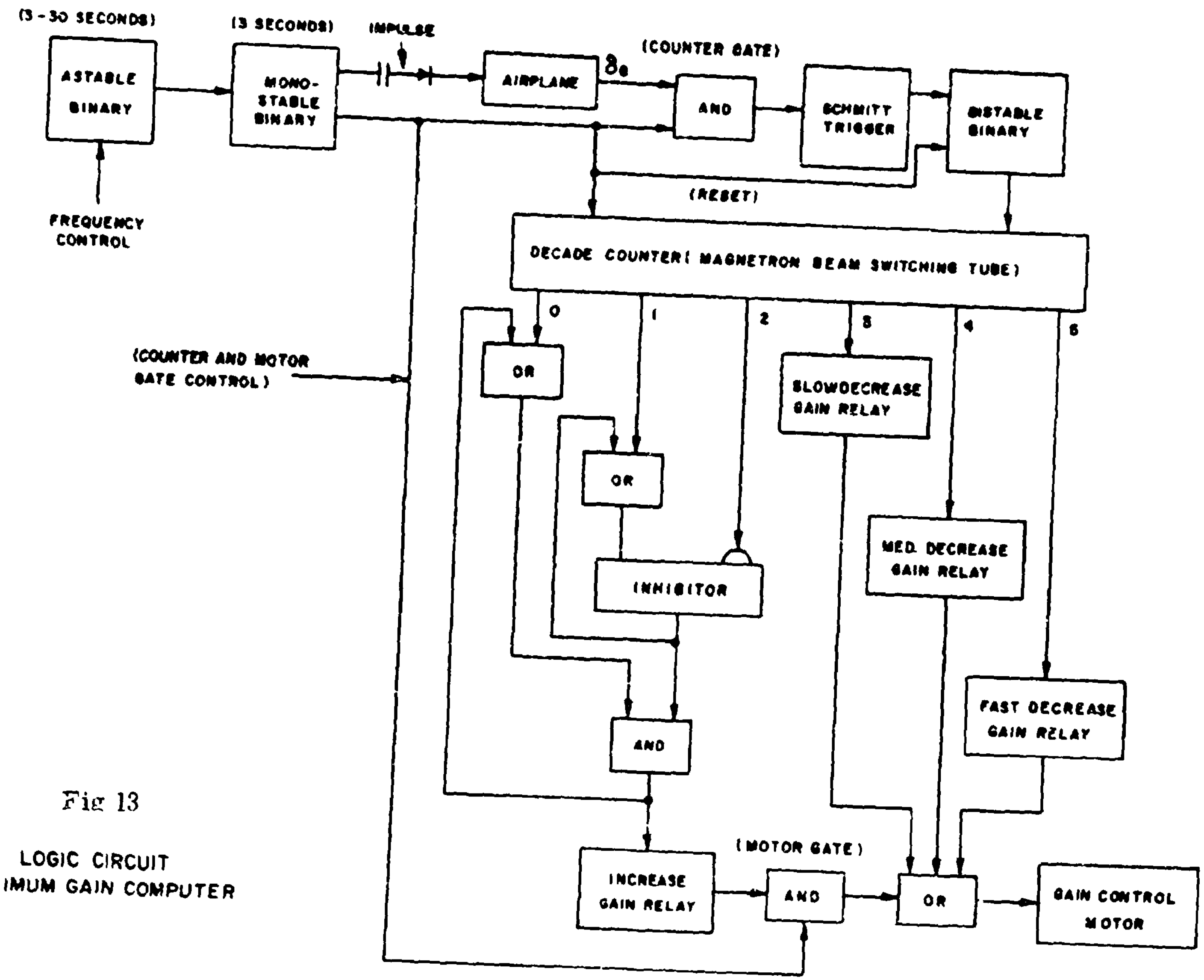


Fig 13

LOGIC CIRCUIT
MAXIMUM GAIN COMPUTER

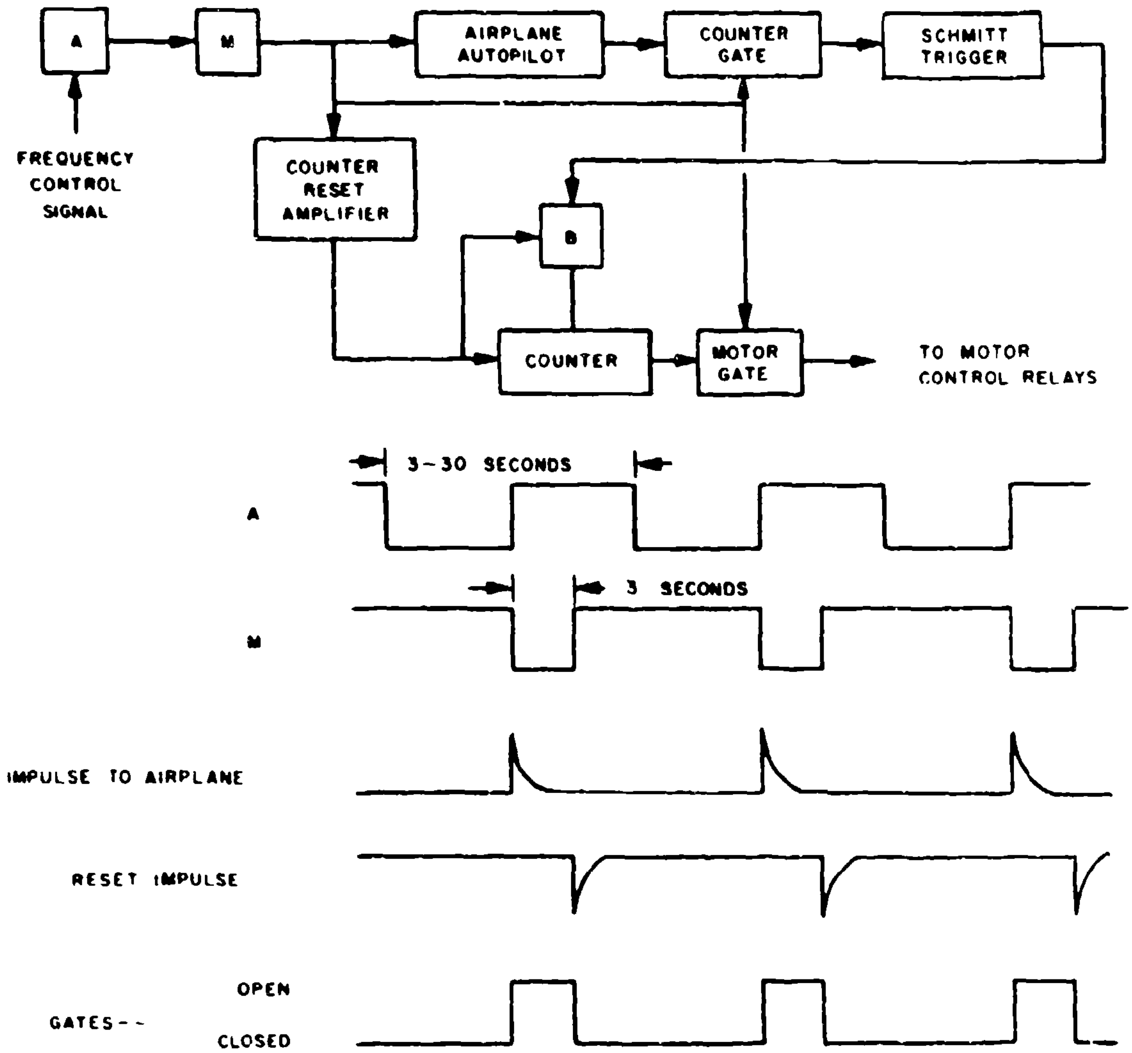


Fig 14

SEQUENCE AND EXCITATION CONTROL BLOCK DIAGRAM

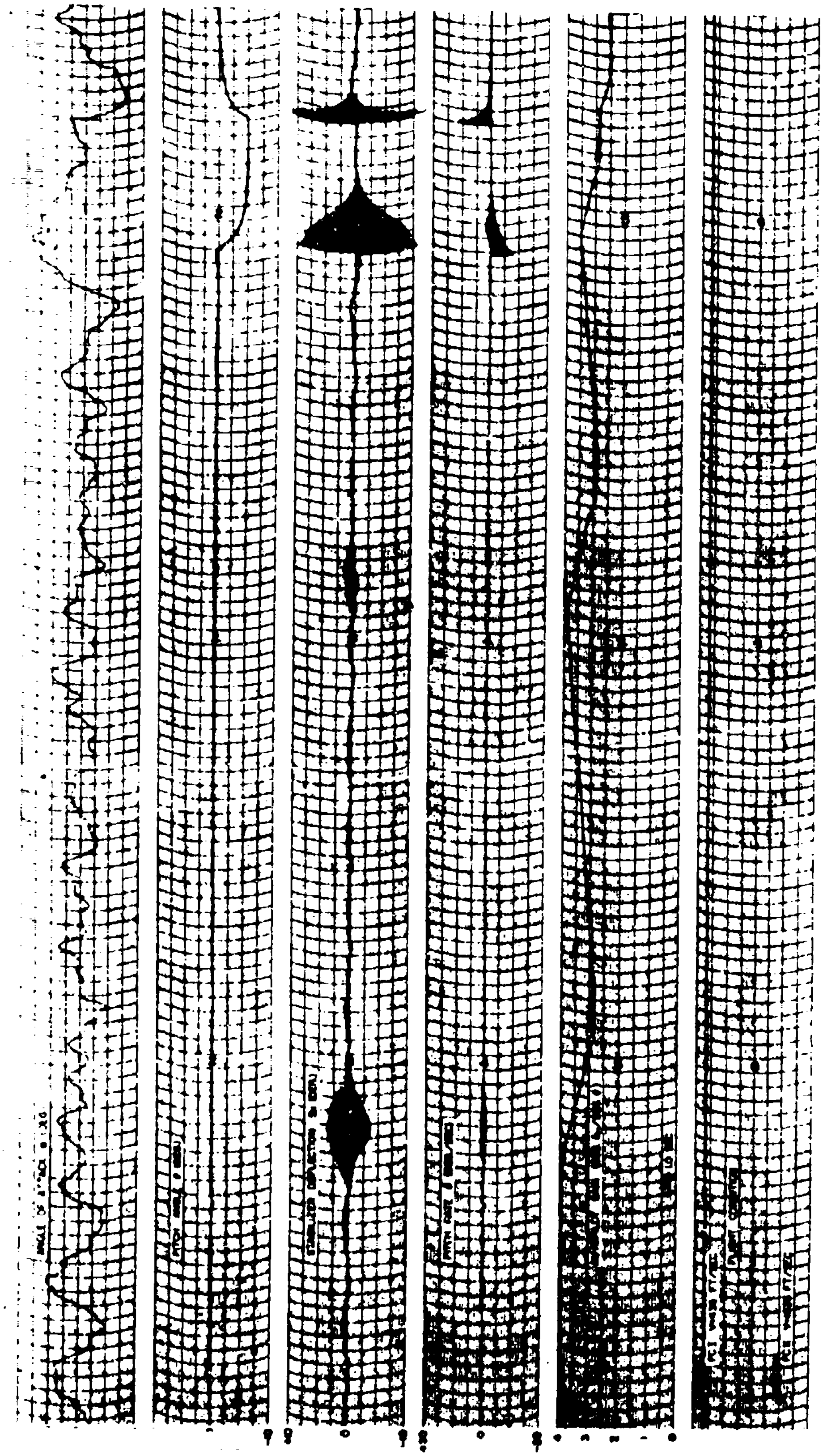


Fig 15

HYPOTHETICAL AIRPLANE - ADAPTIVE AUTOPILOT SYSTEM PERFORMANCE WITH ATMOSPHERIC TURBULENCE AS THE ONLY SOURCE OF EXCITATION

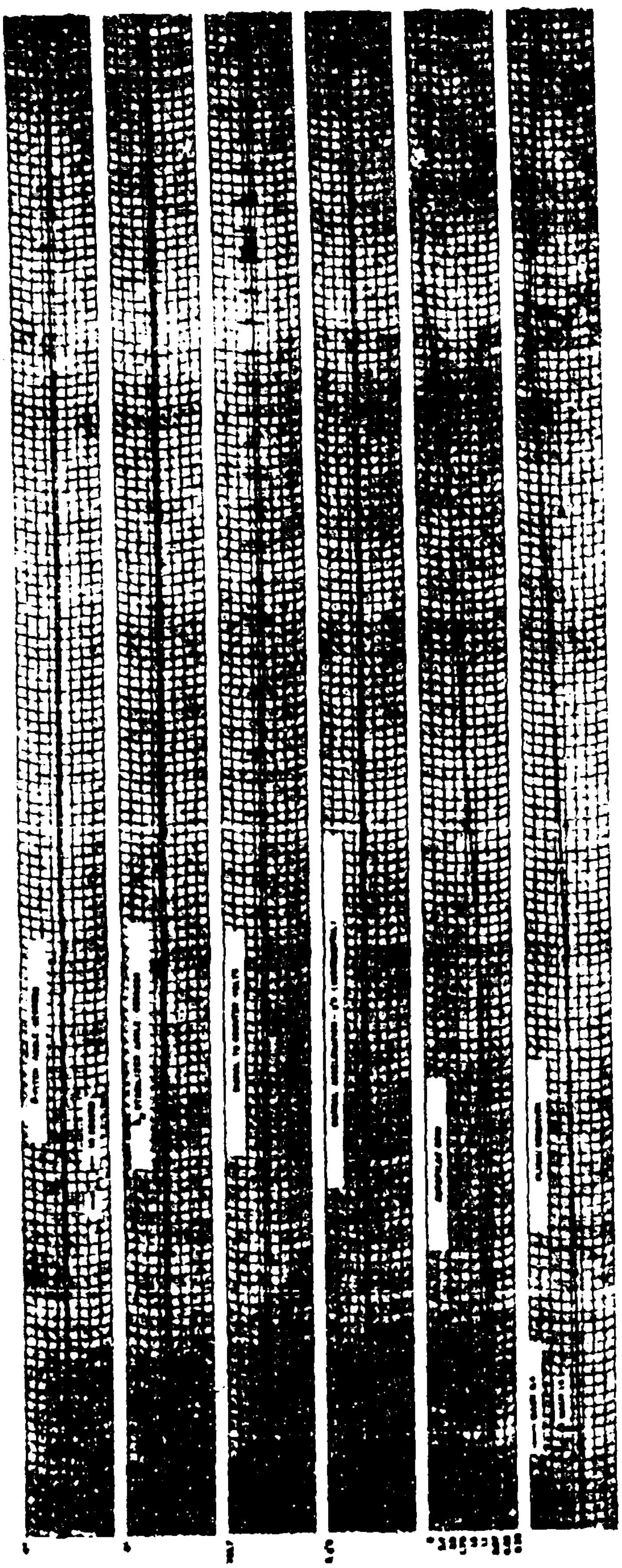


Fig 16

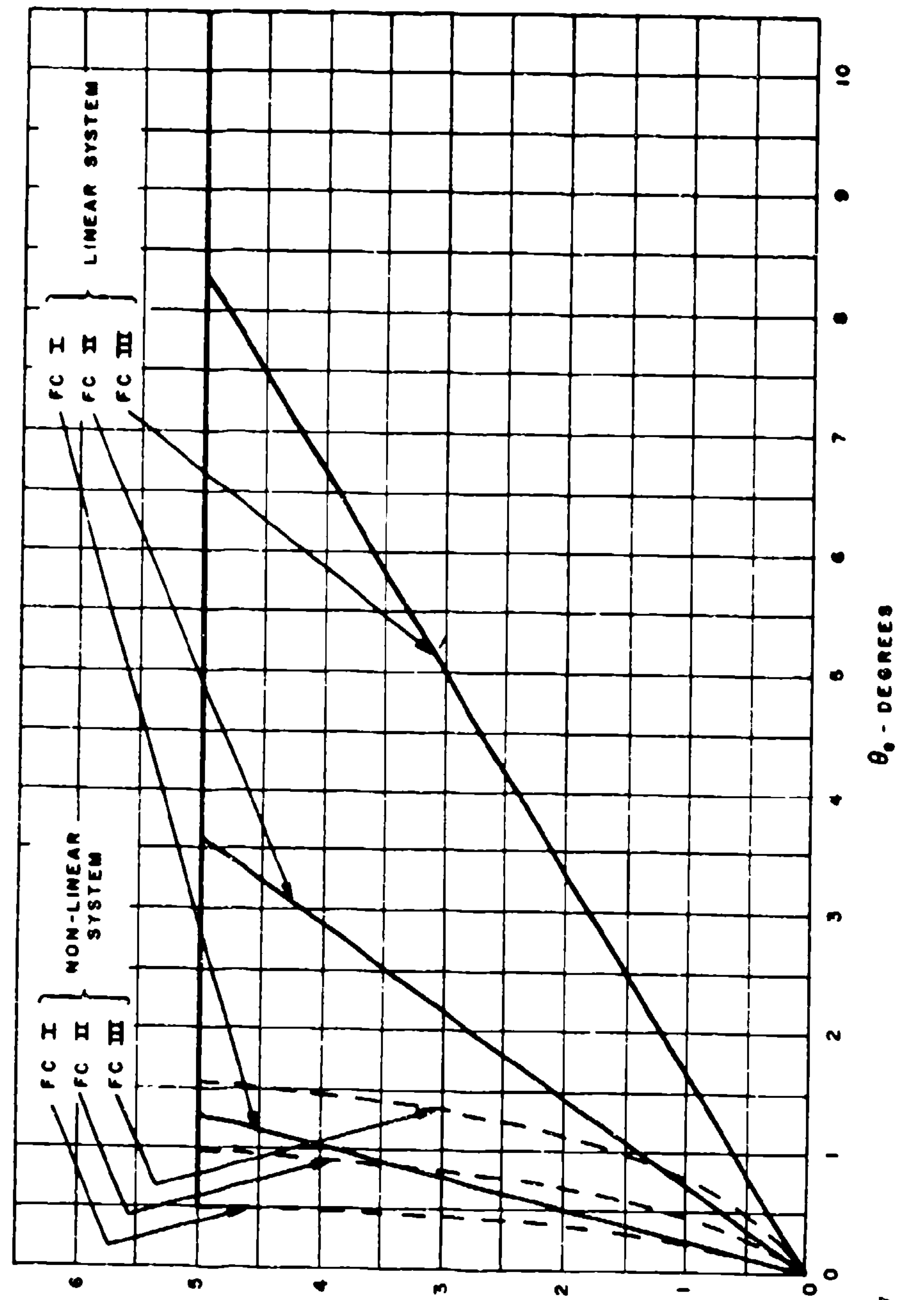


Fig 17
SURFACE DEFLECTION
VS
PITCH ERROR

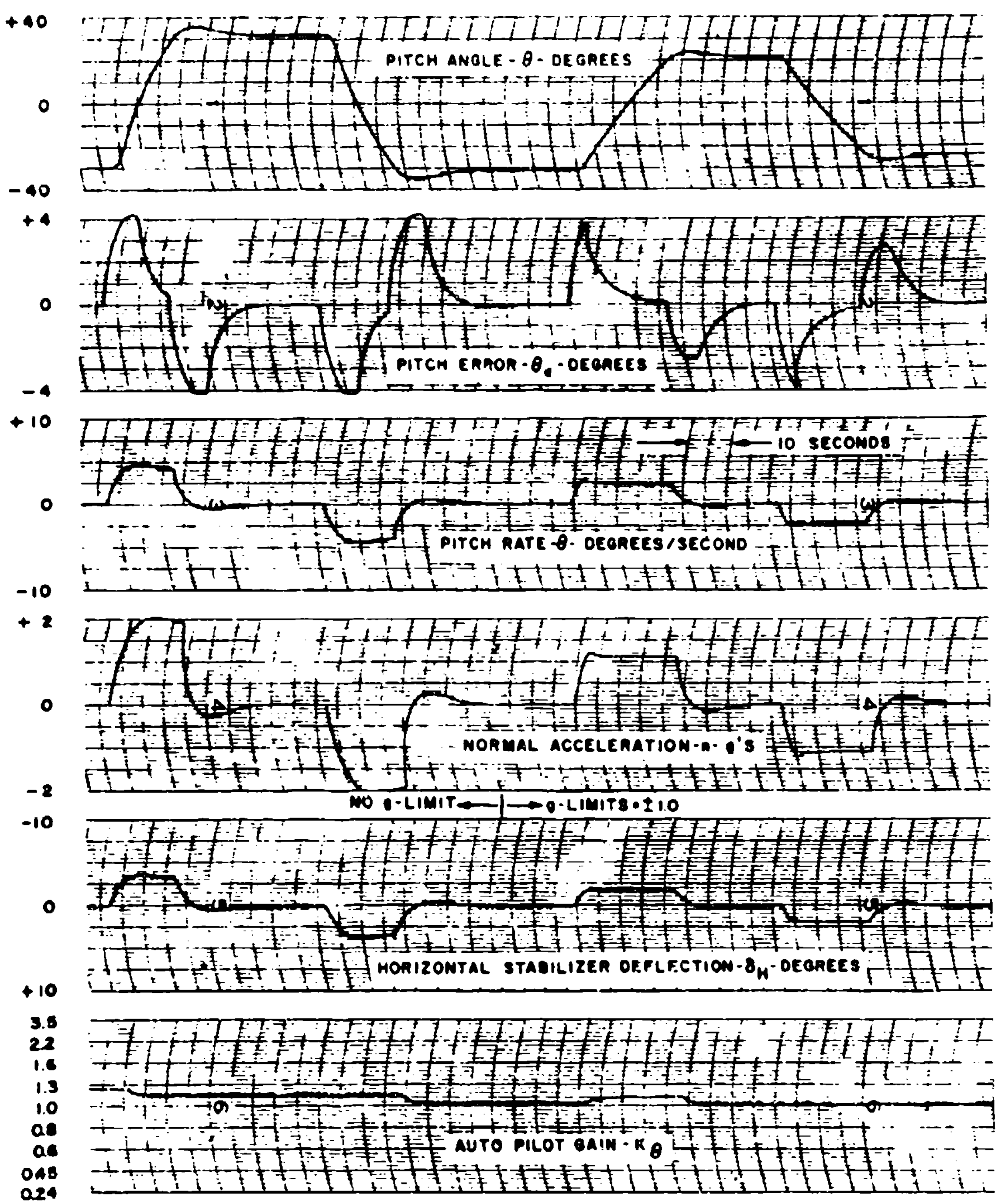


Fig 18

FLIGHT CONDITION II (M=0.95) F104A
PITCH RATE MANEUVERS (2.0g COMMANDS)
WITH AND WITHOUT ±1.0 g-LIMITS
NON-LINEAR ERROR GAIN CONTROLS NOT USED

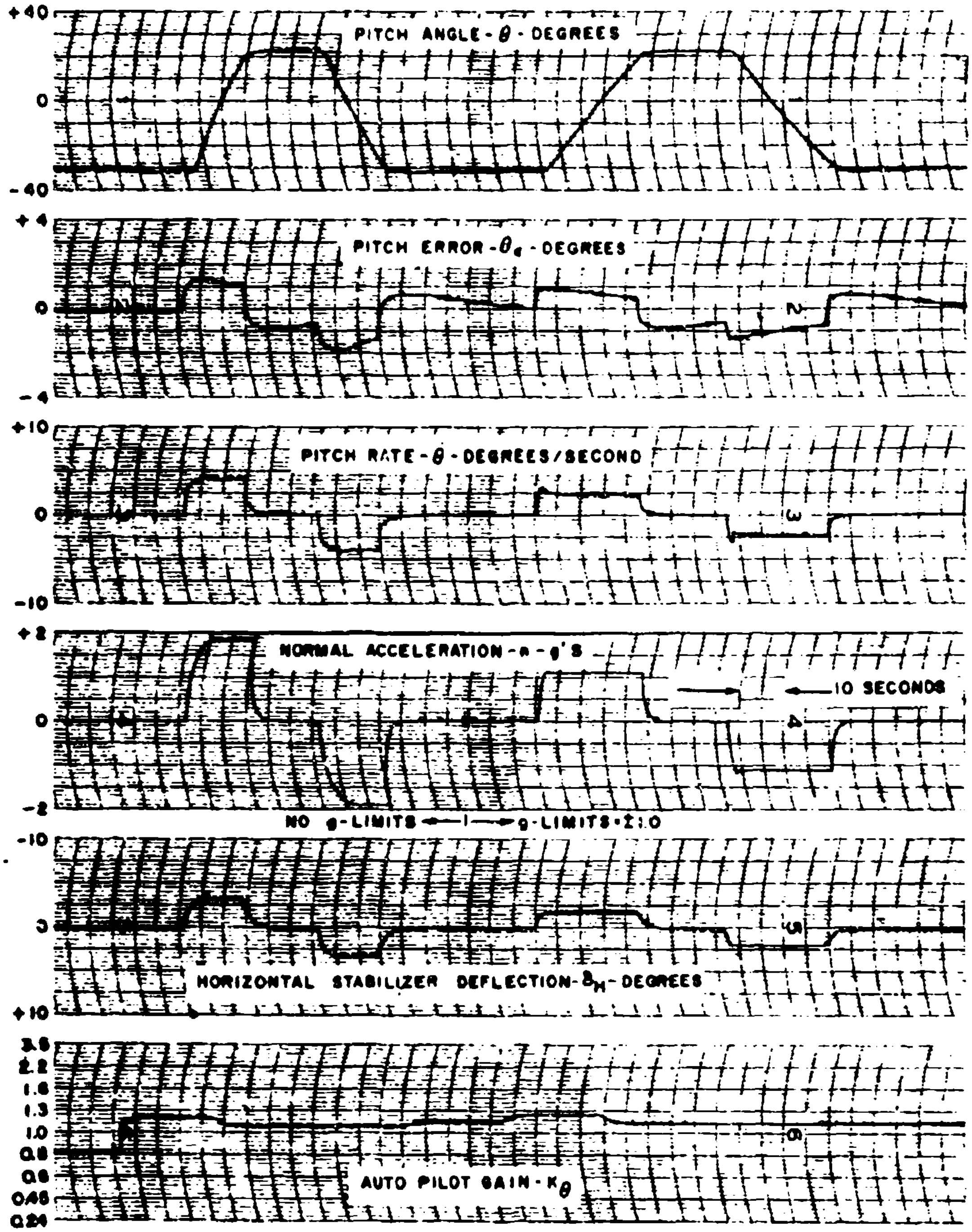


Fig 19

FLIGHT CONDITION II (M=0.95) F104 A
PITCH RATE MANEUVERS (2.0g COMMANDS)
WITH AND WITHOUT ± 1.0g - LIMITS
NON-LINEAR ERROR GAIN CONTROLS

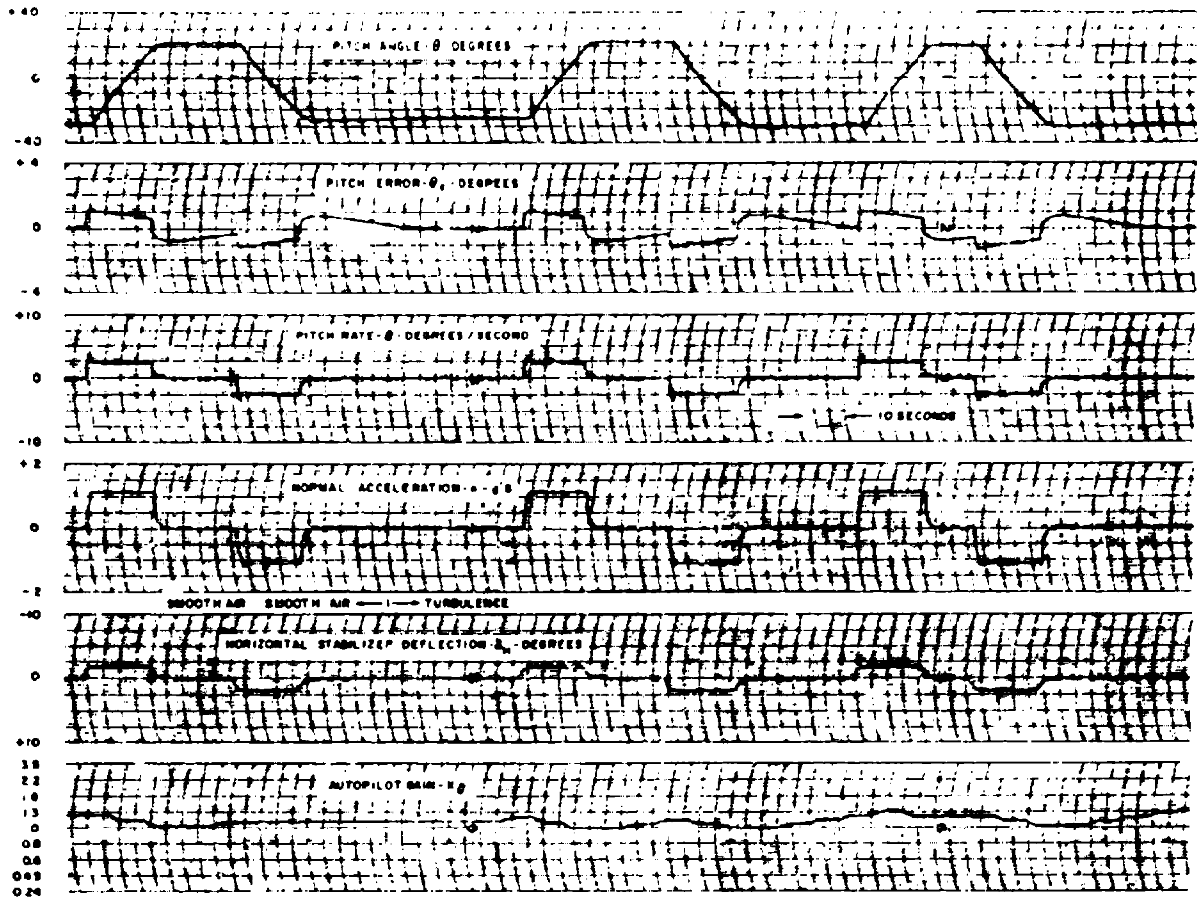


Fig 20

FLIGHT CONDITION II (M-0.95) F104A
PITCH RATE MANEUVERS (2.0 g-COMMANDS)
IN SMOOTH AND TURBULENT AIR WITH
±1.0g-LIMITS AND NON-LINEAR ERROR GAIN CONTROLS

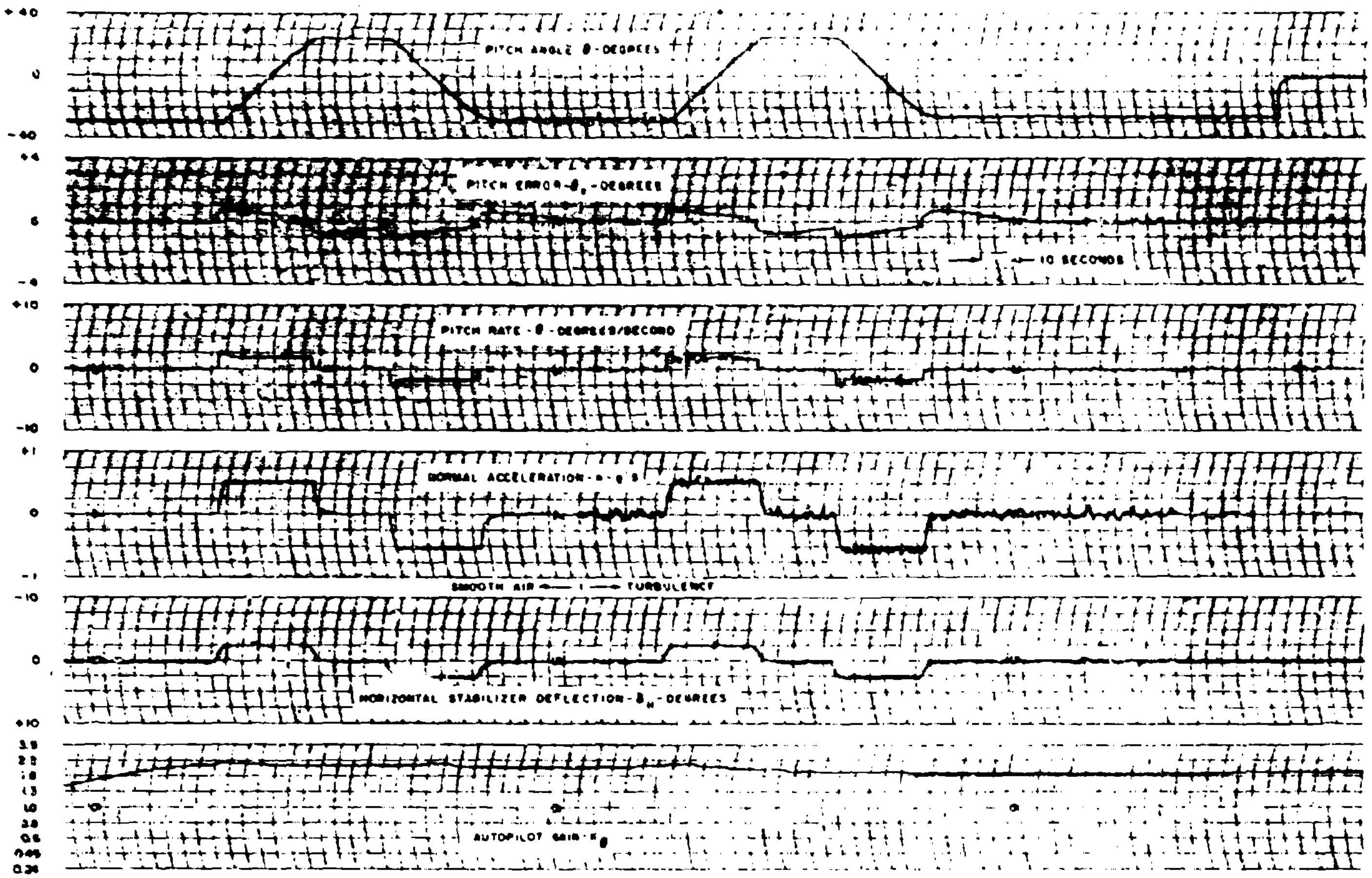


Fig 21

FLIGHT CONDITION I (M=0.6) FIG 4 A
PITCH RATE MANEUVERS (1.0g COMMANDS)
IN SMOOTH AND TURBULENT AIR WITH
±0.5g LIMITS AND NON-LINEAR ERROR GAIN CONTROLS

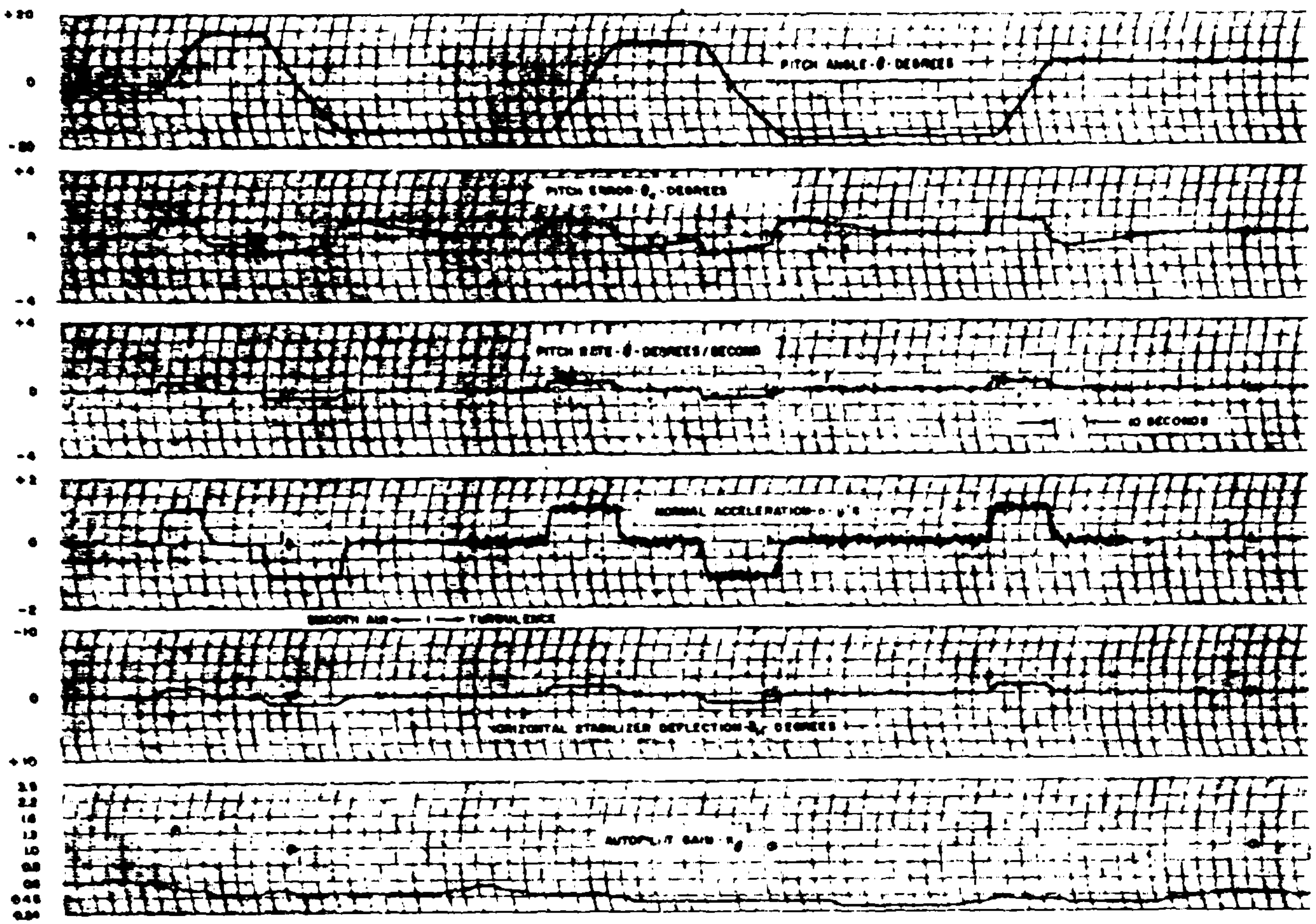


Fig 22

FLIGHT CONDITION III (M=1.6) F104A
PITCH RATE MANEUVERS (2.0g-COMMANDS)
IN SMOOTH AND TURBULENT AIR WITH
±10g-LIMITS AND NON-LINEAR ERROR GAIN CONTROLS

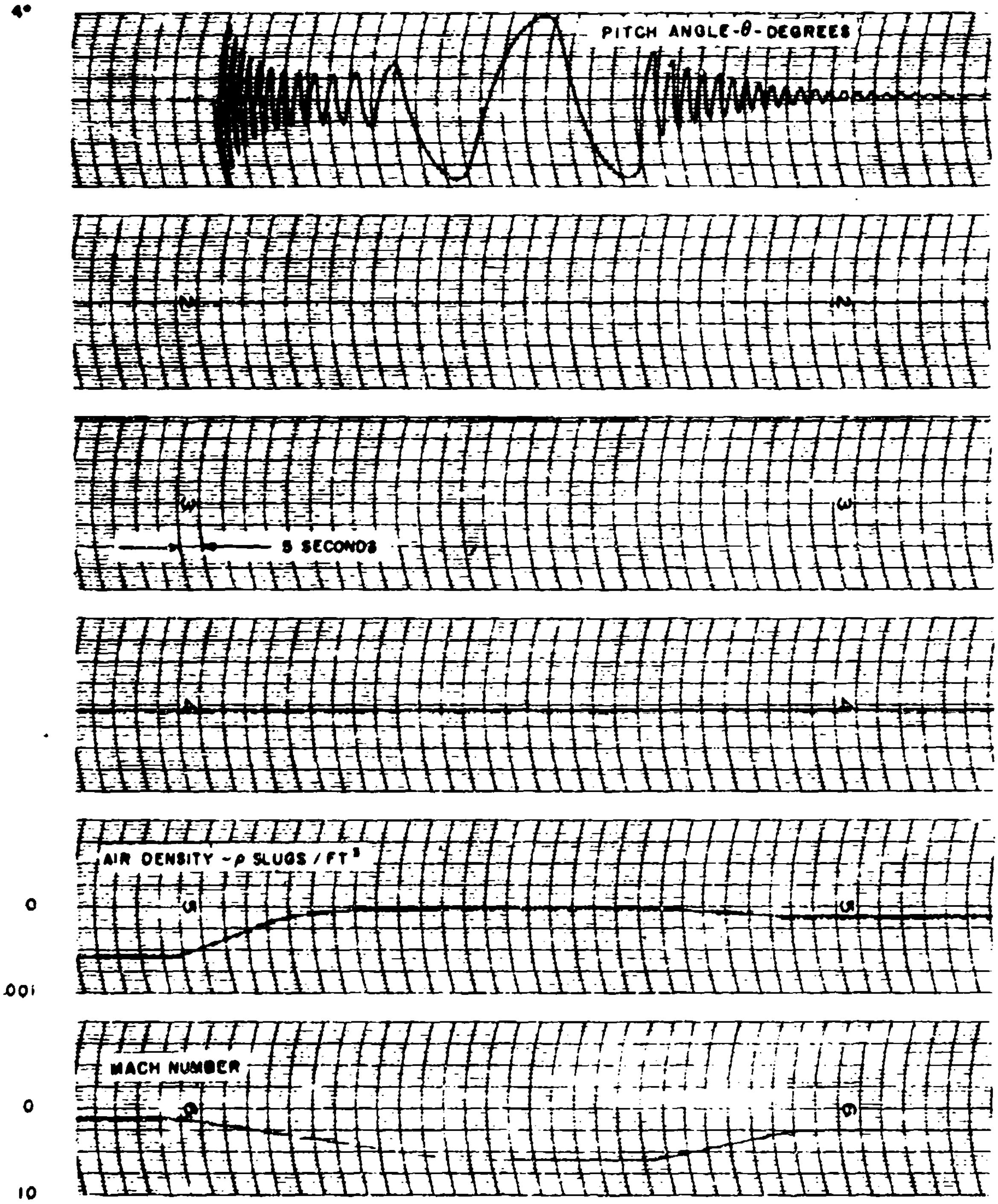


Fig 23

UNCONTROLLED VEHICLE RESPONSE TO DISTURBANCE DURING EXIT AND RE-ENTRY MANEUVER

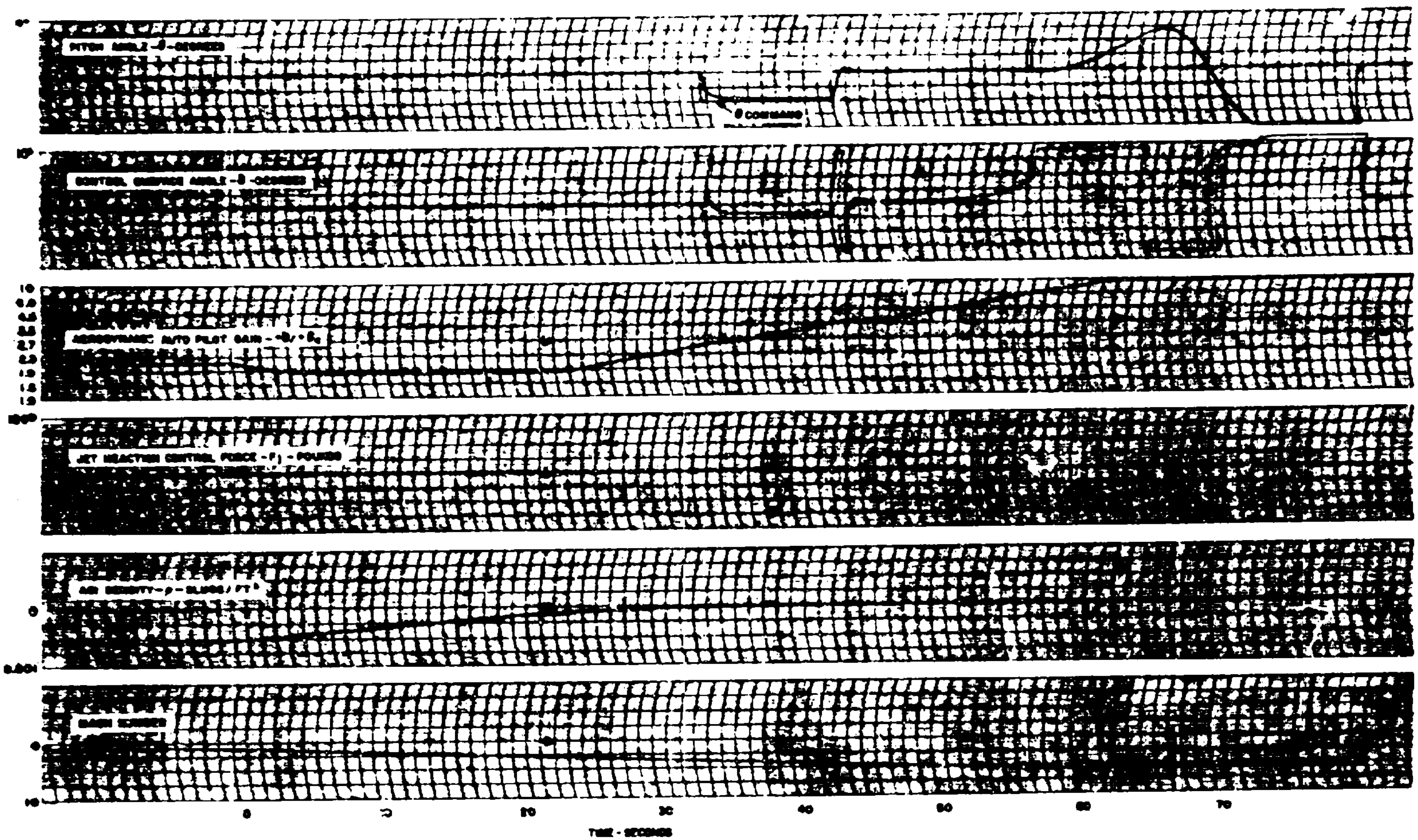


Fig 24

RESPONSE TO 2 DEGREE STEP PITCH COMMANDS DURING EXIT MANEUVER AERODYNAMIC AUTOMATIC PILOT ONLY

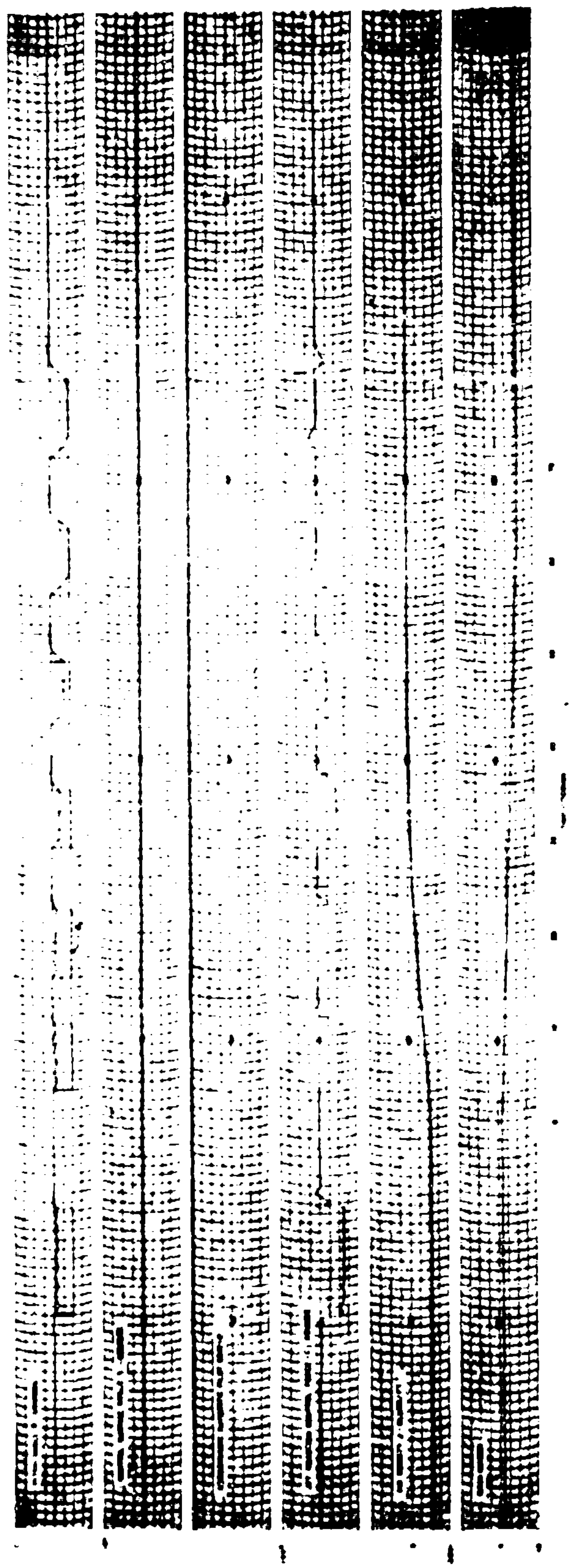


Fig 25

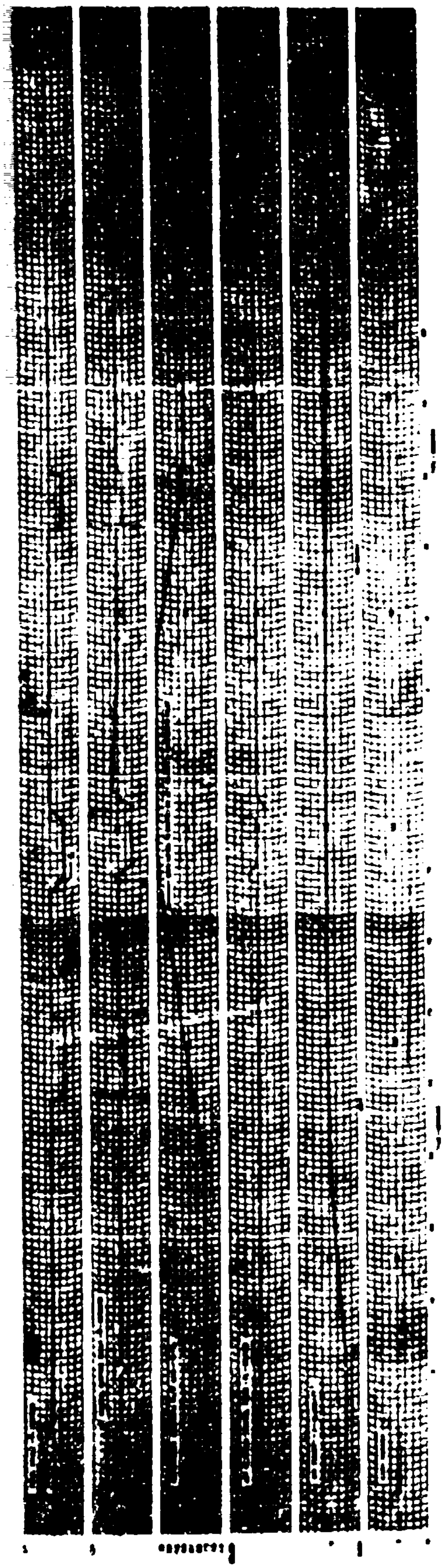


Fig 26

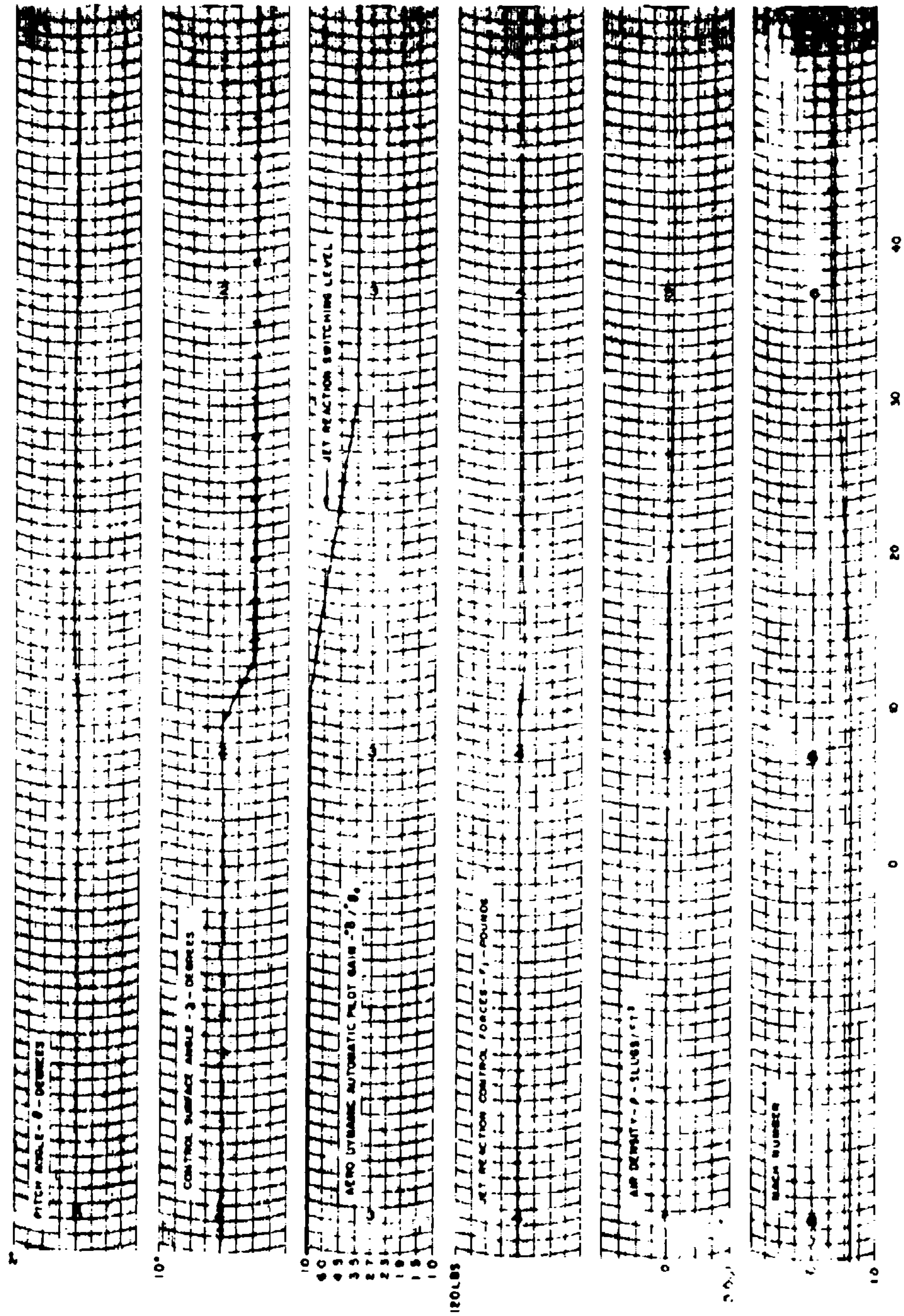


Fig 27

RE-ENTRY MANEUVER WITH CHANGE IN TRIM EQUIVALENT TO FIVE DEGREES OF CONTROL SURFACE OCCURRING IN FOUR SECONDS

HONEYWELL'S HISTORY AND PHILOSOPHY IN THE ADAPTIVE CONTROL FIELD

(Including a description of flight tests of the first Adaptive Flight Control System)

O. Hugo Schuck
Director of Research
Aeronautical Division
Minneapolis-Honeywell Regulator Company

HISTORY

It's rather hard to tell when we at Honeywell first became interested in adaptive controls. Perhaps we didn't use the right words, but certainly the need for adaptive controls has been recognized as being with us for a long time. This has been especially true in two of the areas in which we are vitally interested - flight control and process control.

By now in this symposium it isn't necessary to describe further the need for adaptive flight controls. In reprospect, it seems that we first consciously articulated the need in connection with our early work in automatic approach and landing. There, as you know, the ILS beam has a built-in convergence, leading to a system gain that varies inversely as the distance from the aircraft to the far end of the runway. This gain change plagued us considerably back in 1945 and 1946. If we had the gain set high enough for decent beam entry and following fifteen miles out, we went into a divergent oscillation as we neared the field. Even putting in a manual gain change at the outer marker wasn't enough, but we did manage to work out a useful compromise.

The obvious remedy was a continuous gain reduction as we neared the runway, a technique that we did use successfully in a relative humidity computer. So we were delighted with the advent of the Distance Measuring Equipment (the DME), and assumed that when SC31 recommended it for the Common System our troubles were over. But the DME was not adopted quickly, and anyway, the operators were somewhat less than enthusiastic about getting more electronic gear involved in the critical landing operation. So we sharpened our servomechanism theory pencils and learned to live with the convergence by pushing loci around on Nichols charts.

However, one aspect of this experience stayed with us and became, in fact, an integral part of our flight control system design philosophy. This was the idea of scheduling control system gains as pre-determined functions of measurable parameters. Now, of course, we're trying to get away from such

scheduling.

Another trouble plagued us in the automatic landing problem. The beam converged, not with respect to a straight center line, but to a line that had bends, wiggles and other aberrations with respect to the straight line it was supposed to be. We learned to call these by the scientific name of " noise" - which is a good name because its definition is " anything you wish you didn't have" . We found that the ILS beams at different airports around the country had different degrees of noisiness. The CAA managed to keep the one at Indianapolis pretty clean, but the one at Minneapolis - where we did a lot of test flying - had some pretty bad bends. One, in fact, gave the uninitiated observer the idea that the aircraft was going to land in the third hangar.

Fortunately, the ILS beams all tended to straighten out as the touchdown point was approached. It was in this critical region that we wanted good control. On the other hand, we didn't want to shake up the passengers or stress the airplane unduly. So we saw the problem as one of following clean beams pretty closely, while smoothing out to a considerable extent the noise in those that had a lot of bends. In other words, we posed the objective of doing the best control job possible, considering the signal to noise ratio of the input. This implied a change in system filtering and gain as a function of the input noise.

One approach to this problem was through the use of non-linear techniques. The Air Force was developing an interest in non-linear mechanics, and gave us support for a research project to advance the non-linear mechanics art in its application to automatic controls. One of the specified areas of application was to beam following. After a considerable amount of analytical work, during which we explored many of the techniques being discussed here, we settled for a system that limited the second derivative of the localizer signal as a function of the beam noise. An alternate mode also lowered gain in the presence of high noise levels.

This system was built, it flew the airplane as it was supposed to, but it was unduly complicated. So it never left the Research Department.

In the meantime, we had been in contact with the Research Department in our Brown Instruments Division, and had found that they had their own brand of difficulty in the chemical process control field. Here the problem was not so much one of input noise, but one of changing conditions. They had harrowing stories of working all day in a chemical plant, adjusting gains and limits until the control system was working nicely and stably. Then the five o'clock whistle would blow, a few pressures and temperatures would change with the change in shift load, and the control would go divergent. So they were looking for an approach that would permit greater tolerance to the conditions of operation.

At this time we began to think more clearly in terms we now use, of self-adaptive control. We began to visualize a control system that would produce a desired result, even though there was input noise, and even though the conditions in which it was working changed greatly. After we had been working on this for a couple of years the Air Force found our approach of interest, and gave us support for a study in which adaptive control was one of three approaches to achieving better flight control of high-performance airplanes.

While we did try to keep our approach unbiased, our previous experience tended to point us away from statistical transfer function techniques. We had already become familiar with the discontinuous feedback concept pioneered by Flugge-Lotz of Stanford University. We looked further into its implications, and investigated its applicability to a typical flight control problem. Using an analog computer, we tried various ways of mode switching and evolved a concept combining a model and an intelligently switched bi-stable element. With the results looking quite promising we decided to concentrate on a more definitive check-out of this concept through actual flight test on an F-94C. Equipment was built, thoroughly tested in simulation, and installed. The flight test results were most gratifying. They confirmed the simulation results and led to several engineering programs.

FLIGHT TESTS

Let us look at this adaptive flight control system that was proven in the flight tests on the F-94C. Conceptually it is simple, deceptively so. As shown in Fig. 1, the input is applied to a model whose dynamic performance is what we wish the dynamic performance of the aircraft to be. The actual response of the aircraft is compared with the response of the model, and the difference is used as the input to the servo. If the gain of the servo is very high, the response of the aircraft will be identical to that of the model, no matter what the elevator effectiveness, so long as it is finite and has the right direction. Thus changes in altitude and airspeed have no effect on the response. By building the model to give ideal response, the aircraft is given ideal response characteristics.

Design of the model is fairly simple, provided one knows what kind of response is wanted. Regular network theory can be used. The big problem comes in connection with the need to make the gain of the servo very high. This is essential to making the inner loop sufficiently tight that the response of the aircraft is essentially that of the model. An ordinary linear servo system will not do. It simply cannot be given sufficiently high gain and still be stable. So we go in for non-linearity, the most extreme form of non-linearity, in fact - the bang-bang type. Full available power is applied one way, or the other, depending on the direction of a switching order.

Now it is well known that a simple bang-bang system is oscillatory. And we don't want an oscillatory aircraft. But the bang-bang principle does have

the attractive advantage over any other system of providing full available power to correct even small discrepancies between aircraft response and model response. So we look for ways to tame it down, keeping its high-gain characteristics while reducing its oscillatory activity. This is accomplished through use of a number of techniques in combination, as can better be described in connection with a more detailed diagram.

In Fig. 2 we see the same model, whose output $\dot{\Theta}_M$ is the desired pitch rate. Feedback is provided by a pitch rate gyro. It measures the actual pitch rate $\dot{\Theta}$ of the aircraft, which is represented as a second-order system. Experience has shown that this representation is adequate to describe the short-period motion of the conventional rigid aircraft to elevator deflection. The damping ratio ζ_a, the natural frequency ω_a, the time constant T_a, and the elevator effectiveness M_{δ_e} are all known functions of the aircraft stability derivatives.

The input to the aircraft is elevator deflection δ_e, which is produced by a conventional servo and actuator shown in the block to the left of the aircraft block. It has the usual integration and second-order dynamics of such systems. In the case of the Lockheed F-94C aircraft, the natural frequency is 37 radians per second and the damping ratio is 0.7. The proportional plus integral term in the numerator results from the use of a high-pass network in the feedback loop internal to the servo-actuator system. Cancellation of this numerator term is the primary purpose of the lead-lag filter shown next to the left. Some small lead is introduced to compensate for the normal lost motion of slop in the control gearing.

Next, to the left is the limiter, operating on the output of the relay. It sets the magnitude of the relay's output voltage. Some adjustment of this magnitude seemed desirable, and this is the purpose of the gain changer in the upper block. Its operation is described by the equations in the lower right corner. If the system error is large-larger than B in absolute magnitude, full output voltage is obtained from the relay. After the system error has been reduced below B the output of the relay is decreased exponentially with time in accordance with the second equation. Fig. 3 shows graphically this decrease in available input to the filter.

Returning to Fig. 2, we see that instead of feeding the error signal directly to the relay, a somewhat modified input is provided by the lead -lag network in the switching logic block. Ideally the denominator time constant is zero; the numerator constant is about 0.2 seconds. A further modification of the input to the relay is made by the introduction of a high-frequency sinusoidal dither signal. Its frequency of 2000 cycles per second is so high that it does not appear in the output motion. An averaging process takes place, so that the output of the relay is linearized for very small signals. Use of a

sinusoidal dither signal gives the arc-sin characteristic shown in Fig. 4. Obviously a mechanical relay cannot follow a 2000 cycle per second dither input. An electronic relay can, and such was used in the F-94C flight test equipment.

Returning again to Fig. 2, we see that the pitch-rate inner loop consists of the switching logic's lead network, the relay with dither, a limiter controllable from the error magnitude by the gain changer, a lead-lag filter, the servo and actuator, and the aircraft, with feedback provided by a pitch-rate gyro. The dynamics of the gyro are included for completeness, but they can be neglected. The loop gain is very high, and consequently the actual pitch rate can be maintained acceptably close to the output of the model $\dot{\theta}_M$.

For the F-94C flight test program a model was used having a natural frequency of 3 radians per second and a damping ratio of 0.7. These values have been established by the NACA and Cornell Aeronautical Laboratories as acceptable for manned aircraft of the type used.

To obtain pitch attitude control the switch shown at the left in Fig. 2 is closed. Since the inner loop transfer function is unity, the attitude control system can be described by a third-order transfer function: second order from the model and an integration from pitch rate to pitch attitude.

This, then, is the nature of the system we're talking about. Before going on to describe its mechanization for the flight test program, I want to admit - in fact, emphasize - that it did not reach this form by a process of pure, abstract cerebration. Each portion is there because it was found necessary in the course of a long simulation program, and the various constants were worked out before the mechanization was undertaken. It was the apparently successful performance obtained in simulation that led to the decision to undertake flight test verification. Furthermore, since this was a research project and the objective of the flight tests was primarily to provide a feasibility check of the adaptive flight control technique that had been evolved, minimum modifications were made to the aircraft and its equipment. Most of the equipment used was from the old Honeywell E-10 Autopilot development program of several years before.

Fig. 5 shows the amplifier that was built for the flight tests. It incorporated the various circuits that were unique to the adaptive system. All of this equipment was given careful hanger testing, with analog simulation of aircraft flight dynamics, before initiating the airborne tests. It is interesting to note that these tests showed the previous simulations of the hardware to be quite valid.

Fig. 6 shows the flight test engineer's test panel. In designing the flight test program we decided to go beyond just verifying the simulation, and try to

get some comparisons with the operation of the standard E-10 Autopilot - both quantitative and pilot reaction. Available to the flight test engineer, accordingly, there are on the test panel various switches and controls to allow a considerable variety of configurations to be set up. These included changes in some of the significant parameters of the adaptive system, such as the dither amplitude, filter characteristics, and gain changer operation. Provision was also made for the introduction of standardized pitch rate and pitch attitude commands.

Let us look at a few of the results of the flight test program, as carried out at Minneapolis in April of 1958. The first series covered operation without the gain changer, and with limiter output set to provide a maximum elevator deflection rate of 4.6 degrees per second. In Fig. 7 we see a typical recording. At the top is the command input, third down are the model response and the aircraft response. We find it convenient to record in opposite sense those quantities that are to be directly compared.

The performance shown is typical of the results obtained throughout most of the flight envelope. In the steady-state condition the traces are acceptably smooth, that is, the residual motion is quite small - on the order of what is obtained with the usual linear control system. The model-following capability is excellent, even at relatively high control frequencies. In other words, the aircraft's response does agree with the model's response. This is true for small-scale maneuvers; it is also true for violent maneuvers.

Fig. 8 shows flight test results of one of a series of violent maneuvers. This is a complete loop. During the short time taken to carry out the maneuver the altitude changed between 10,000 and 20,000 feet and the Mach number between 0.4 and 0.8. Note that the pitch rate is quite constant throughout the loop, as specified by the output of the model. Even cutting out the afterburner at the top of the loop produced only a small bump in the pitch rate record. We see here a very significant feature of an adaptive system of this type, that it can adapt quickly to rapid changes in operating conditions. There is no waiting for a scheduling device or computer to "catch up".

In Fig. 2 a switch was shown that provided pitch attitude control. This mode of operation was also checked in the flight testing program. The pitch attitude feedback gain was set by use of third-order charts to give an overshoot of 30%. As it turned out, the pilots' thought this overshoot a bit excessive, and we wished we had designed for a lower value. However, that was what we specified, and that was what we got, as the recordings in Fig. 9 show. At least, we got it at 0.6 Mach. At 0.4 Mach the initial overshoot was still about 30%, but the transient took a somewhat longer time to damp out.

This is not surprising, since the performance of the pitch-rate inner loop was expected to deteriorate somewhat at the lower dynamic pressures. The

object of all this work was, of course, to have the aircraft response always agreeing with the model response, no matter what the flight conditions were. A large part of the flight test program was, accordingly, devoted to determining just how the agreement did depend on flight condition. For small variations the measured discrepancies were found to be quite small. However, at the extreme ranges of the flight envelope, certain deteriorations were noted.

Fig. 10 shows results obtained at landing speeds. Under such low dynamic pressure conditions the response is that of a system with lower damping than that of the model. When the input is cycled rapidly there are both amplitude changes and phase shifts.

Fig. 11 shows results obtained at high speeds. With high dynamic pressures the model-following capability is quite good, but the amount of residual motion in the steady state is only marginally acceptable.

Fig. 12 shows a summary of the first tests covering the flight envelope. Throughout most of the operating range excellent control and stability were obtained. However, at low dynamic pressure there is a small following error, and at high dynamic pressure there is an objectionable amount of residual motion.

The results described so far all show operation without the gain changer, that is, the output of the relay was always limited at 4.6 degrees per second elevator rate. This was the compromise value that had been worked out in the simulation studies. Better results at low speed would be expected if the limiting rate were higher. Flight test results confirming this are shown in Fig. 13. At low dynamic pressures a limiting rate of 9.2 degrees per second gives noticeably more precise following of the model. Likewise, at high dynamic pressures a lower limiting rate of 2.3 degrees per second reduces the steady-state residual motion to an entirely acceptable degree.

These flight test results confirm the simulation results, and show that a four-to-one change in limiting elevator rate will provide fully acceptable performance of the adaptive flight control system over the entire envelope of the F-94C. It was for this reason that the gain changer previously mentioned was added in the simulation studies. With properly chosen constants it produced the desired results in simulation. Unfortunately, the aircraft's availability schedule did not permit adequate investigation of gain-changer performance in the flight-test program reported here. Subsequent tests at Wright Field by the Flight Control Laboratory showed that the gain changer did, in fact, do what it was supposed to do in cleaning up performance at the edges of the flight envelope. A slightly different way of accomplishing this function is used in the later equipment described in a companion paper.

PHILOSOPHY

What I've been giving is a highly summarized review of our activities up to and through the F-94C flight tests. I haven't tried to mention all the disappointments and blind alleys. Some of these are described in our various reports, others we would just as soon quietly forget. I do think, however, that it's worth pointing out the research role, since it is over five years since we consciously set our hands to learning how to make a self-adaptive automatic control system. Since that time there has been maintained, on company funds, a consistent effort in this direction. Furthermore, it's worth pointing out the value of trying to help the military solve its problems, in that the support thus made available facilitated getting an early physical proof of the research theory.

How, then, do we view the future? We see adaptive controls as typifying the nature of the future of automatic control. From a practical point of view we see adaptive control advantageous primarily as a means of achieving a desired level of equipment and performance with superior reliability. After reliability and reliability and reliability, there come weight, size, and cost. These are all considerations that apply strongly to airborne equipment, but they're also of significance in ship-borne and ground-borne equipment, including process control and environment control.

From a theoretical point of view, adaptive controls form a particular class in the broad field of non-linear controls. We feel we must continue to do research work in this field, and particularly in the adaptive class. Our current research program is so aligned, and we are hoping to expand it. There are two prongs to our research effort. One is to extend our theoretical understanding of the present approach in regard to higher order systems. The other is to continue to investigate approaches other than the one we are now using.

As a sort of side line, we have been intrigued for quite a while with the relationship of our mechanistic controls to those highly refined controls we find in the biological organism. A recent paper by our Charles Johnson, delivered at the Eleventh Annual Conference on Electrical Techniques in Medicine and Biology, discussed this relationship. Johnson showed that, when one lines up the characteristics of the human being as a controller, and those of the adaptive servomechanism, that has been described here, the parallelism is inescapable.

It is easy to say that the automatic control art has much to learn from biology, whose control art has been refined over millions of years. But it is somewhat dangerous, we feel, to take such a statement too seriously. The mechanisms involved are different, radically different. True, the biological controls are non-linear, they are digital, they are adaptive. But they use

physical elements that are not appropriate to our control devices. One cannot hope to design a flight control system by dissecting a bumble bee, and there is no known biological organism that is capable of space flight. Where we can learn from biology, and perhaps support the biologists themselves, is in terms of concepts. Conceptually, our adaptive control is parallel to that of the organism, but the mechanisation is quite different. Our aim, then, is to learn conceptually, and mechanize practically.

That statement tends to summarize Honeywell's philosophy in regard to automatic control and particularly adaptive control. We want to learn from every possible source, and we want to apply the knowledge we obtain to produce improved performance in practical situations. And the primary measure of practicality is reliability.

REFERENCES

1. Markusen, David L., Norton, John S., Pomeroy, Orville, P., " The Application of Some Non-Linear Techniques for the Improvement of Aircraft Beam Following" , WADC Technical Report 53-74, Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota, February 1953.

2. Young, Turrittin, Loud, Hess, Culmer and Putzer, " Non-Linear Control System Applications to Beam Following and Other Flight Control Systems" , WADC Technical Report 53-520, Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota, December 1953.

3. Stone, C. R. (ed), " A Study to Determine an Automatic Flight Control Configuration to Provide a Stability Augmentation Capability for a High-Performance Supersonic Aircraft" , WADC Technical Report 57-349 or Minneapolis-Honeywell Aero Report 48312, Final, Minneapolis, Minnesota, 30 May 1958.

4. Johnson, Charles W., Adaptive Servomechanisms, Paper No. 58 presented at the 11th Annual Conference on Electrical Techniques in Medicine and Biology, Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota, 25 November 1956.

ACKNOWLEDGMENTS

Many individuals have contributed to the developments discussed in this paper. Among those whose contributions were most direct must be mentioned: R. C. Alderson, D. L. Markusen, R. W. Bretoi, C. Ling, A. L. Ljungwe, J. T. Van Meter, R. C. McLane, L. T. Prince, R. C. Lee, R. C. Stone, C. W. Johnson, D. L. Mellen.

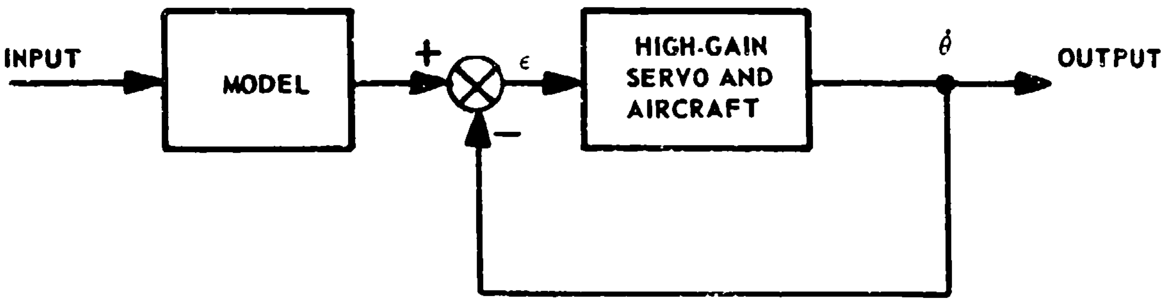


Figure 1. Adaptive Flight Control System (Elevator Channel Only)

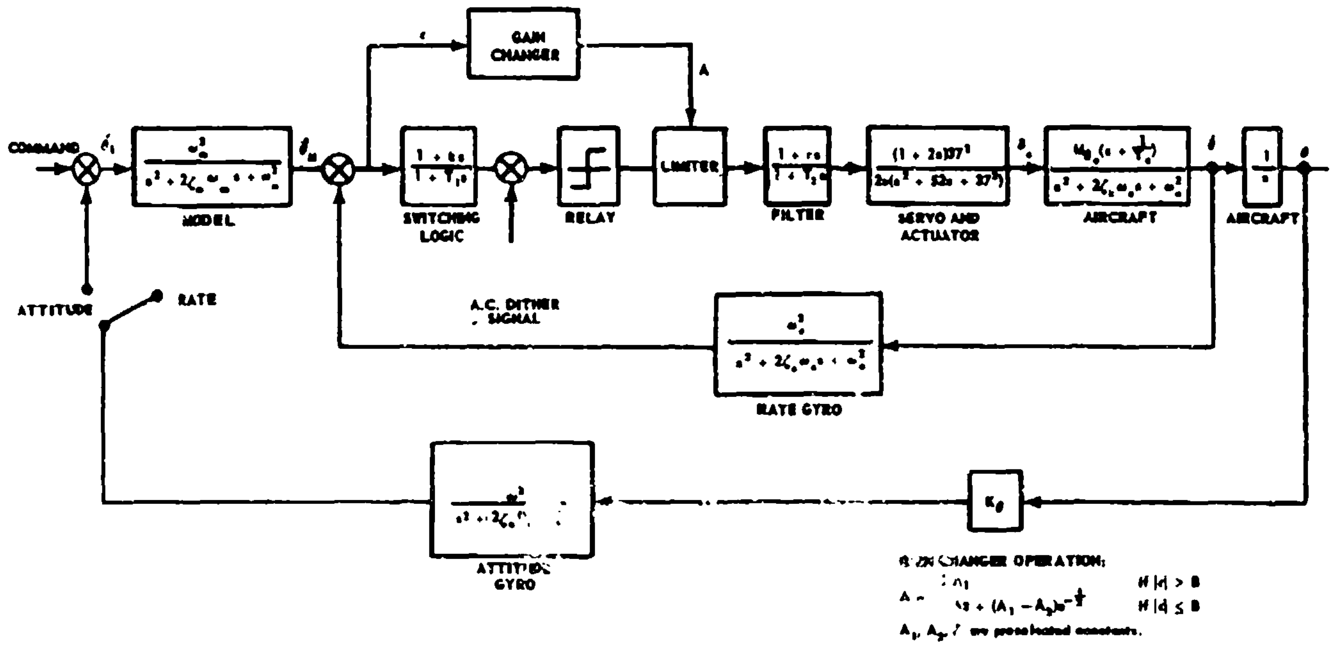


Figure 2. Block Diagram of Adaptive Control System for F-94C

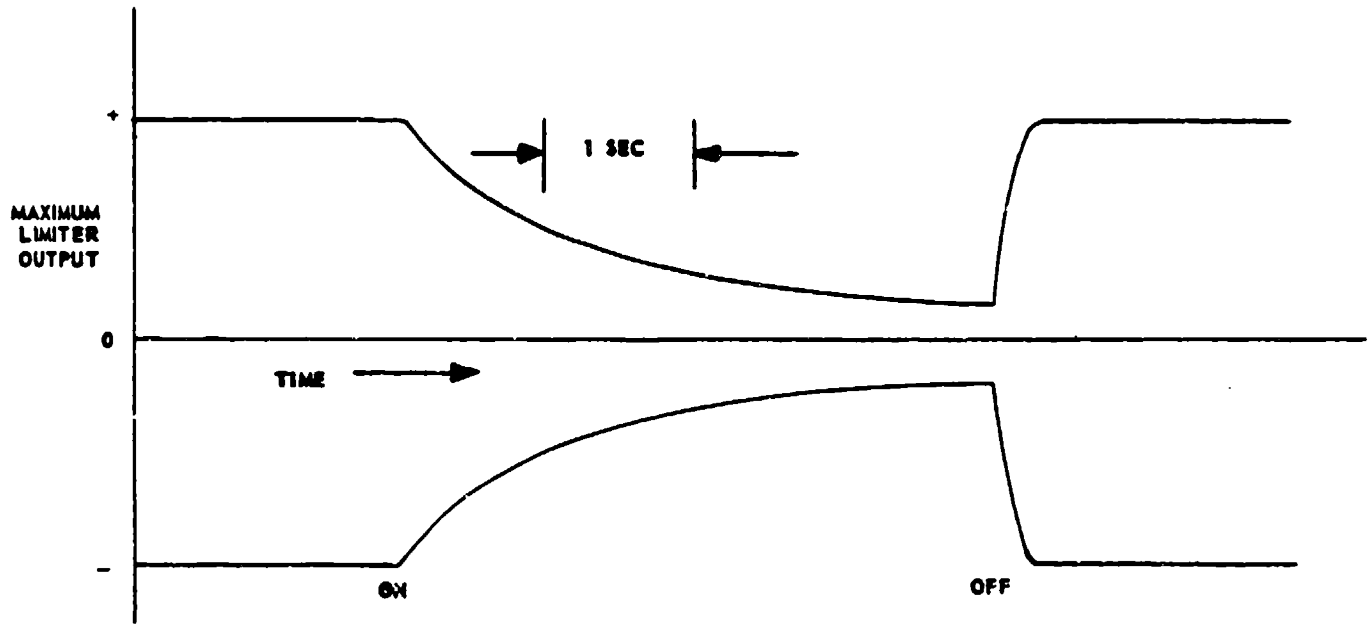


Figure 3. Effect of Gain Changer on Limiter

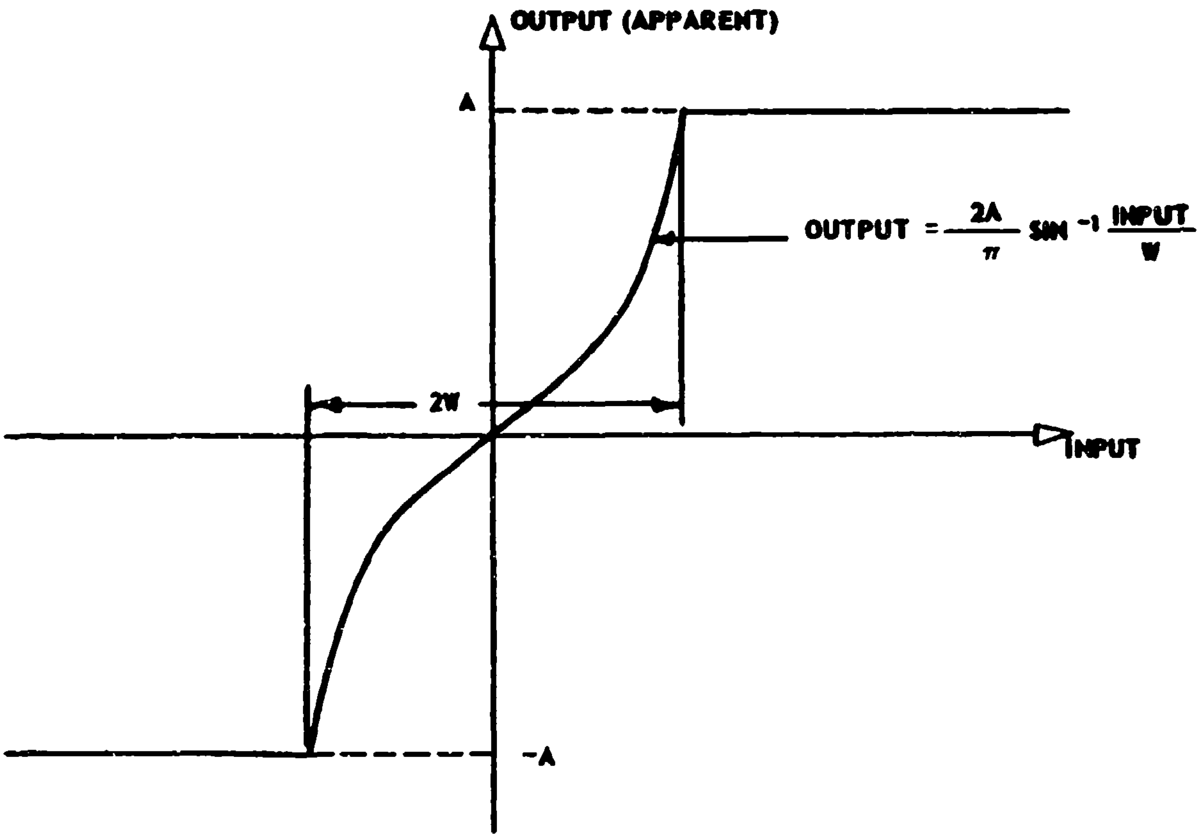


Figure 4. Relay Characteristic - Sine-wave Dither

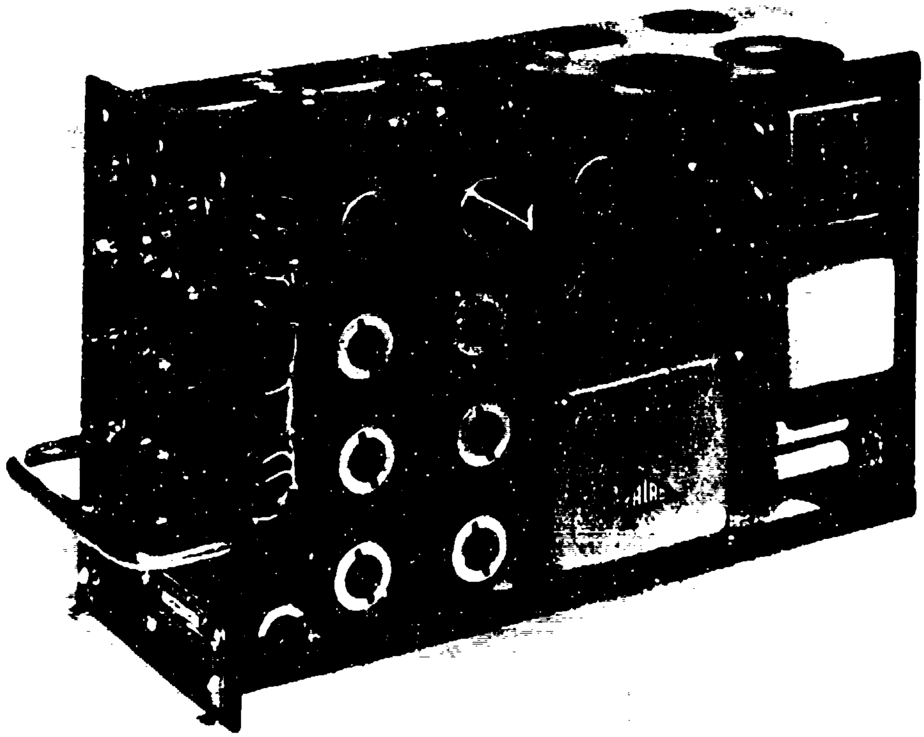


Figure 5. Adaptive Control System Amplifier

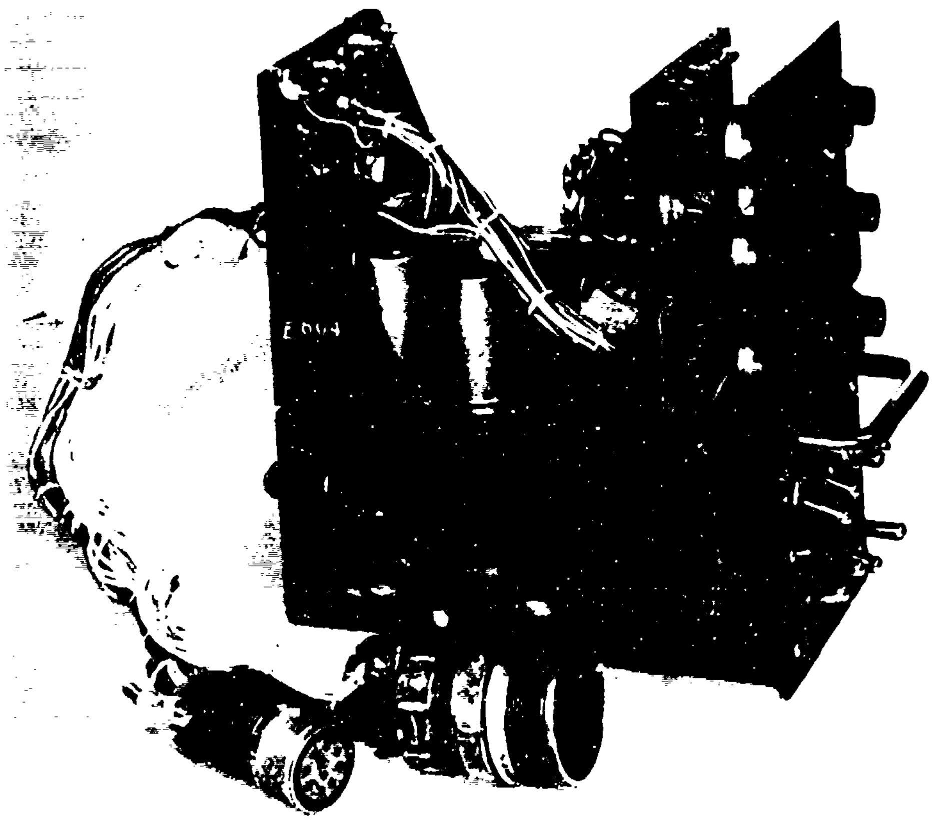


Figure 6. Engineer's Test Panel

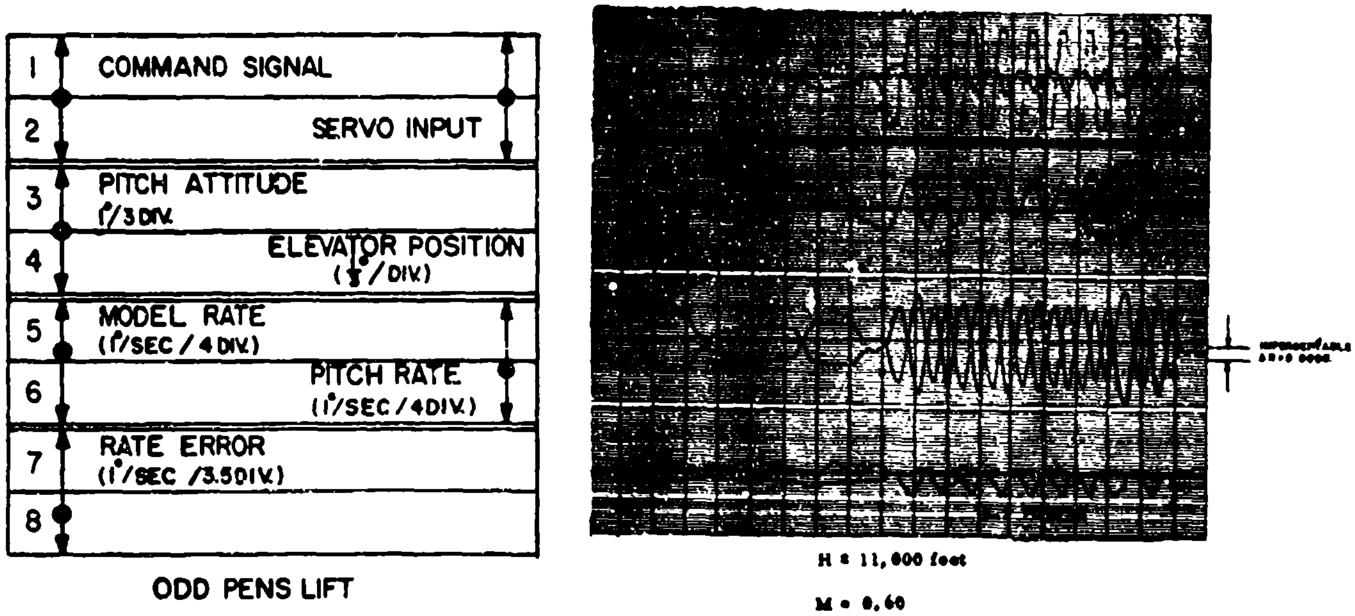


Figure 7 Typical Flight Test Response of F-94C with Adaptive Control System; Pitch Rate Commands

$\dot{\delta}_{e_{max}} = 4.6$ deg/sec

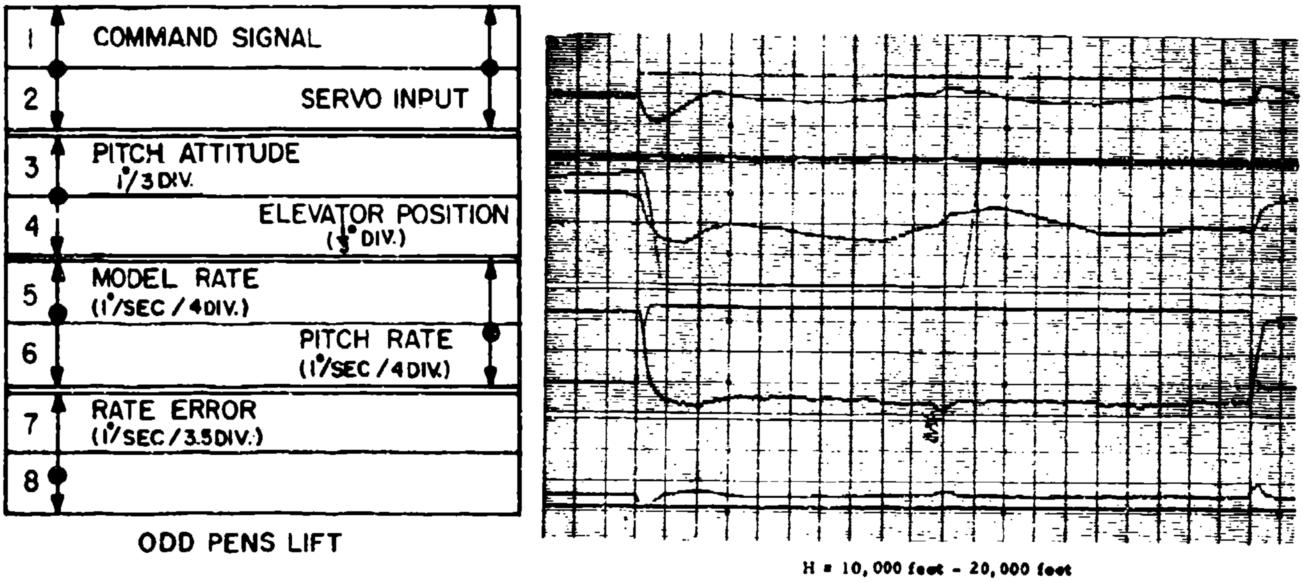


Figure 8 Typical Flight Test Loop

$\dot{\delta}_{e_{max}} = 4.6$ deg/sec

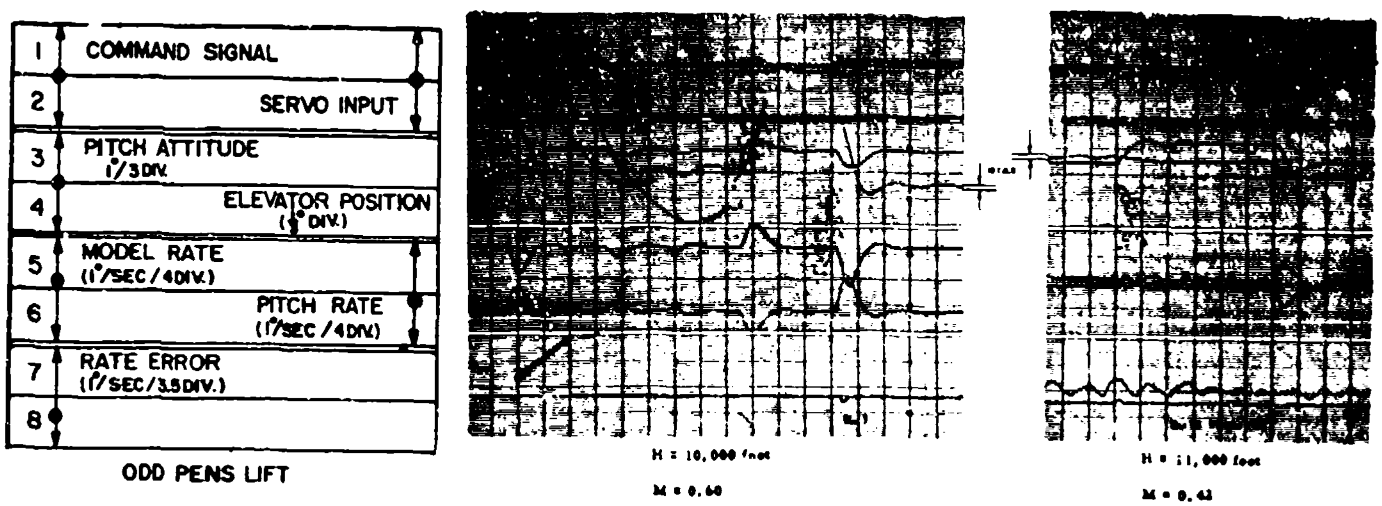


Figure 9 Typical Flight Test; Pitch Attitude Response

$\dot{\delta}_{e_{max}} = 4.6$ deg/sec

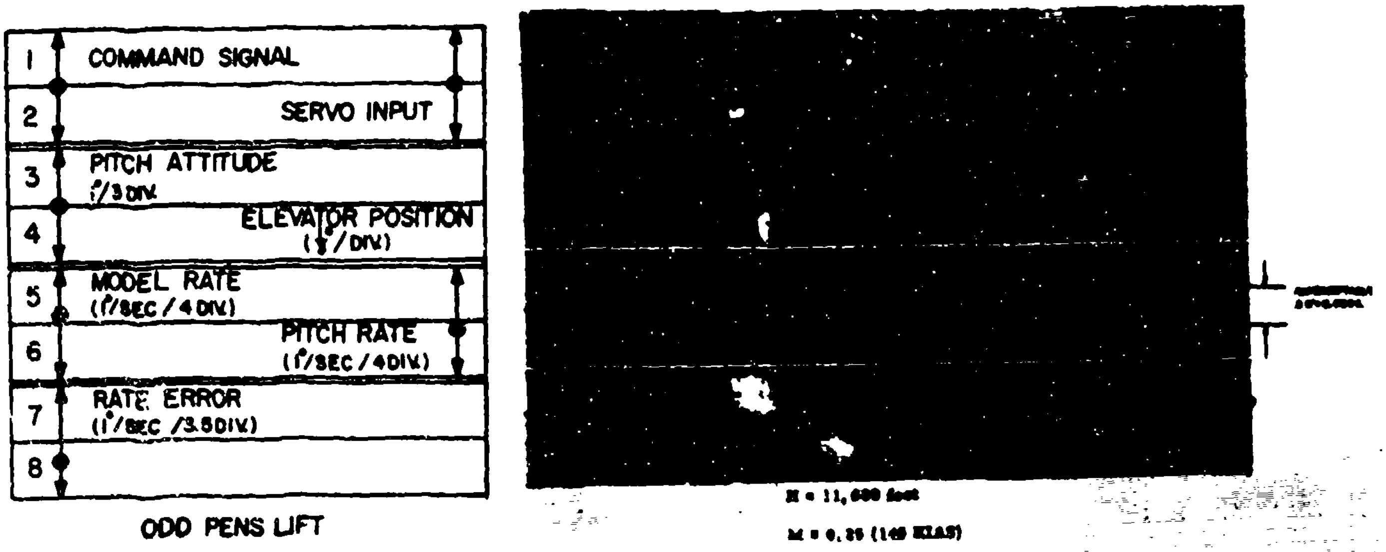


Figure 10 Typical Flight Test Response of F-94C with Adaptive Control System; Pitch Rate Commands

$\dot{\delta}_{e_{max}} = 4.6$ deg/sec

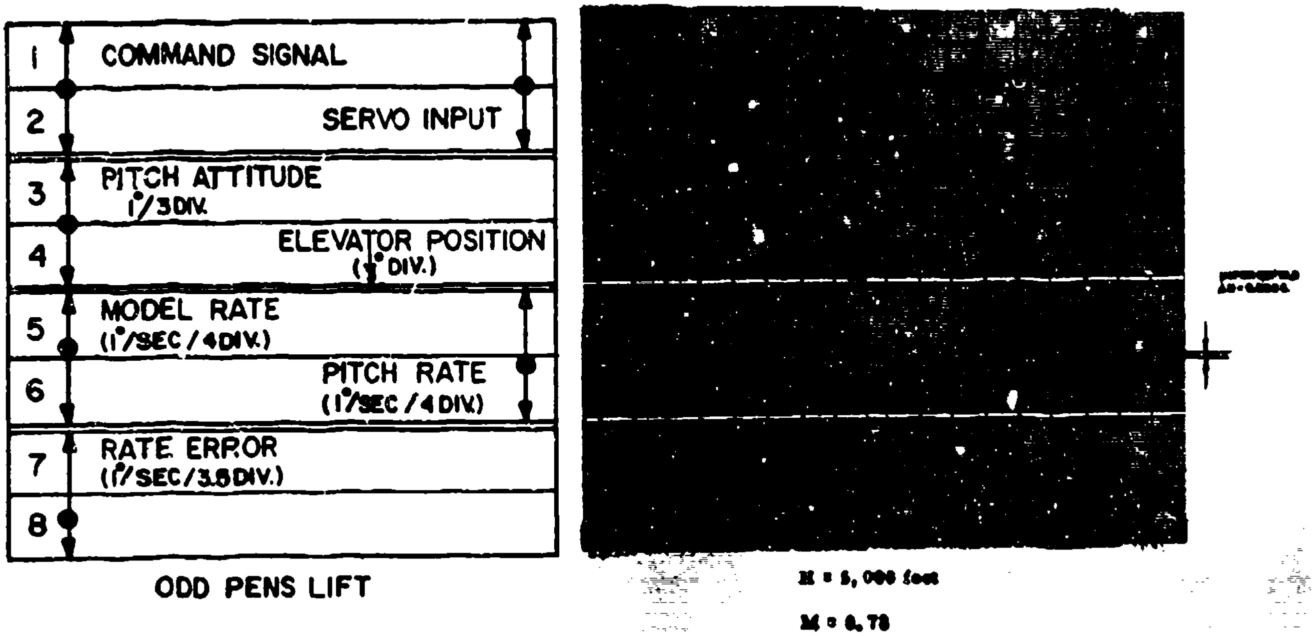


Figure 11 Typical Flight Test Response of F-94C with Adaptive Control System; Pitch Rate Commands

$\dot{\delta}_{e_{max}} = 4.6$ deg/sec

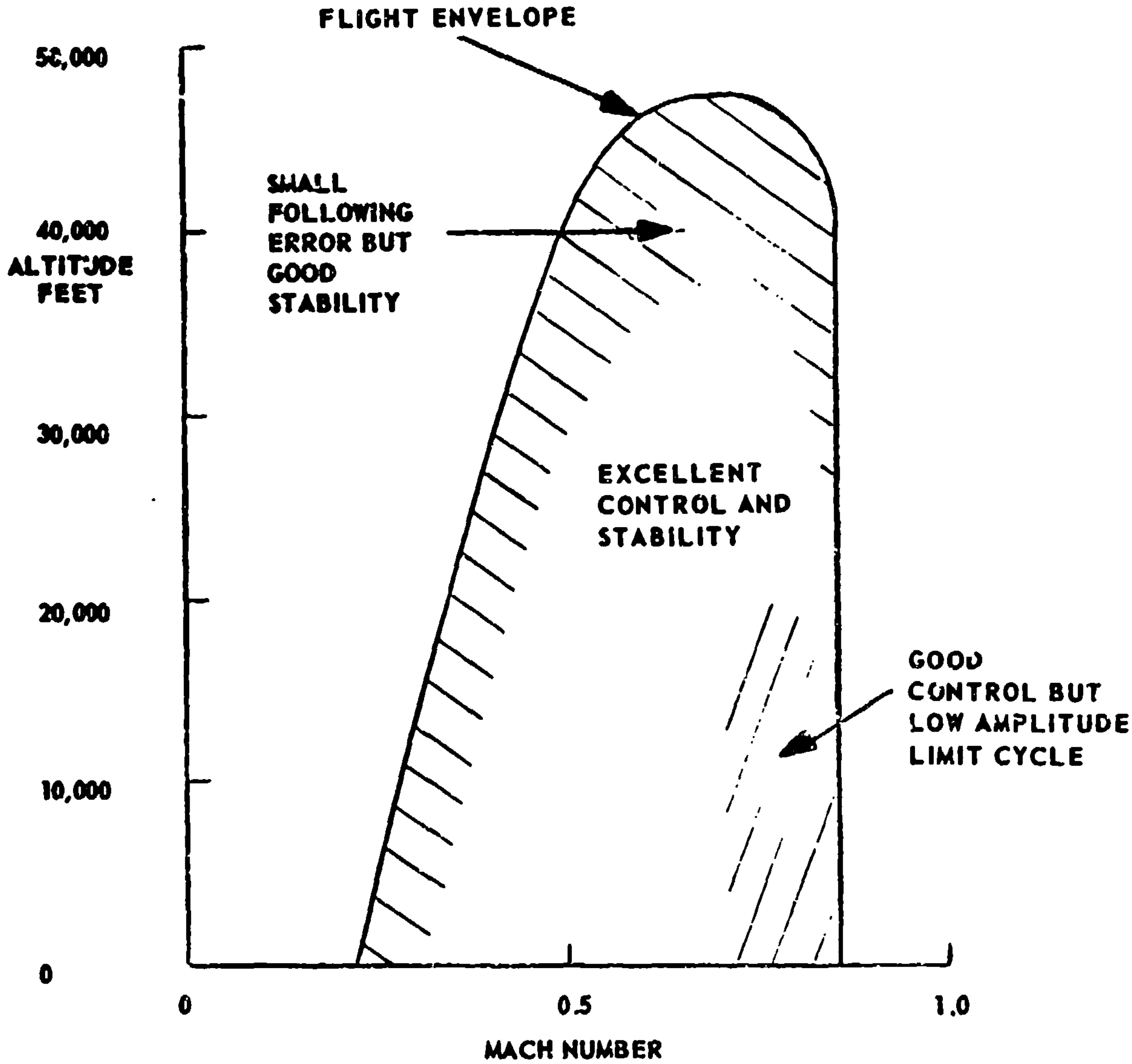
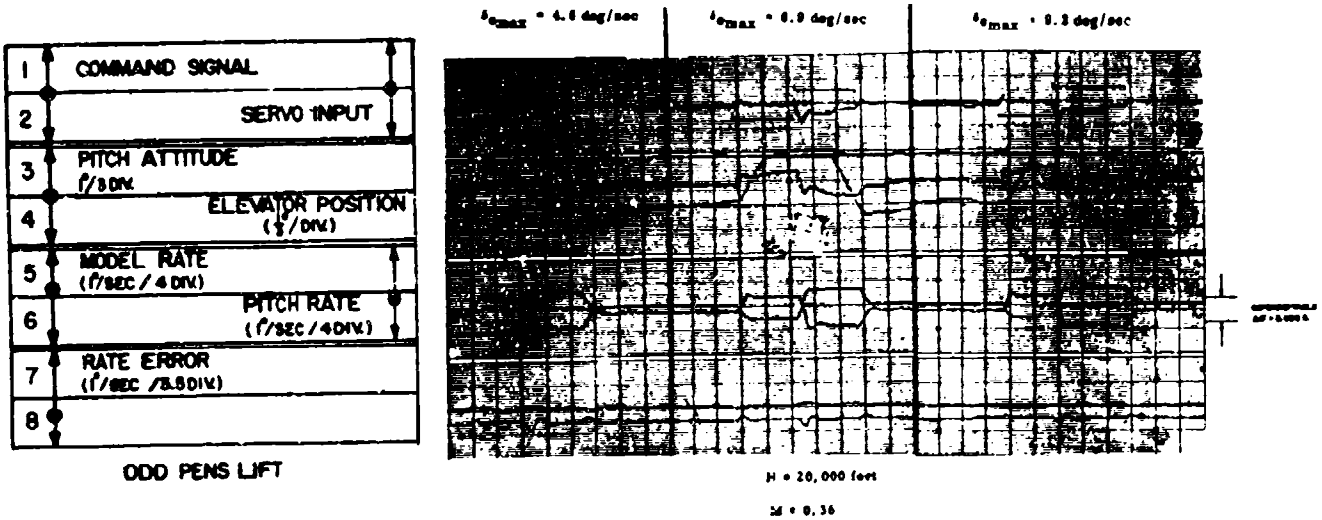


Figure 12 Summary of Adaptive Pitch-rate Control F-94C Flight Test Results (Minimum System - No Gain Changer)



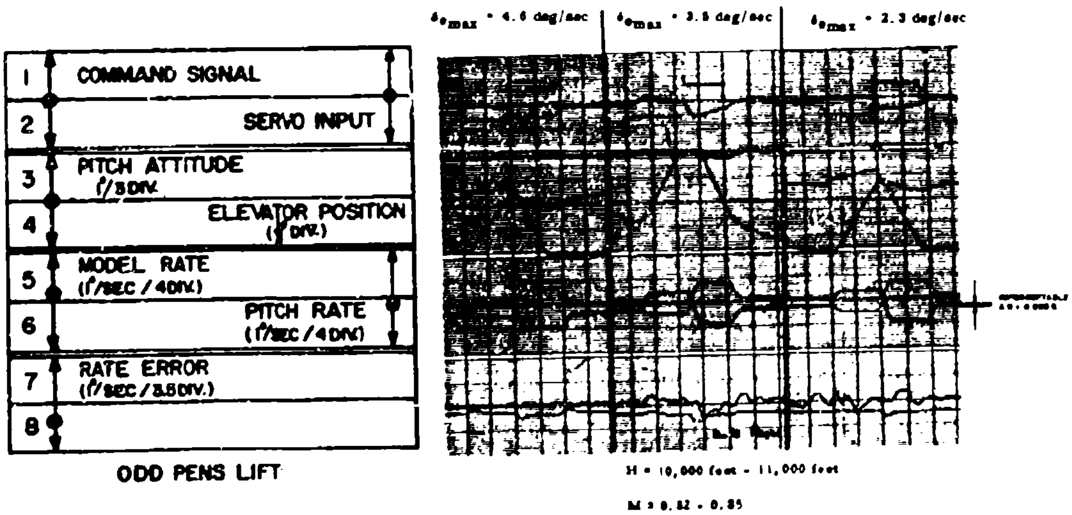


Figure 13. Flight Test Results of Effect of Limiter Amplitude

THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS SYMPOSIUM

SESSION IV

Dr. C. S. Draper, Chairman
Massachusetts Institute of Technology

Dr. C. S. Draper
Head, Department of Aeronautics
Head, Instrumentation Laboratories
Massachusetts Institute of Technology

I might tell one little story. Hugo has talked about mechanizing the human being or taking the human being as a model. Seeing General Davis sitting down here reminds me of how we determined the stability margin to put in the Day Fighter Sight, which became the A-4 sight. This was done by the two of us going up in various airplanes and making passes at targets on the ground such as barns and rocks. I suppose the most important accomplishment is that we scared a few people almost to death, including me a couple of times. As a reward for working for about two or three weeks we came up with the conclusion that the stability number should be two-tenths. I don't know what this cost but it didn't cost much. After big computors were turned loose on the job for two or three years, the conclusion was that the right number should be point two three and I will defy anybody to find a difference between these figures in practice. This is using a real cheap piece of machinery, a human being, to find out what you should do with these complicated systems.

A TECHNIQUE FOR ANALYZING AN ADAPTIVE FLIGHT CONTROL SYSTEM CONTAINING A BI-STABLE ELEMENT

Luther T. Prince, Jr.
Minneapolis-Honeywell Regulator Company

A highly desirable design objective for an adaptive control system is that the system have the capability for satisfactorily controlling aircraft which exhibit large and rapid changes in performance characteristics in certain flight envelope regions. This should be accomplished along with a significant improvement in the reliability of the flight control system. Minneapolis-Honeywell's approach to the realization of this objective utilizes nonlinear devices to obtain improvement in reliability by simplifying the basic control loop. The use of a bi-stable controller, as previously mentioned by Mr. Schuck, provides the means for accomplishing this as the possibility exists for combining a major portion of the system into one package. With this type of nonlinear controller the use of any type of mechanical gain scheduling in order to achieve the desired performance characteristics has not been necessary, and if a bi-stable servo actuator is used the adaptive controller and servo can be combined in one unit.

One of the greater difficulties associated with using nonlinear devices appears to be the lack of useful analytical techniques which provide a better understanding of the characteristics of such systems and insight into ways of more effectively utilizing such concepts and improving them. As a result of this difficulty, the development of a better understanding of this type of control system has been dependent, to a great extent, on using an imperical approach utilizing the results of computer studies and observations made during the actual operation of the system.

Realizing the value of mathematical analysis as a tool for synthesizing, evaluating, improving, and more thoroughly understanding various types of control systems, an attempt has been made to utilize nonlinear analysis techniques to gain a better understanding of the Honeywell adaptive concept. The results of this effort have been very encouraging and will be subsequently discussed.

Of the known techniques for analyzing nonlinear systems, the frequency response method proved to be the most promising. This can be attributed to several factors: (1) This method does not become increasingly complicated as the order (or complexity) of the system increases as does the phase space method, (2) the engineer is able to build on a technique commonly used in the analysis of linear systems which readily provides a better physical feel for the operation of the system, and (3) a simple technique for computing the closed loop frequency response of nonlinear systems already exists.

This technique is described in detail in Reference 1.

DISCUSSION

In order to demonstrate how this method can be used, consider the block diagram of the basic F-94C adaptive pitch rate control shown in Figure 1. Since the time response, $\dot{\Theta}(t)$, to a step input, $\dot{\Theta}_c$ (t), closely resembles the time response of the model, $\dot{\Theta}_M(t)$, when the system is operating satisfactorily, an investigation was made to determine if the frequency response of the complete system closely resembled the frequency response of the model over the same range of command inputs and flight conditions. To do this, the frequency response of the closed loop following the model is calculated and combined with the frequency response of the model. The results of this mathematical operation are compared with the model frequency response to determine what conditions must exist for the two functions to be essentially identical at control frequencies.

The successful application of this technique to the analysis of nonlinear systems is based on the validity of the following assumptions:

1. The bi-stable characteristic of the adaptive controller can be adequately described mathematically by an " equivalent gain" for sinusoidal inputs, i.e. the higher harmonics generated at the output of the nonlinear element can be neglected.

2. The bi-stable element is the only significant nonlinearity in the system.

3. Some correlation does exist between the frequency response and transient response of a control system containing a nonlinear element.

Since these assumptions do not generally hold for nonlinear systems, the valid application of this technique must be established by the degree of agreement between theoretical and experimental results. The correlation between experimental and theoretical results for the Honeywell adaptive control system flight tested in an F-94C is sufficiently good to justify the use of this technique as a means for gaining insight into the basic characteristics of the system.

Before discussing the results of this investigation, a brief outline of the approach used will be presented. Details of the basic approach can be found in Reference 1.

In order to conduct a sinusoidal analysis of the pitch rate adaptive control system shown in Figure 1, the bi-stable characteristic of the adaptive controller is represented by a variable gain, $k(\gamma)$, which is defined in Equation 1.

$$k(\gamma) = \frac{\alpha}{\gamma} \tag{1}$$

where γ is the peak value of any steady state sinusoidal signal which may exist at the input to the bi-stable element, and α is the peak value of the output, which will be approximated as a sinusoid. Since the output of this device has a constant magnitude and the same polarity as the input, the gain, $k(\gamma)$, is inversely proportional to the magnitude of the input, as seen in Figure 2. If a constant amplitude sinusoidal signal is introduced at the input to the model during closed loop operation, another sinusoidal signal, γ, of some undefined magnitude but the same frequency, will exist at the input to the bi-stable element. The magnitude of this signal defines the gain $k(\gamma)$ for a constant α, as seen in Equation 1. If the input signal frequency is changed, γ and thus $k(\gamma)$ will have a new steady state value.

In fact, a unique value of $K(\gamma)$ will exist for each magnitude and frequency of the input sinusoidal signal to the model. If the exact values of this nonlinear gain can be established for specific values of the input signal, then the closed loop frequency can be calculated with linear equations at each frequency by substituting the appropriate value of $K(\gamma)$. The equation used to perform the necessary calculations is shown below and can be derived very simply from Figure 1.

$$\frac{\dot{\Theta}}{\dot{\Theta}_c}\left[j\omega,\ k_n,\ (\gamma)\right] = \frac{\left[K_m G_m\ (j\omega)\right]\ K_1 G_1\ (j\omega)\ \ k_n(\gamma)\ \ K_2 G_2\ (j\omega)}{1 + K_1 G_1\ (j\omega)\ \ k_n(\gamma)\ \ K_2 G_2\ (j\omega)} \tag{2}$$

where the subscript n denotes the particular value of $k(\gamma)$ that exists for a specific value of the magnitude and frequency of the input sinusoidal signal. Two distinct operations are necessary to calculate the closed loop frequency response for a given input signal. First, the correct gain of the bi-stable element must be established, and second, the gain and phase of the frequency response function are then determined from Equation 2. These operations must be repeated for as many points as are needed to obtain adequate information. The resulting function will be valid for only one particular amplitude of the input signal because the response of a nonlinear system is amplitude dependent; consequently the procedure must be repeated for each different magnitude of the input signal.

The usefulness of this technique as an engineering tool would be somewhat limited if separate mathematical operations were required to obtain each point on the many frequency response functions that may be needed. The basic method used during this investigation utilizes a graphical technique to greatly reduce the number of necessary calculations.

In order to demonstrate how the closed loop frequency response of the nonlinear portion of the Honeywell adaptive control system is obtained, it is

useful to examine a family of linearized frequency response functions of that portion of the system following the model in the block diagram of Figure 1. These curves are calculated using several constant values of the nonlinear gain k(γ). A typical family of these curves, shown in Figure 3, was calculated from the F-94C pitch axis configuration. (The criteria used to select the maximum gain will be discussed later).

If a sufficient number of these curves is plotted over the range of possible values of k(γ), then points on the actual closed loop frequency response for a specific input signal will exist somewhere in this group of curves. These points can be located exactly by establishing the correct value of k(γ) at specific frequencies of the fixed amplitude input signal.

OPEN LOOP FREQUENCY RESPONSE

Before actually determining the frequency response of the closed loop following the model, it is useful to open this loop and consider some of the parameters in the open loop frequency response <u>exclusive of the model</u>. The magnitude of the open loop frequency response at the <u>particular</u> frequency where the open loop phase lag is 180 degrees is a factor of major importance in determining the closed loop response. It is useful to consider the open loop gain at this frequency to be composed of a constant term and a variable term. The later quantity will be the adaptive controller gain, k(γ). Open loop frequency responses of the F-94C pitch rate system of Figure 1 are shown in Figure 4 with all quantities included except the bi-stable element and the model. Aerodynamic data for flight conditions 1, 3, and 10 have been used, representing the landing condition, sea level - Mach .86, and 22,000 ft - Mach .86 respectively. From Equation 1 it is seen that the bi-stable element gain, k(γ), can assume any magnitude from infinity to values approaching zero. In Figure 4, it can be seen that, exclusive of k(γ), the gain margins of the system for the three flight conditions are 28, 36, and 51 db respectively. When this loop is closed the bi-stable element gain will establish itself at some steady state value. It can be shown that the actual steady state gain established in the bi-stable element is exactly equal to the magnitude of the gain margin at each flight condition.

At the frequency where the open loop phase lag is 180 degrees the system will exhibit a stable limit cycle. Since this residual motion always exists, if the adaptive controller characteristics are as shown in Figure 2, <u>the steady state gain of the bi-stable element can never exceed the value which establishes this limit cycle.</u> Consequently, the inclusion of this device provides the system with a variable gain which will always establish itself under steady state conditions at the maximum value that the system can have at any given flight condition.

In Reference 3, Ljungwe shows that the amplitude of the limit cycle motion is directly proportional to the product of bi-stable element output and the

surface effectiveness, $M_{\delta e}$. It can further be shown that the limit cycle amplitude decreases as the limit cycle frequency increases if $k(\gamma)_{Max}$ remains constant.

During satisfactory operation of the adaptive system the limit cycle can be superimposed on the response to input sinusoidal excitations so that no further consideration of its existence is needed to calculate the closed loop frequency response other than to establish the maximum value of $k(\gamma)$. This means that the gain characteristics of the bi-stable element used in the F-94C can be accurately represented as shown in Figure 5 for sinusoidal signals less than or equal to twice the cut-off frequency of the model.

Before concluding the discussion of the open loop frequency response it should be noted that the characteristics of the airplane have no effect on the limit cycle frequency in a pitch rate system. This is true because the 90 degrees of phase lag introduced by the airplane at high frequencies is essentially cancelled by the phase lead in the switching logic; consequently, the limit cycle frequency does not change with flight condition, and it is determined primarily by the control lags in the system.

CLOSED LOOP FREQUENCY RESPONSE

As mentioned previously, a graphical technique is used to calculate the closed loop frequency response of the nonlinear part of the Honeywell adaptive system. A detailed discussion of the method will be presented in a forthcoming paper. The actual procedure is as follows:

(1) Calculate and plot families of linearized frequency responses of the closed loop portion of the adaptive control system. These are obtained by replacing the nonlinear gain $k(\gamma)$, by a constant for each curve. One of these should be the largest gain that can exist at each flight condition to be investigated.

(2) Calculate an open loop frequency response from the transfer function $K_2G_2(S)/K_mG_m(S)$, where $K_2G_2(S)$ represent the linear portion of the system between the bi-stable element and the output of the system.

(3) Overlay the open loop frequency response obtained in step 2 on the family of curves obtained in Step 1. The zero db lines for the two graphs are separated by the ratio $\alpha/\dot{\theta}_c$, where α is the output of the bi-stable element and $\dot{\theta}_c$ represents the magnitude of the sinusoidal input.

Typical results for Step 3 are shown in Figure 6 with the dashed lines representing the calculation made in Step 2.

The amplitude of the actual closed loop frequency response is determined by following the maximum k(γ) curve until this function is intersected by the open loop function for a specific value α/Θ_c. The response then follows the dashed line for higher frequencies as long as this function is less than the amplitude of the k(γ)$_{M_{ax}}$ curve. Curves for three values of this ratio are shown in Figure 6. Points on the phase curve can be found at each frequency where k(γ) is known. This occurs wherever there are intersections between the open loop function and members of the family of linearized closed loop curves. Typical results are shown in Figure 7. To obtain the total response of the adaptive control system, the frequency response of the model is combined with the frequency of the nonlinear system.

RESULTS

The simulation and flight test results of the F-94C system show that the response of the adaptive flight control system closely resembles the model over most of the flight envelope of the airplane. Under certain conditions, however, the response of the airplane differs from the model. Results of the theoretical investigation show that the frequency response of the complete system closely resembles the frequency response of the model under the same conditions that the transient response of the system is essentially identical to the transient response of the model. In Figure 8, typical re sults are shown for 3 values of the ratio α/Θ_c. It can be seen that very good correlation exists between the theoretical and experimental results and that both are almost identical with the model. Frequency responses of most flight conditions are essentially identical to the ones shown in Figure 7. Frequency response analyses of two other flight conditions are also included for which the transient response of the system does not closely follow the model for 6 and 12 db ratios of α/Θ_c. Frequency response for these two flight conditions are shown in Figures 9 and 10. It should be noted, however, that these frequency responses do closely resemble the model if the ratio α/Θ_c is equal to or greater than 20 db.

CORRELATION BETWEEN TRANSIENT AND FREQUENCY RESPONSE OF THE SYSTEM

Transient responses obtained from an analog computer study are shown in Figures 11, 12, and 13. It should be noted that the particular trend in the transient responses for different values of the ratio α/Θ_c is also evident in the frequency responses for the same ratios. An attempt was made to calculate the transient response directly from the frequency response of the adaptive control system using the technique described in Reference 2. The results were accurate only when the model dominated the frequency response at all

frequencies below the natural frequency of the model. Investigations of the correlation between frequency and transient response of non-linear systems is continuing.

CONCLUSIONS

The use of a bi-stable element in an adaptive control system greatly enhances the possibility of obtaining greater reliability through simplification of the control system. Inclusion of a bi-stable element in a feedback control system provides the system with a variable gain device which will always establish itself at the maximum value that the system can have at any given flight condition. Thus it can be said that the system is self compensating.

With this type of nonlinear device the tight loop required to minimize errors to commands is obtained without the stability problems normally encountered in high gain linear systems. This system exhibits two unique characteristics: for small errors the gain is large, for large error signals the gain is low. This means that the system will operate at maximum bandwidth for small errors (good following of the model) and yet will operate at low bandwidth with sufficient phase margin to provide good response to disturbance inputs such as gusts. In addition wide variations in the airplane static stability and damping have negligible effect on the performance of the system.

Uniform performance characteristics can be obtained for the F-94C system over the complete flight envelope if the ratio of $\frac{\alpha M_{\delta_e}}{\dot{\theta}_c}$ is equal to or greater than approximately 30 db, where α is the output of the bi-stable element, M_{δ_e} is the magnitude of the elevator surface effectiveness, and $\dot{\theta}_c$ is the magnitude of the pitch rate command. Since this ratio was not greater than 30 db for all F-94C flight conditions some deteriorations can be expected at flight conditions where the surface effectiveness is low.

Whenever a bi-stable element is included in a feedback loop, a limit cycle will exist. The limit cycle magnitude is directly proportional to the product αM_{δ_e}. Since α was held constant in the F-94C system the limit cycle increases in magnitude for high dynamic pressure flight conditions. In fact, the output of the bi-stable was established on the basis of the maximum acceptable limit cycle amplitude. Since this occurs at the highest elevator effectiveness flight conditions, a compromise in performance exists at the flight conditions having low elevator effectiveness.

One possible improvement is to vary the output of the bi-stable element so as to keep the product αM_{δ_e} constant at all flight conditions. If the limit cycle

amplitude is kept constant by varying the output of the bi-stable element as the surface effectiveness changes, the necessary value M_{δ_e} can be obtained. Extensive studies have shown a significant improvement in performance with this addition to the system. The principal disadvantage of this approach is that the surface motion required to sustain the limit cycle becomes large at flight conditions where the surface effectiveness is low.

A more desirable modification is to keep the adaptive controller maximum gain at the largest possible value at all flight conditions, with a very small limit cycle at the control surface, but with an adequate value of αM_{δ_e} so that the responses to commands will follow the model. A method for accomplishing this is being included in an adaptive 3 axis automatic flight control system which will be flight tested in the near future. A description of this system is to be presented by Mr. David Mellen of Minneapolis-Honeywell.

REFERENCES

1. A Generalized Method for Determining the Closed Loop Frequency Response of Nonlinear Systems, Luther T. Prince, Jr., AIEE Paper 54-280.

2. A Series Method of Calculating Control-System Transient Response from the Frequency Response, D. V. Stallard, AIEE Paper 55-192.

3. WADC Technical Report 57-349 (Part 3) 10 December 1957.

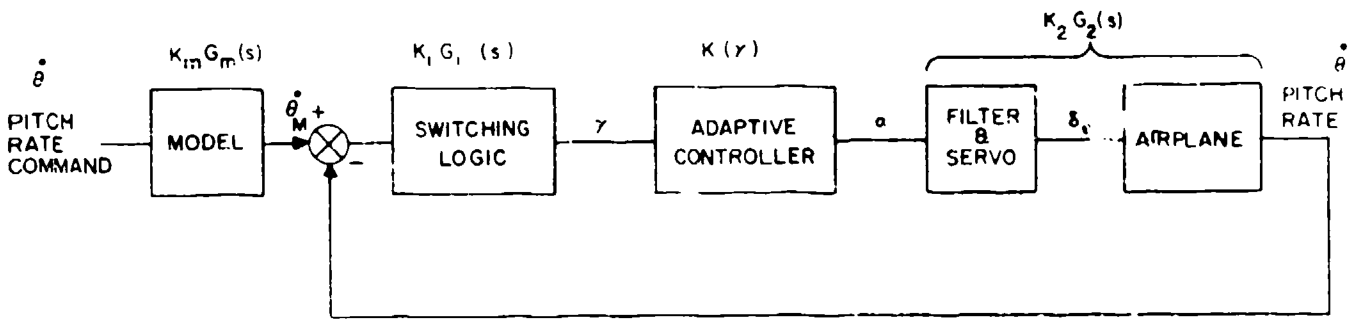


Figure 1. F-94C Adaptive Pitch Rate Control System

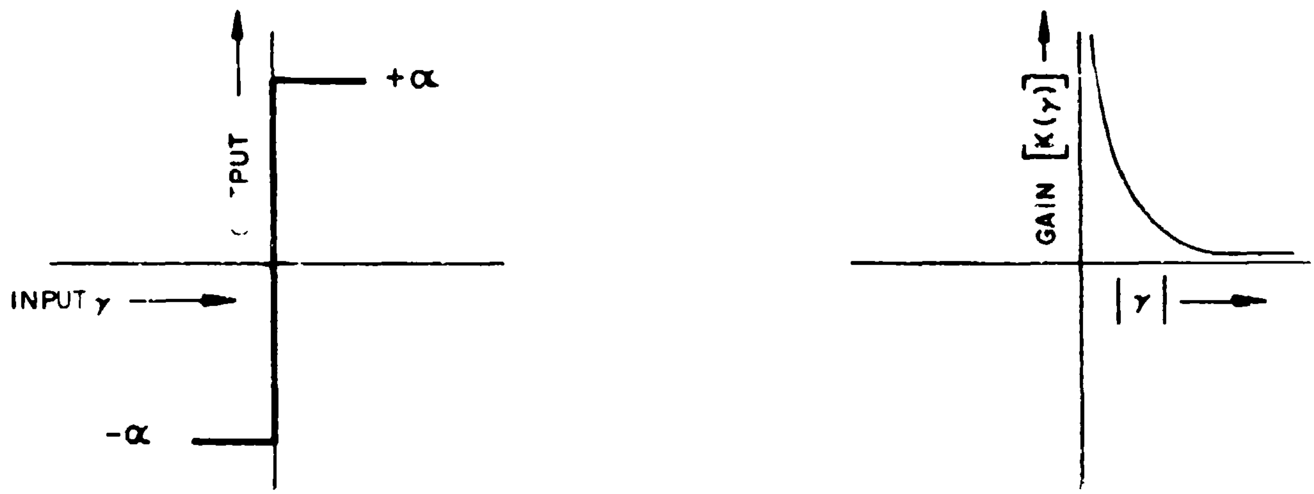


Figure 2. Bistable Element Gain Characteristics

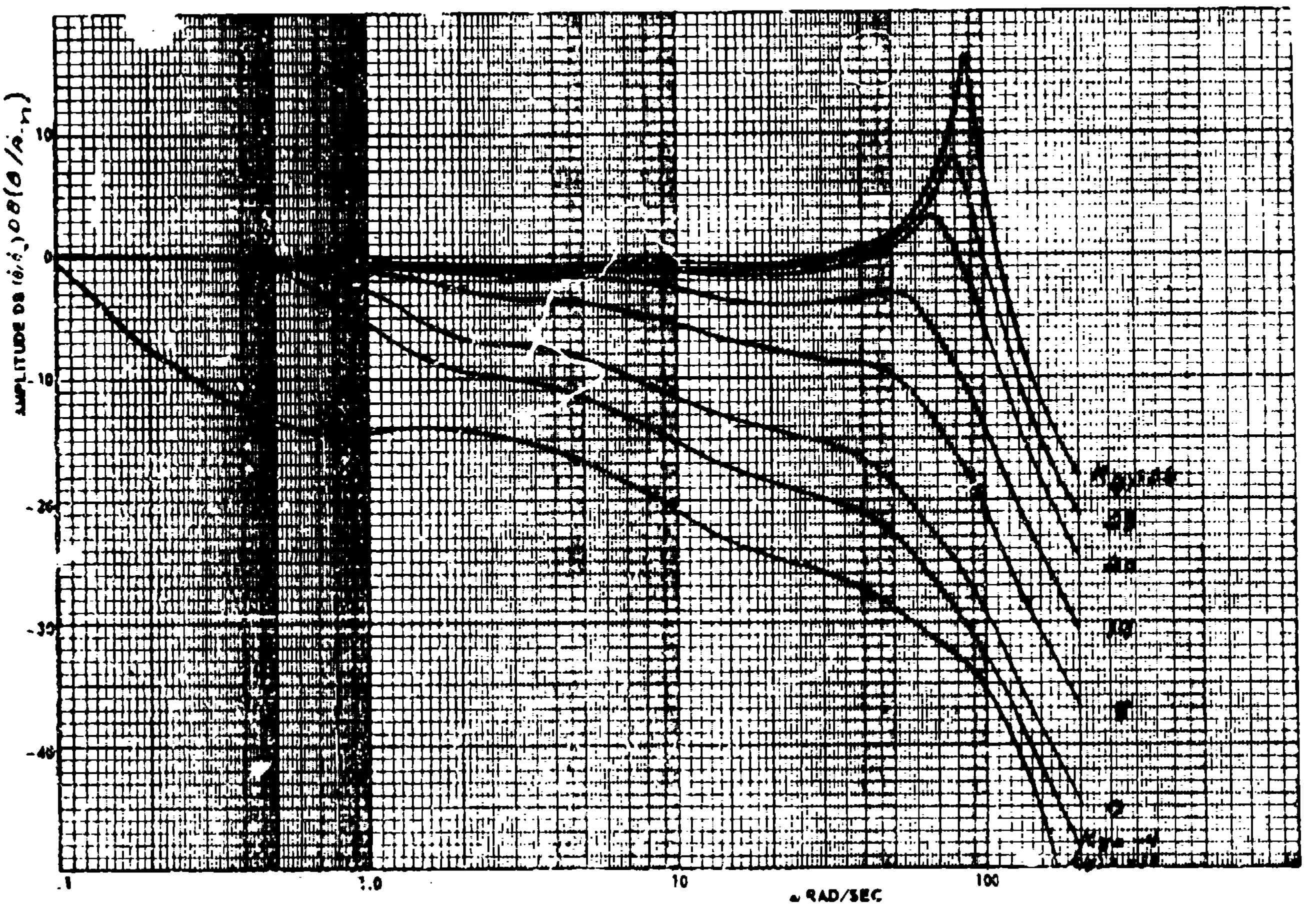


Figure 3a. Typical Family of Linearized Closed-loop Frequency Responses - Amplitude

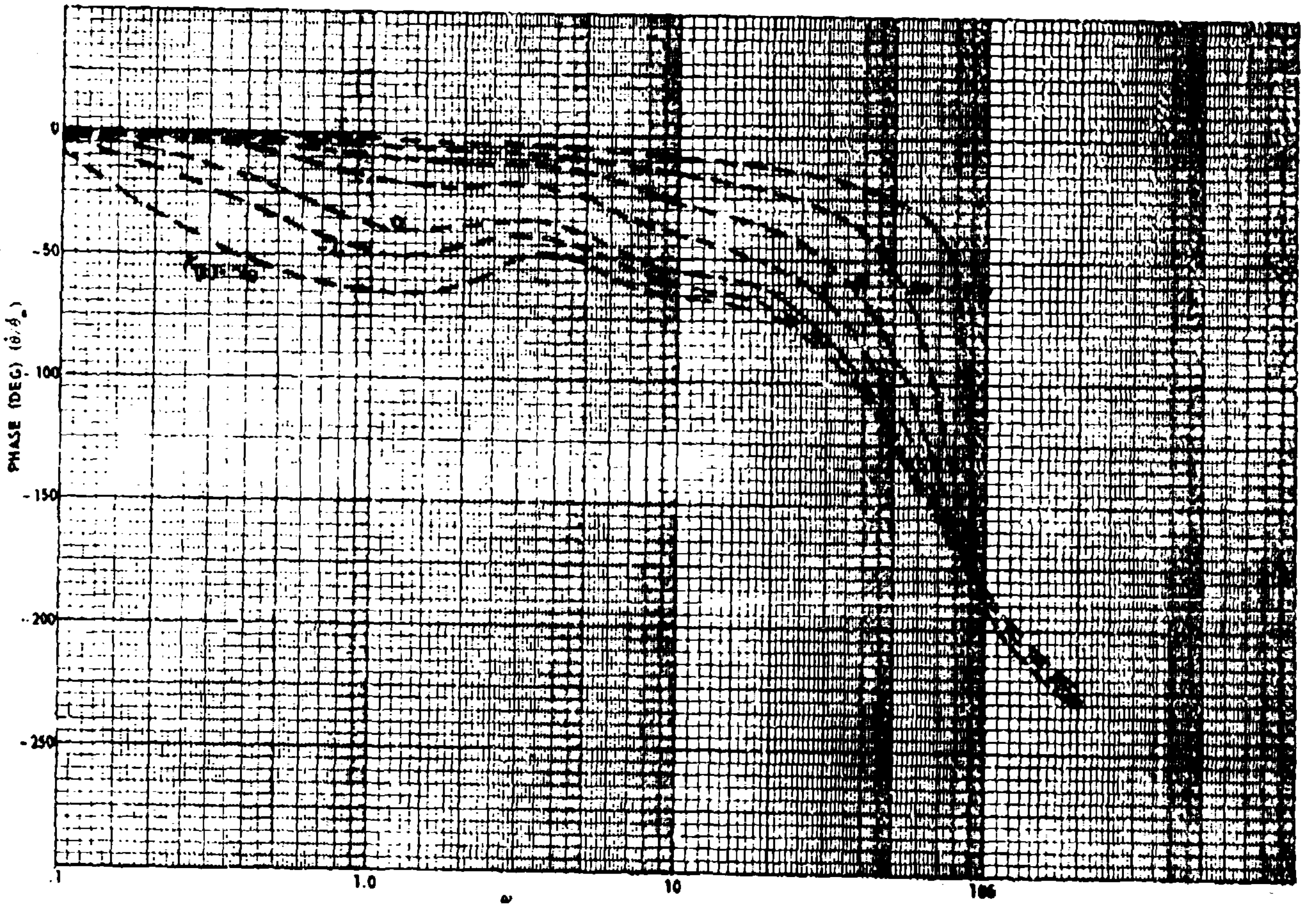


Figure 3b. Typical Family of Linearized Closed-loop Frequency Responses - Phase

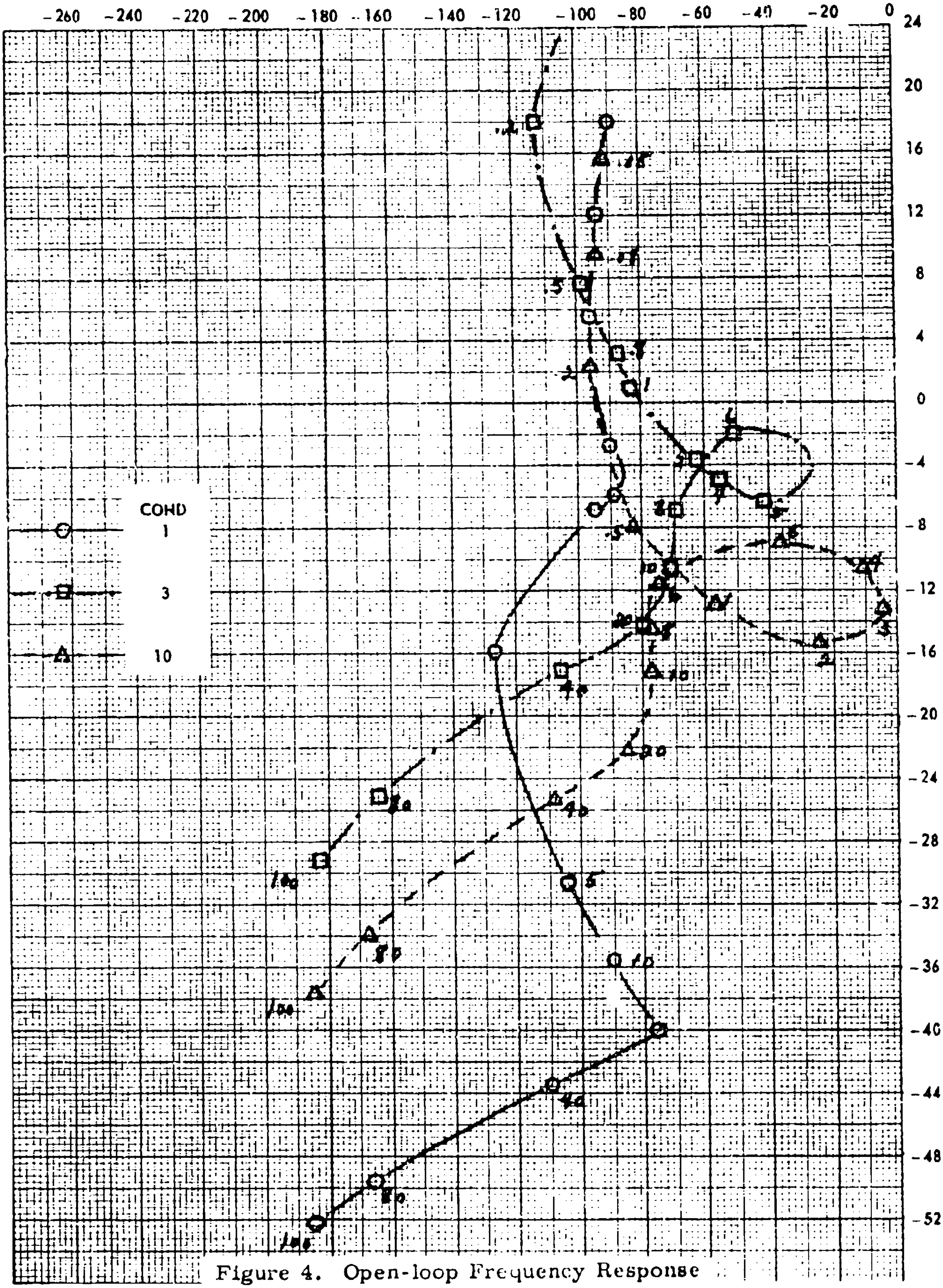


Figure 4. Open-loop Frequency Response

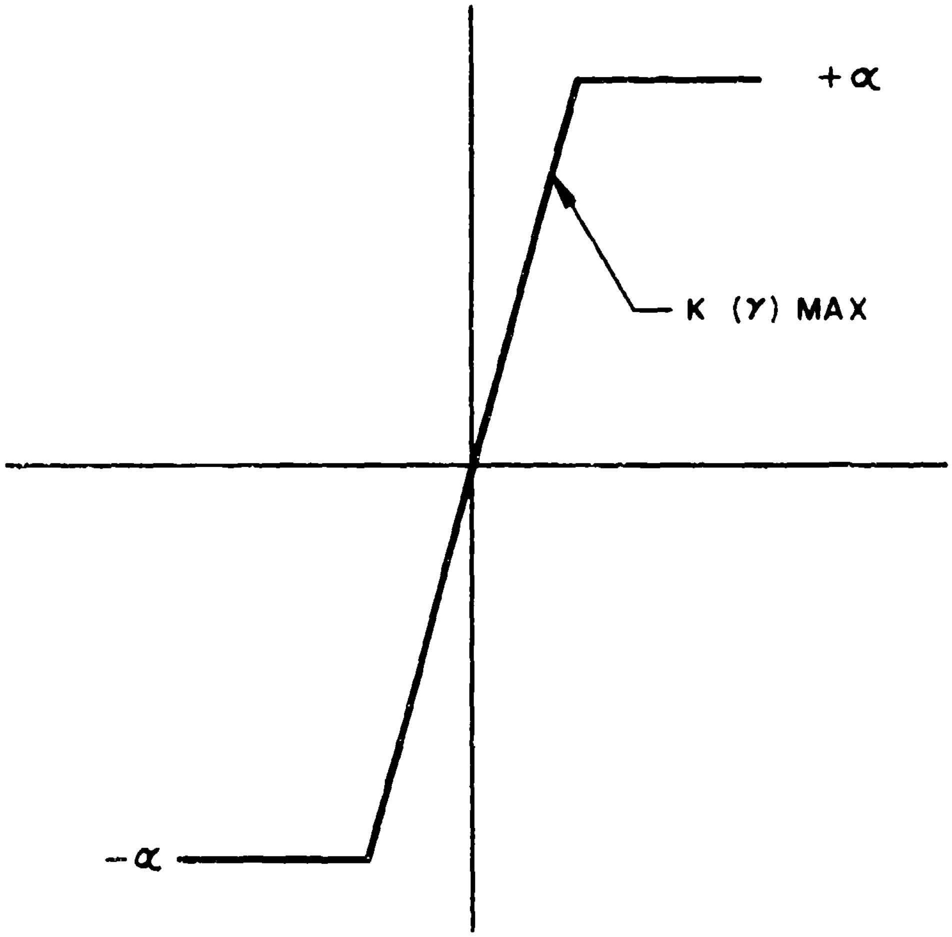


Figure 5. Modified Gain Characteristics of a Bistable Element

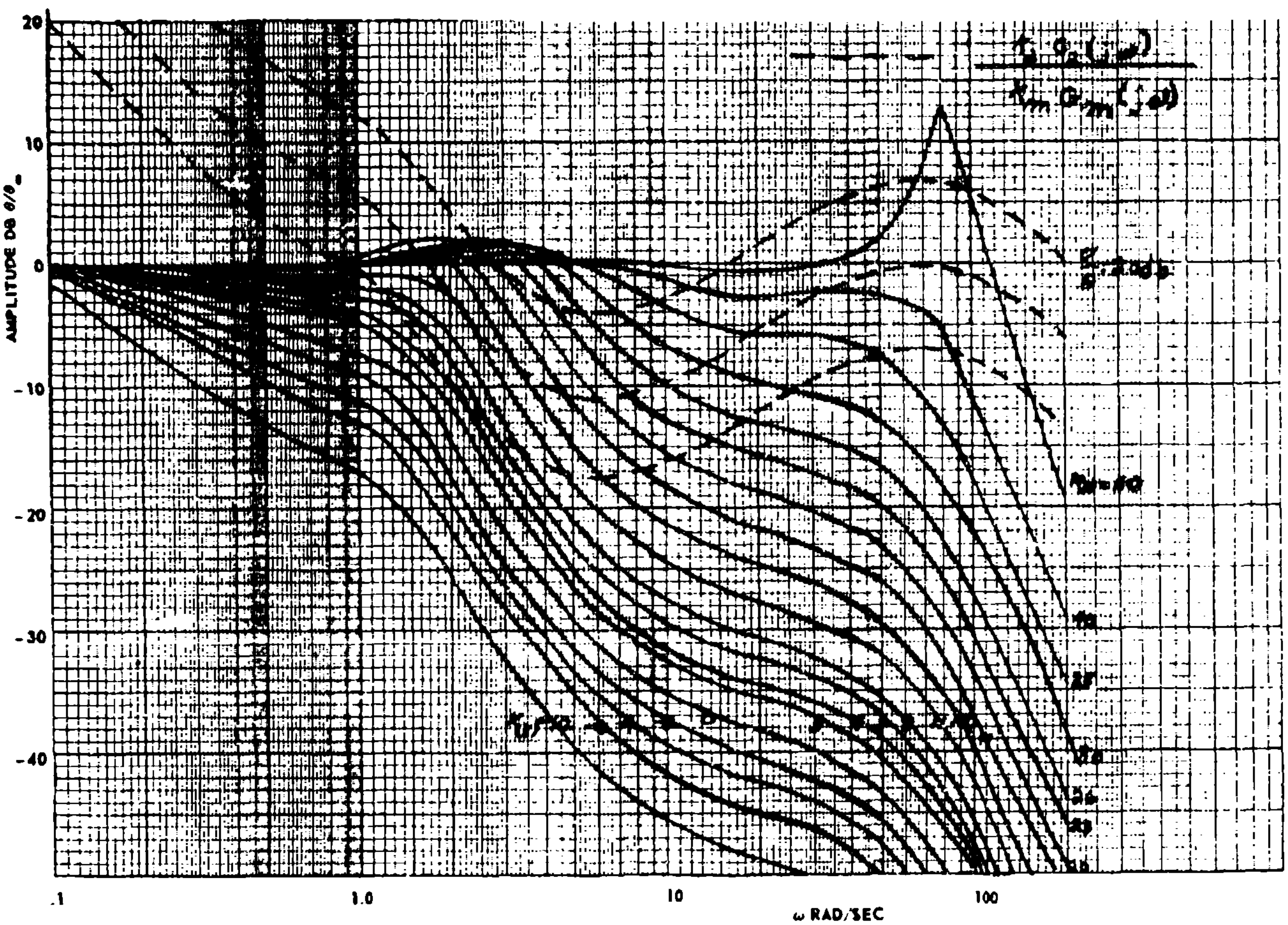


Figure 6. Graphical Solution of Bistable Element Gain during Closed-loop Operation

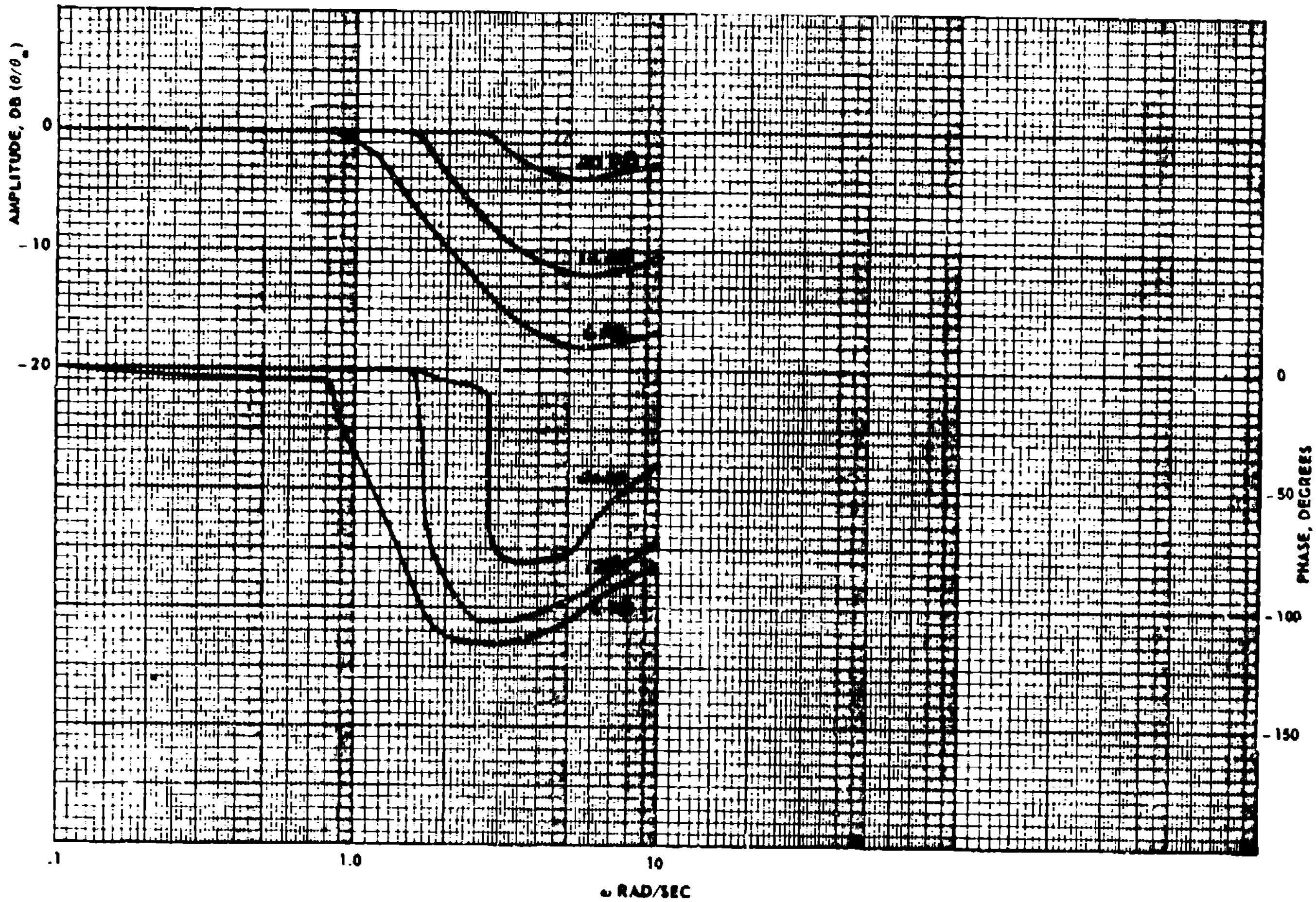


Figure 7. Typical Nonlinear Closed-loop Frequency Responses Excluding the Model

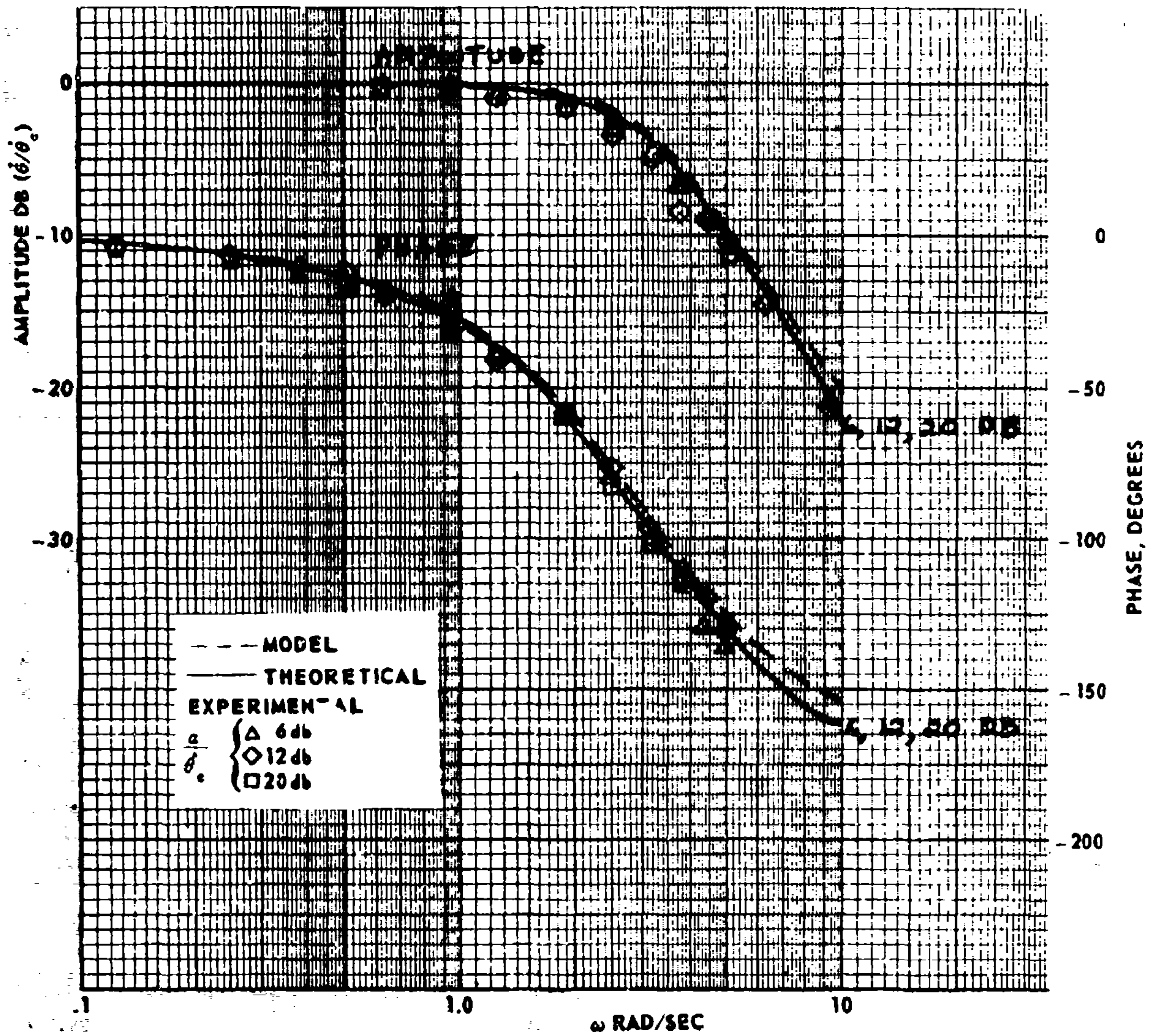


Figure 8. Frequency Response of the Complete Adaptive Control System - Condition 3

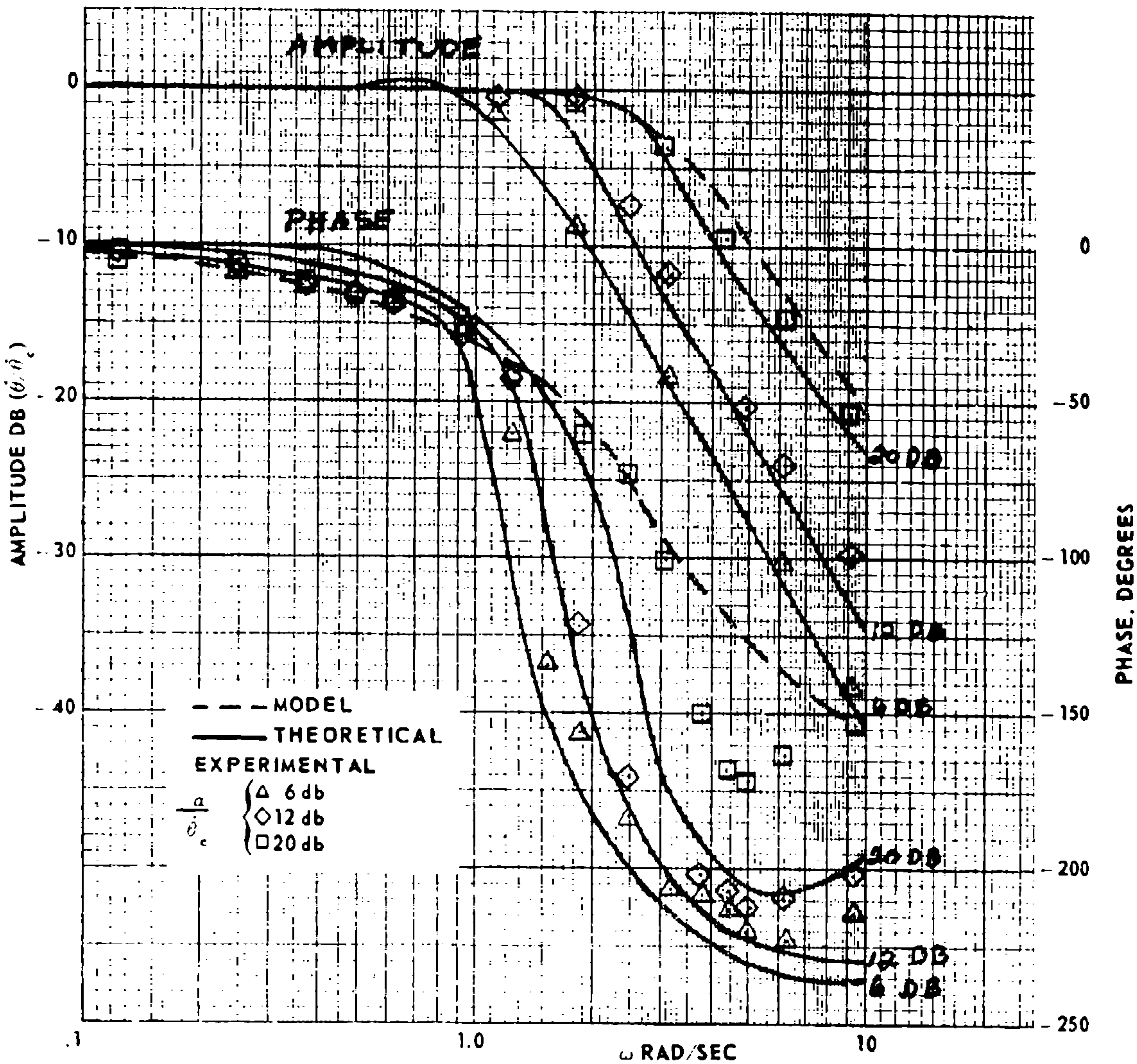


Figure 9. Frequency Response of the Complete Adaptive Control System - Condition 1

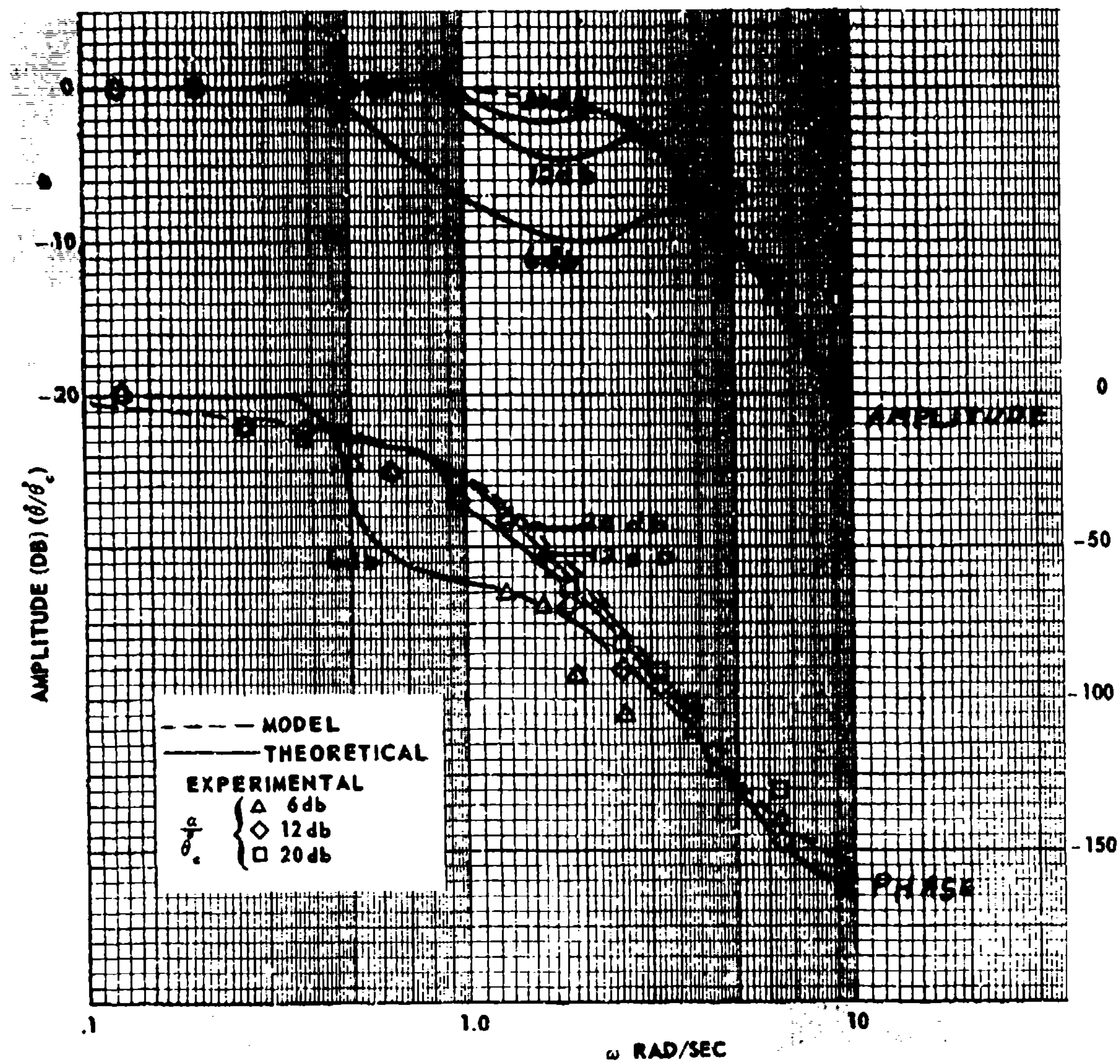


Figure 10. Frequency Response of the Complete Adaptive Control System - Condition 10

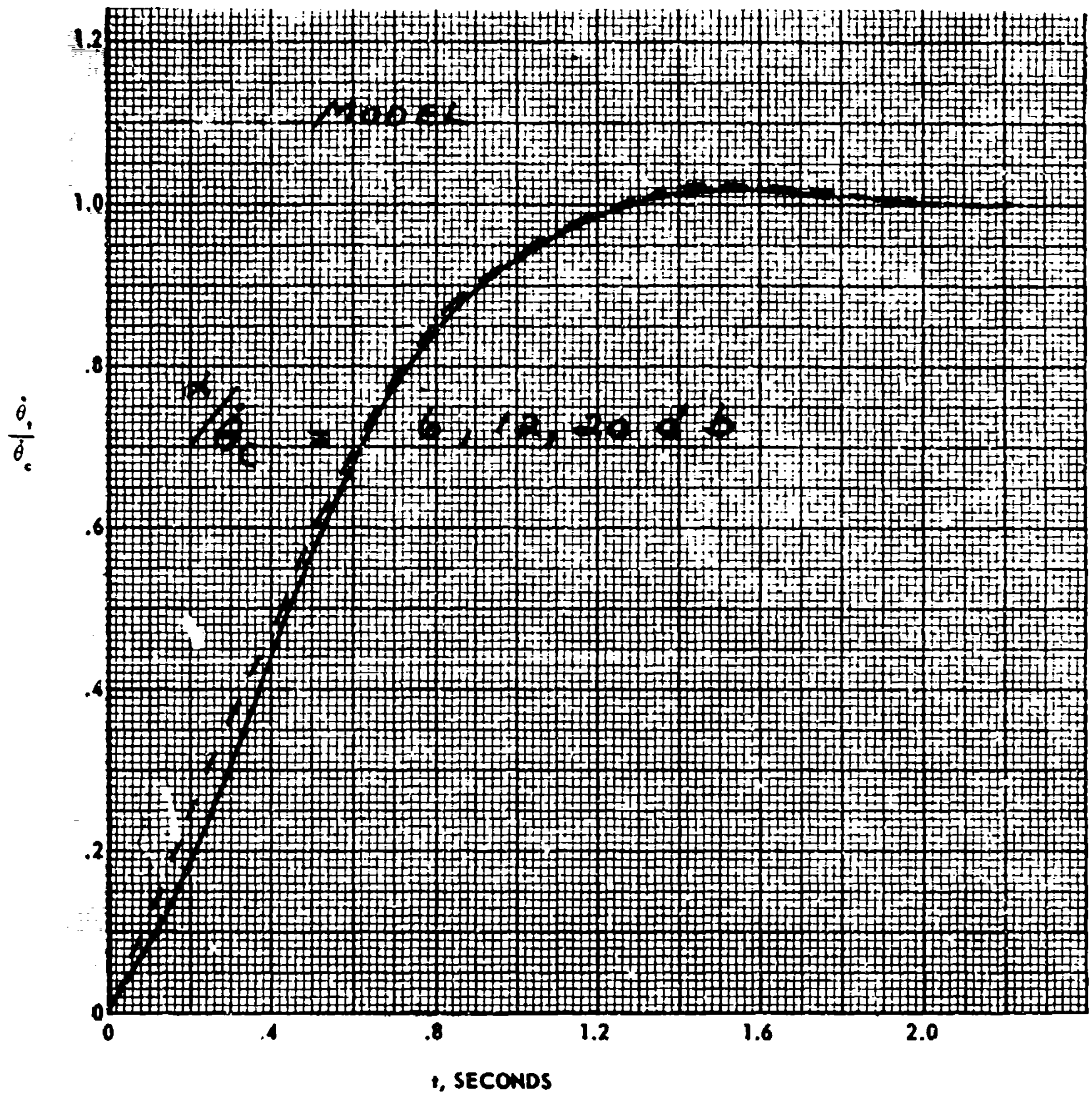


Figure 11. Transient Response of the Complete Adaptive Control System - Condition 3

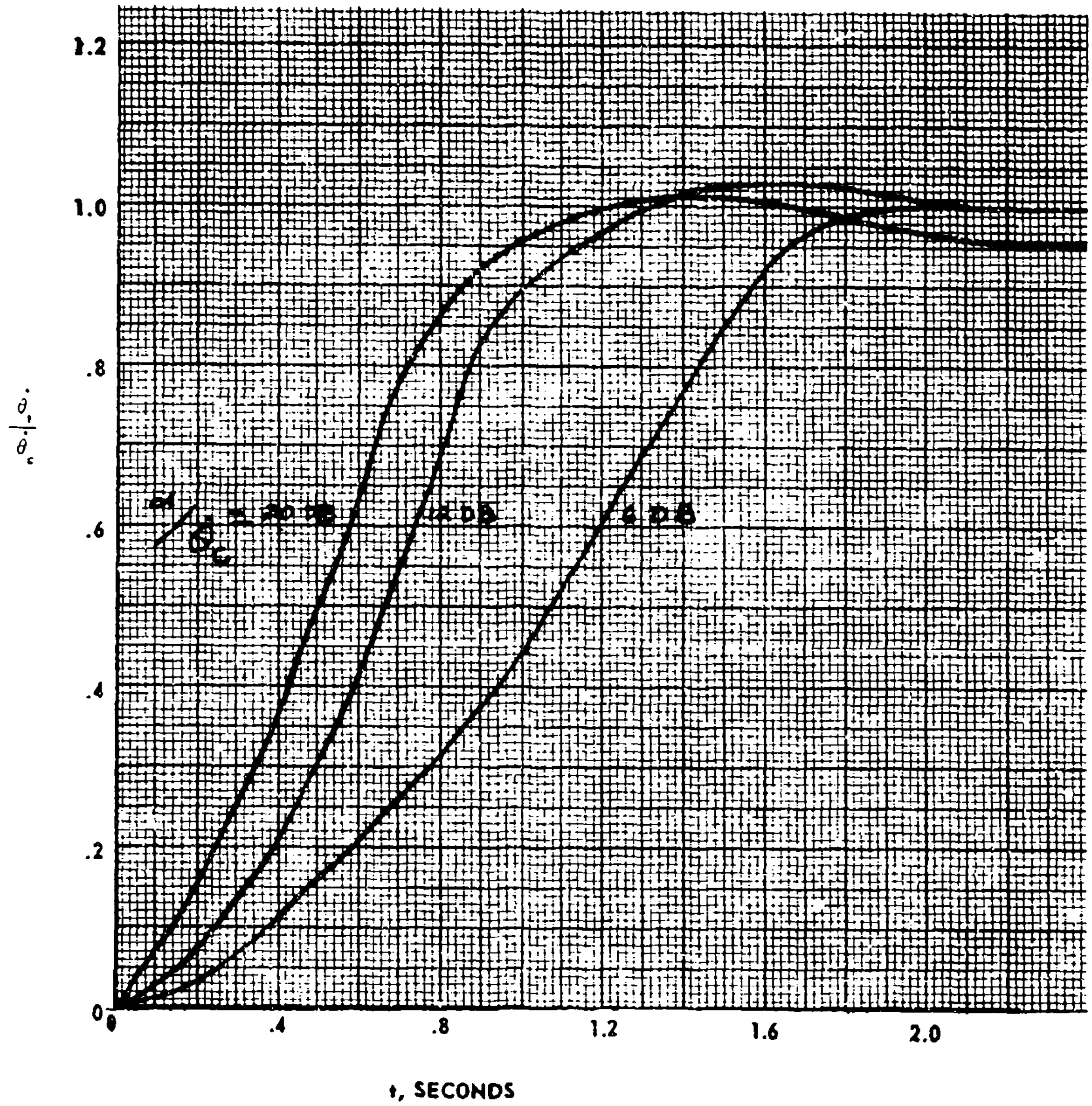


Figure 12. Transient Response of the Complete Adaptive Control System - Condition 1

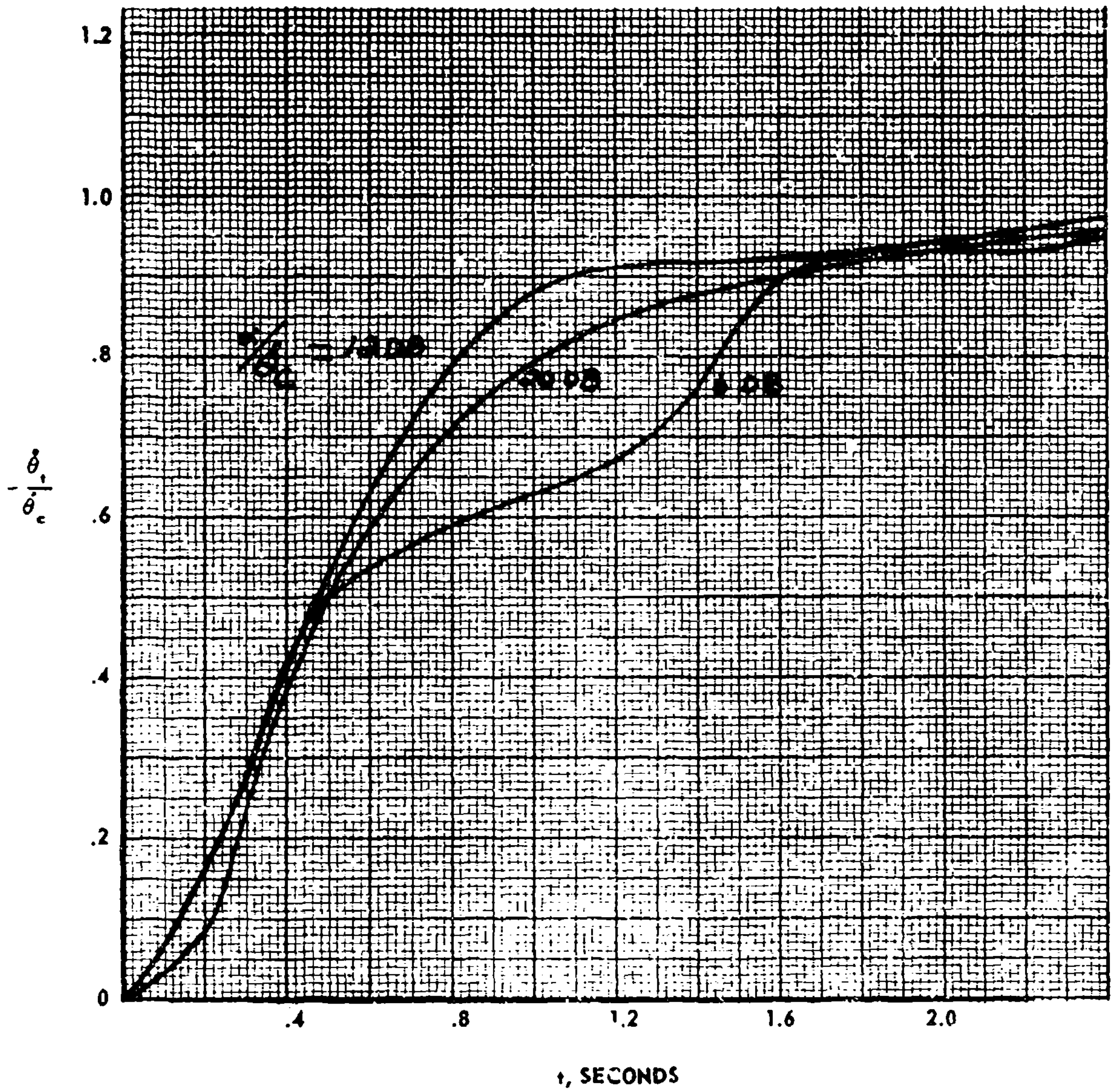


Figure 13. Transient Response of the Complete Adaptive Control System - Condition 10

DISCUSSION OF THE HONEYWELL ADAPTIVE FLIGHT CONTROL SYSTEM FOR HIGH-PERFORMANCE AIRCRAFT

D. L. Mellen and F. Boskovich

1. INTRODUCTION

Encouraged by the F-94C flight test results and bolstered by a better analytical understanding of nonlinear control systems, we at Honeywell initiated two new adaptive control system development programs:

1. The development and flight test of a complete three-axis adaptive flight control system for application to high-performance aircraft.

2. The development and flight test of a simplified adaptive control system.

It is appropriate to discuss in this paper only the first of these two systems as this will provide a comprehensive coverage of the Honeywell adaptive flight control concepts.

Since the F-94C flight tests, much effort has been directed toward extending the adaptive concept to all axes of control and in overcoming the deficiencies found in the flight test of the F-94C system. As a result, a complete adaptive flight control system has evolved and is being readied for flight tests in a F-101A fighter aircraft.

2.0 DISCUSSION OF THE ADAPTIVE INNER LOOP

The model and the adaptive controller form the kernel of the Honeywell adaptive flight control system. Figures 3 and 4 are block diagrams of the pitch and lateral axes showing these units in their proper perspective to the complete system.

Being common to all axes of the system, the model and adaptive controller will be discussed in some detail. Consider the block diagram shown in Figure 1.

2.1 Model

The model in the adaptive flight control system provides the system with the selected standard of performance. It represents what the pilot wants in his aircraft handling characteristics. Hence, the model is an analog simulation of an ideal aircraft.

Specifically, in the pitch axis where normal acceleration is the major adaptive loop feedback, a second order model with a damping ratio of unity and a natural frequency of 2 radians per second is used. In the roll axis a first order lag with a time constant of 0.5 second is used.

Electric commands to the model are obtained from a stick force transducer located on the control stick of the airplane. As the pilot applies force to the control stick, an electrical voltage, which is proportional to the applied force, is generated and transmitted to the model. The output of the model represents the desired airplane response. In other words, the output of the model is the specific performance the pilot would like to have for the given input.

2.2 ADAPTIVE CONTROLLER

If the aircraft response could be made to duplicate the response of the model, then it will, of course, respond as the pilot would like it to respond. This is precisely the job of the adaptive controller, that is, to force the aircraft to follow closely the output of the model.

Studies conducted at Honeywell have shown that the " following error" can be kept reasonably small if the inner loop natural frequency is at least 5 to 10 times that of the model. Hence, the function of the adaptive controller is to provide the compensation and gain adjustment necessary to meet this criteria at all flight conditions.

The adaptive controller which contains no moving parts consists of a compensating network, a modified bi-stable element, an electronic integrator and an automatic amplitude modulator (see Figure 1). The compensating network is a phase lead network which operates on the error signal to provide anticipation and to maintain the frequency of the limit cycle relatively constant over the flight regime of the aircraft. How this is accomplished is discussed in the previous paper by Luther Prince.

The modified bi-stable element is essentially a high gain linear amplifier with limited output. The gain and limits are variable in the manner shown in Figure 2.

The electronic integrator is used to obtain a proportional plus integral signal in the forward path of the inner loop.

The automatic amplitude modulator senses servo motion at the limit cycle frequency only and varies the gain of the modified bi-stable element to maintain the forward loop gain at the highest stable value for all flight conditions. Hence the automatic amplitude modulator compensates for changes in aircraft control surface effectiveness.

Now the system operates with an error sufficiently small to stay within the linear portion of the modified bi-stable element. In accomplishing this, a small controlled residual motion exists at the frequency corresponding to the neutral stability point. The amplitude of this residual motion is kept constant at the servo output by the automatic amplitude modulator. If the amplitude of motion is larger than that designated by a bias voltage, the gain of the modified bi-stable element is decreased, and vice versa.

In the yaw axis there is a deviation from the above discussion as the gain of the modified bi-stable element in yaw is adjusted by the roll automatic amplitude modulator. Hence, no automatic amplitude modulator is included in the yaw axis. No characteristic residual motion exists in the yaw axis as the gain of the modified bi-stable element is adjusted to be less than that required to sustain a limit cycle.

2.3 GENERAL CHARACTERISTICS

2.3.1 COMMAND AND GUST INPUTS

Since the system gain for small error signals is maintained at that gain required to produce a limit cycle at a frequency at least five to ten times that of the model, the system output will follow the model very closely with the error signal rarely becoming large enough to exceed the small linear band of the modified bi-stable element.

However, if the aircraft is excited by a gust, the transient error in the inner loop becomes quite large, and, as can be seen in Figure 2, the effective gain of the modified bi-stable element is sharply reduced during the transient. This is a desirable and necessary feature of the system; because it increases the system phase margin, and hence damping, for external disturbances without compromising system " following error" for signals fed through the model.

2.3.2 CHARACTERISTIC RESIDUAL MOTION

The size of the characteristic residual motion is set just large enough to overcome the various thresholds of the system. In the case of the F-101A, the amplitude of the surface residual motion will be kept under 0.1 degree at a frequency of 6 cps. The resulting pitch and roll rate and acceleration experienced by the aircraft are well under the pilots threshold. The pitch attitude residual motion is kept under 1 mil.

3.0 GENERAL DISCUSSION OF BLOCK DIAGRAMS

The objective set up for the F-101A system is optimum performance with no air data scheduling of parameters.

3.1 PITCH AXIS

As it can be seen in Figure 3, three outer loop modes in the pitch axis are provided in the F-101A system. They are control stick steering, attitude and altitude hold. Automatic glide slope and flare-out systems can be added easily.

To eliminate the need for scheduling in certain outer loop controls (such as, altitude hold, Mach hold, etc.), a pitch rate inner loop is desirable; and for certain other outer loop controls (such as, altitude hold, flight path commands, control stick steering, etc.), a normal acceleration inner loop is preferred. The F-101A system mechanization is such that it allows the effective utilization of either a normal acceleration inner loop or a pitch rate inner loop depending upon the mode of operation. A consequence of this is the complete elimination of air data scheduling in all modes of operation.

In aircraft handling qualities, a pilot prefers a constant stick force per "g" characteristic. For control stick steering then, normal acceleration plus high-passed pitch rate are fed back as the primary inner loop signals. The ratio of normal acceleration to pitch rate is adjusted to provide adequate damping and an essentially uniform normal acceleration response throughout the aircraft flight regime.

For the attitude hold mode, the adaptive inner loop system is essentially switched to a pitch rate system which is as earlier mentioned, the ideal inner loop system for attitude control. The addition of a high-passed pitch attitude feedback and a negative normal acceleration signal, fed through the model, effectively change the inner loop to a pitch rate system. It can be shown mathematically that the result of adding these two feedbacks is to effectively remove the high-pass from the pitch rate signal and to attenuate the normal acceleration feedback in the range of model control frequencies. Hence, it is possible to achieve uniform dynamics over the flight range without resorting to air data scheduling. A low gain integration through the trim synchronizer is used to ensure a droopless system.

For the altitude hold mode, the normal acceleration adaptive inner loop is preferred because both rate of change of altitude and normal acceleration are functions of forward velocity. No pitch attitude loop is in the system when on altitude hold. For stability purposes this signal is replaced by altitude rate from an inertially augmented altitude controller which will provide a good altitude rate signal. A low gain integration on the altitude displacement signals is provided as in the pitch attitude case.

Input command signal limiting is achieved by a diode limiter just ahead of the model.

In the pitch axis, the series servo and parallel servo are driven by the output of the adaptive controller through a splitter network. The function of the splitter network is to separate the incoming signal into a high frequency band and a low frequency band. The high frequencies are fed to the series servo and the low frequencies to the parallel servo. In this manner, the characteristic residual motion and high-frequency damping is accomplished by the series servo while the parallel servo will, for the most part, reflect only the gross surface motions encountered during a maneuver. Selection of the splitter network time constant is governed by the available series servo authority and desired stick feel.

3.2 ROLL AXIS

As shown in Figure 4, roll rate is used for the primary inner loop feedback and four outer loop modes are included: control stick steering, altitude and heading hold, and heading select.

For the control stick steering mode, summing the stick force signal with roll rate only, provides a constant stick force per degree per second roll rate throughout the flight regime.

Roll attitude hold is provided by merely feeding a roll attitude signal to the constant characteristic roll rate adaptive loop. Roll attitude is automatically engaged when the roll stick force is below a certain limit.

The heading hold and heading select modes are obtained by adding a heading feedback to the roll attitude mode. The heading hold mode is automatically engaged when the bank angle is less than 7 degrees and when the stick is centered laterally. Heading select is obtained through the heading synchronizer by a manual input selector. A unique feature of the heading select mode is that the gain of the heading error signal is a nonlinear function of heading error. Thus, the heading response at high speeds is improved because the commanded bank angle will be held at its limit value until the aircraft is nearly at the selected heading.

As in the pitch case, a command signal limiter is provided to prevent excessive outer loop commands.

All signals out of the adaptive controller are fed into the two series type servo actuators located at each aileron.

3.3 YAW AXIS

The yaw adaptive control loop (Figure 4) utilizes high-passed yaw rate, and lateral acceleration to provide dutch roll damping and transient turn coordination for the yaw axis. No outer loop modes are used in the yaw axis.

Since the rudder pedals apply direct mechanical inputs to the rudder actuator, manual inputs may be made by the pilot. This manual input establishes a transient command in lateral acceleration which will eventually be reduced to zero, within the limits of the series servo authority, through the integral action of the adaptive controller.

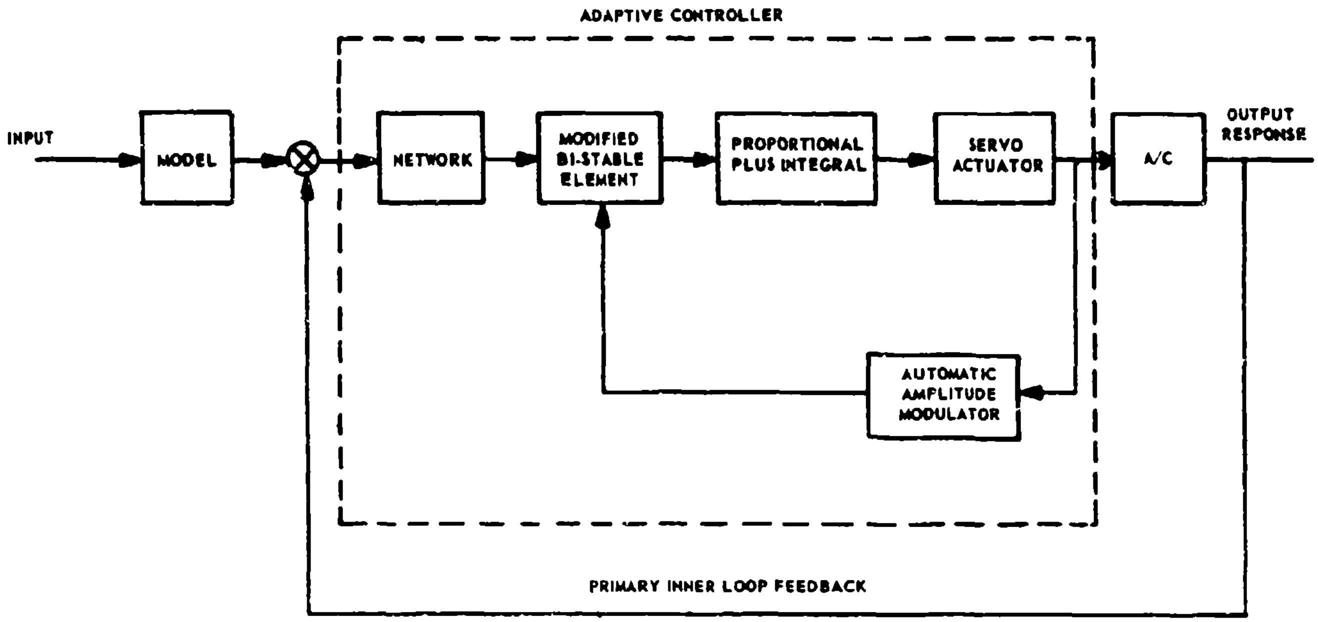


Figure 1. Typical Adaptive system

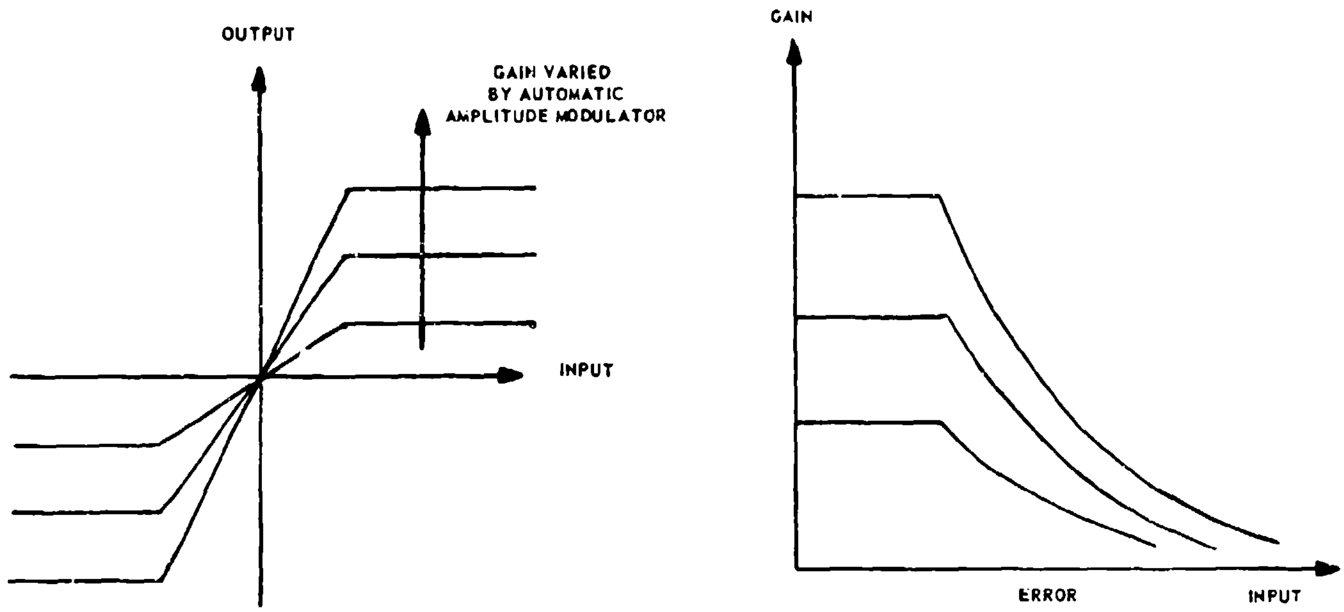


Figure 2. Characteristics of Modified Bistable Element

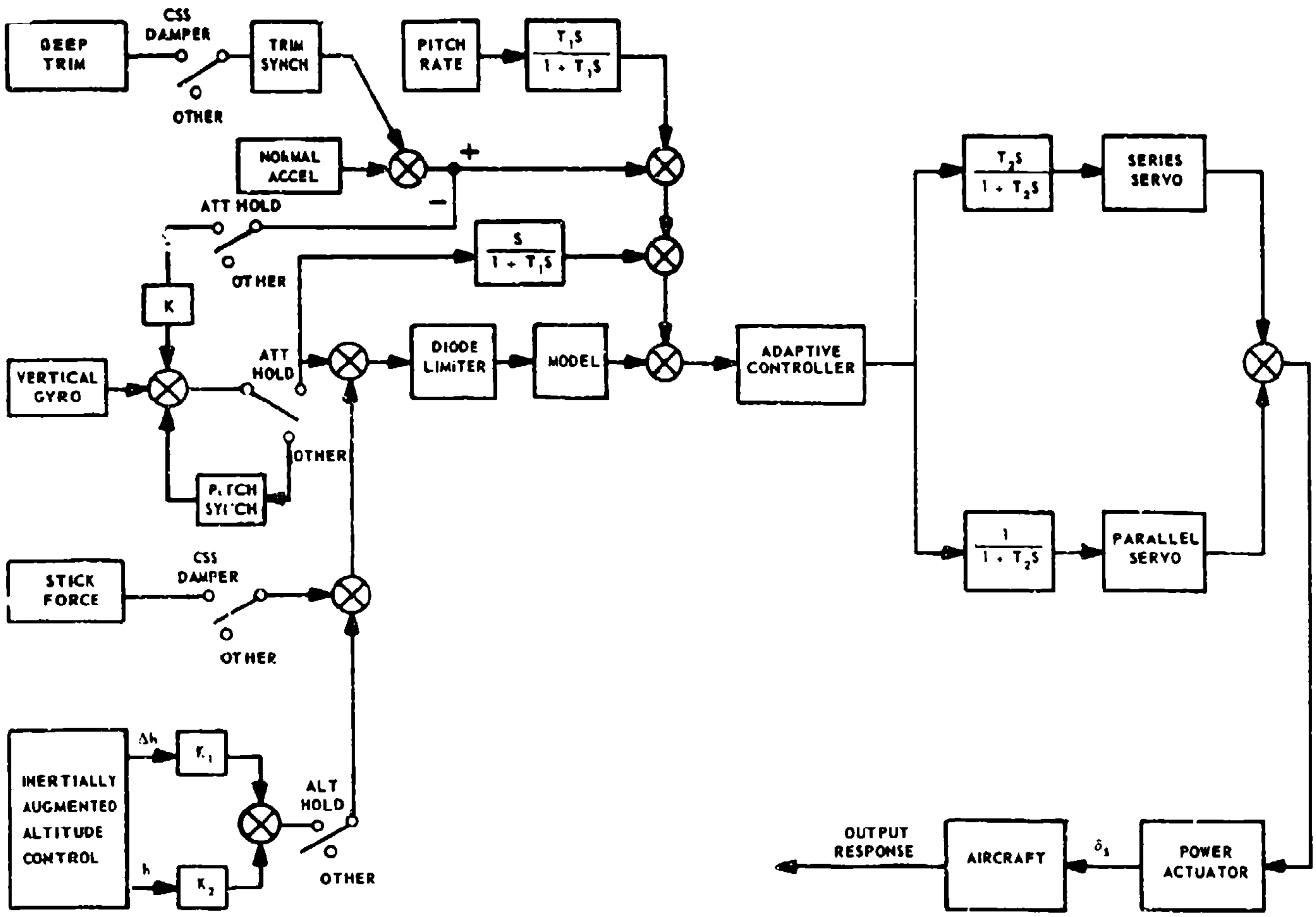


Figure 3. Pitch Axis Block Diagram - F101A

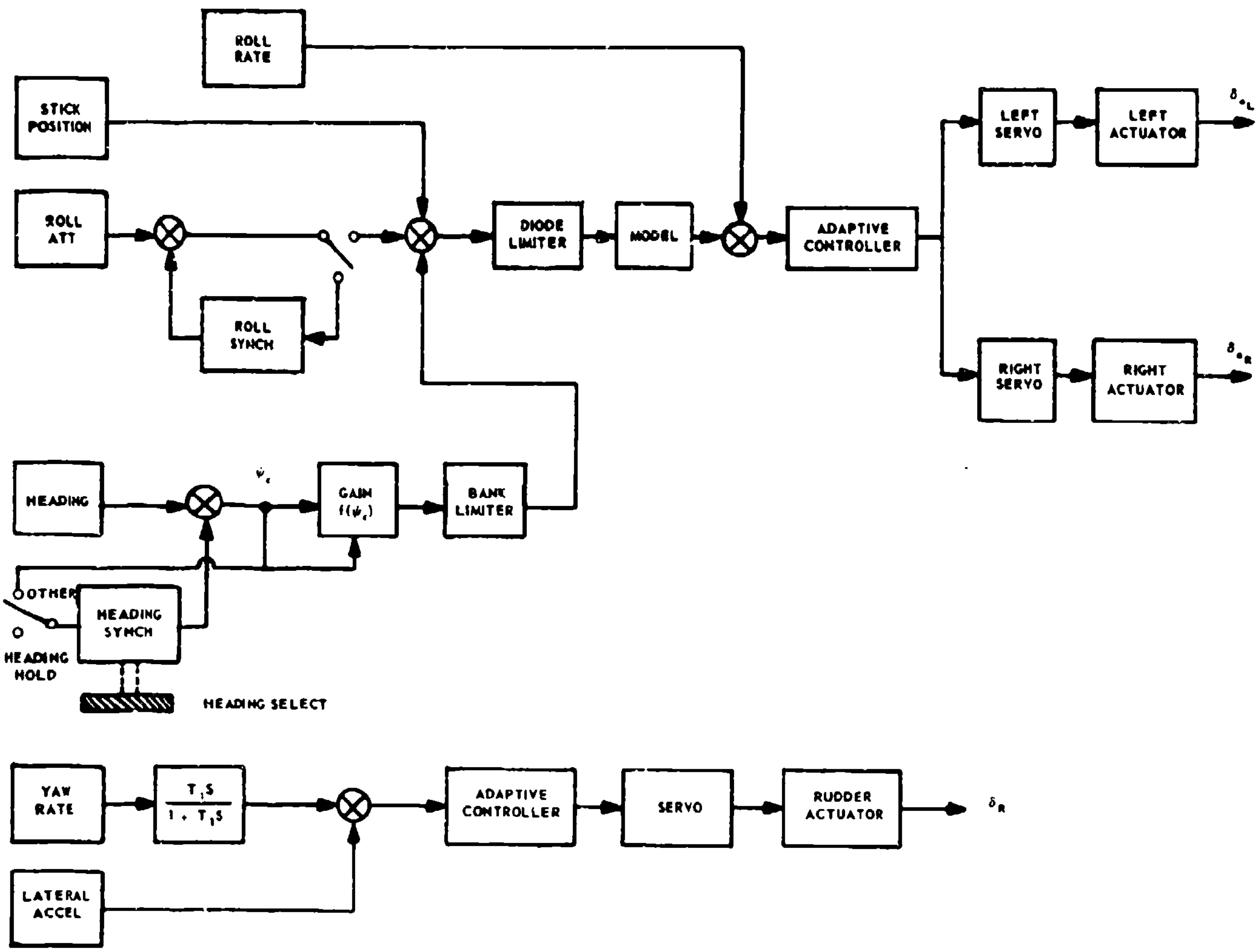


Figure 4. Yaw and Roll Axis Block Diagram - F101A

A REVIEW OF OPTIMIZING/COMPUTER CONTROL

I. Lefkowitz and D. P. Eckman
Case Institute of Technology
Cleveland, Ohio

ABSTRACT

Optimizing control has as its principal objective the maintenance of the optimum performance of a multi-variable system subject to both disturbing and constraining influences. First, the system performance criteria must be defined. Then optimum performance can be achieved by either of two basically different methods: one, a direct approach in which the output performance is compared with the input manipulation to determine the system behavior and thus provide the direction for optimizing control of the system; this may be done with or without a perturbation or test signal. Two, a model method in which the model provides the basis for analytical definition of the optimizing control conditions for the system. The model is manipulated such that its behavior agrees with the observed behavior of the system.

Optimizing computer control is gaining attention as a further means for achieving better control of a complex physical system. The single negative feedback loop no longer suffices for control of systems with widely varying parameters or systems with parameters which are either undefined or not completely known. Large disturbances or load changes also emphasize the need for more advanced control.

The development of reliable computer techniques in both analog and digital form have made it possible to consider much more complex working methods in automatic control than the conventional proportional, integral,and derivative effects. These include nonlinear and higher order control functions, application of filtering, prediction and correlation techniques, repetitive computer methods, etc.

PERFORMANCE COMPUTATION

The system under control can generally be described, as in Figure 1, as having k outputs under the influence of i independent inputs (determined by factors external to the system under consideration), and under the influence of j dependent inputs which may be manipulated.

The system presumably has a utility of some sort; that is, the system is put into use with the outputs forming a valuable product or service. It is assumed that the utility can be judged by some appropriate method so that the performance of the system can be computed.

There are two general methods of specifying performance and these are first, economic and second, technical. Economic performance is often expressed as a linear combination of system variables

$$p = \sum_i K_i u_i + \sum_j K_j m_j + \sum_k K_k q_k$$

where p = performance criterion

u_i = independent input variables

m_j = dependent (manipulated) input variables

q_k = system output variables

K_i, K_j, K_k = Appropriate profit or cost coefficients

On the other hand, technical performance is often specified in such terms that an optimum or best value may exist. For example, in many industrial processes,

$$p = f(m_j, u_l, q_k)$$

The necessary conditions for the optimum are determined by setting

$$\frac{\partial p}{\partial m_j} = g(m_j, u_l, q_k) = 0 \qquad j = 1, 2, \ldots$$

Another example is the formulation of best performance in the traditional sense of minimum mean-square error criterion,

$$p = \lim_{T \to \infty} \frac{1}{T} \int_0^T \sum_k (w_k - q_k)^2 \, dt$$

where w_k represents the desired functions corresponding to system outputs, q_k. Whether performance is optimized as an extremum control in which a maximum or minimum value is maintained or is optimized as a feedback control in which a near-zero difference is maintained is not important because one form can be readily converted into the other.

Constrainin influences are very important and it is often necessary to subject the control system to bounds of the form,

$$Q_{1k} < q_k < Q_{2k}$$

or of the form

$$q_k \geq h(q_1, q_2, \ldots)$$

These constraints often make it necessary to find system performance at a limit which is in turn a function of other variables. The performance computer of Figure 1 is therefore employed to compute the necessary functions and obtain the variable (p) on which to base system control.

DIRECT OPTIMIZATION

Direct optimization proceeds with a minimum of knowledge about the system under control. As shown in Figure 2, the optimizer receives data on the variations in the manipulated variables (m_j) and the resulting changes in performance (p) and in turn manipulates each of the m_j inputs in the indicated direction for improving the performance. Sometimes a perturbation or test signal is employed in order to initiate changes upon which the control measurements are based.

Direct optimization is thus exploratory or experimental in nature in that the results of each manipulation is assessed and another manipulation is made. These may be done sequentially or simultaneously in a number of manipulated variables, depending upon the type of exploring scheme in use.

The direct method may be achieved through continuous measurements or by the use of sampled and/or quantized data. In either case, the general principle of the optimization system may be the same.

DIRECT OPTIMIZATION WITHOUT PERTURBATION

Direct optimization may proceed without employing a perturbation or test signal; one such method is shown in Figure 3. The divider generates the derivative of performance with respect to the manipulated variable (m_j) and thus determines when the performance is maximum. Analysis of the dynamics shows that the system is stable for a number of practical applications. When several manipulated variables are involved, the divider-integrator circuit is repeated for each variable and the manipulations are performed either sequentially or simultaneously.

This method has the very great advantage of simplicity and easy realizability and has been applied to a number of industrial processes. The disadvantages of this method are that only a relatively few (four or five) manipulated variables can be accommodated and that its speed of response is limited by the dynamics of the system.

DIRECT OPTIMIZATION WITH PERTURBATION*

The perturbation or test signal method shown in Figure 4 employs the perturbation to disturb the system. The response of system performance (p) is then correlated to the perturbation to generate control signals for the system inputs (m_j). The particular method shown in Figure 4 employs the continuous time integral of the product of system performance and the perturbation signal. The perturbation signal may be a sinusoid or other periodic wave, white noise in continuous or discret form or an impulse sequence. Each of these forms of the signal has a particular use depending upon the system and control method employing it.

The perturbation method also has good stability in a number of practical applications. When several manipulated variables are involved, the multiplier-integrator circuit is repeated for each variable. In this case, it is sometimes convenient to use perturbation signals of differing time scale or frequency in order to discriminate the effects of simultaneous manipulations.

* See references 1 and 5

This method has the advantage of simplicity and may be used very adequately in multi-dimensional problems. It has been applied to engine control as well as industrial processes. The disadvantages of this method are that the perturbations may be undesirable (as in machine tool control) and often the system becomes unstable for too small a perturbation signal amplitude.

MODEL METHODS

Model methods provide an alternate approach to optimizing control. In general, the necessary conditions for optimum performance of the system under control are determined on the basis of an appropriate system model. The model may form an integral part of the control system or it may only be present in concept. It may range from some physical simulation or analog of the process to a mathematical abstraction manifested as a set of equations or a multi-dimensional surface describing the system behavior.

PREDETERMINED OPTIMIZATION

If the model is complete and exact, then the conditions for optimization can be determined completely and exactly. In particular, these conditions may be predetermined for any given set of constraints and boundary conditions.

A conceptual approach to Predetermined Optimization is given in Figure 5. An optimizing computer determines paths or functions for the manipulated variables, m_j, based on the predicted behavior of the system as described by the model variables, w_k. The actual behavior of the system is described by q_k; system performance is then gauged in terms of the q_k variables.

It is apparent that, once computed, the optimizing conditions can be stored on punched-tape, magnetic drum or even, in the simple two-dimensional case, on a mechanical cam. The system variables are then manipulated according to the playback of the appropriate stored program. Note that in systems manually operated or supervised, the operator is often guided by a predetermined optimizing program stored graphically or in tabulated form or stored mentally in the guise of experience.

The control scheme described above is essentially open-loop; i.e., there is no feedback of information to verify either that the resulting system performance is as specified, or that the model accurately describes the system behavior. Accordingly, if there are any factors tending to cause the system to deviate from the model as, for example, the influence of disturbances u_i, the system performance may be expected to deviate from the computed optimum.

REPETITIVE COMPUTED OPTIMIZATION

The predetermined optimization concept is modified by repetitive feedback of information describing the state of the system. Thus, as shown in Figure 6, the q_k variables are periodically sampled into the optimizing computer providing the basis for repetitive recomputation of the optimizing conditions. In this way, each computation is based on the most recent information describing the state of the process. As a result, deviations of the system from the postulated model do not cause cumulative errors. Indeed, the repetitive computer action tends to force the system to the desired performance despite significant inadequacies of the model.

SELF-CHECKING

In practical applications of optimizing computer control, it is expected that the postulated model will deviate significantly from the actual system behavior. There are several reasons for this:

1. The system may not be known in complete and accurate analytical form.

2. Some of the variables describing the state of the system may not be readily measured to provide feedback to the optimizing computer.

3. The system may be so very complex that a control program based on the complete model would be impractical from the standpoint of computer capacity.

The repetitive control concept described in the preceding section is effective in substantially reducing the effect of model deviations on system performance. Limitations are introduced, however, in terms of the repetitive period, measurement dead-time, limiting of the manipulated variables, and the general order of the approximations employed.

The self-checking concept is superimposed on the repetitive control action to extend its effectiveness and range of applicability. This is illustrated in Figure 7.

Self-checking refers to the periodic adjustment of the parameters of the model (y_k) such as to force the model to agree (at least within the neighborhood of the operating point) with the observed behavior of the system. The behavior of the system is represented by variables q_k (see Figure 7); the predicted behavior based on the model is represented by w_k.

An important conceptual distinction is noted here: whereas most of

the discussions on adaptive control considered a fixed model and adjusted the parameters of the control system to fit this model, the basic idea presented here is that of a model adjusted to fit the system.

EXAMPLE: OPTIMIZING CONTROL OF A BATCH CHEMICAL PROCESS

The model method concept has been applied to the optimizing control of a batch chemical process. A simple prototype of the process studied is described very briefly* as follows:

The reaction mixture is made up of three chemical components identified as X, Y, and Z. Hydrogen under pressure and in the presence of catalyst reacts with X and Y according to the following reaction scheme:

$$X + H_2 \xrightarrow{k_1} Y$$

$$Y + H_2 \xrightarrow{k_2} Z$$

where k_1 and k_2 represent kinetic reaction coefficients.

A reasonable approximation to the kinetic behavior of this process is given by the equations,

$$\frac{dx}{dt} = -k_1 x \tag{1a}$$

$$\frac{dy}{dt} = k_1 x - k_2 y \tag{1b}$$

$$x + y + z = 1 \tag{1c}$$

where x, y, z represent molar concentrations of components X, Y, Z, respectively.

The kinetic coefficients are functions of the operating conditions: pressure, temperature, catalyst, agitation, etc. Assuming only pressure is to be manipulated and that all other influencing factors are relatively constant, the coefficients may be expressed,

* See references 4, 7, and 8 for a more detailed description of the process and the computer control application.

$$k_1 = A_1 p^{N_1} \quad (2a)$$

$$k_2 = A_2 p^{N_2} \quad (2b)$$

where A_1, A_2, N_1, N_2 are assumed constant

p = process pressure.

Based on a mathematical model consisting of Equations (1, 2), the necessary conditions for optimum process performance may be derived. In the particular case under study, control to a specified product composition consistent with minimum processing time is established as the performance criterion. By means of the calculus of variations, the following optimizing control equations are derived:*

$$\frac{du}{dv} = (1 - k)u + 1 \quad (3a)$$

$$\frac{dk}{dv} = \left(\frac{N_1}{N_2} - 1\right)\frac{k}{u} \quad (3b)$$

where $u - y/x$

$$v = \log_e x_0/x$$

$$k = \frac{k_2}{k_1} = \frac{A_2}{A_1} p^{N_2 - N_1}$$

Equations (3a, b) are solved by the optimizing computer such as to satisfy the boundary conditions (x_0, y_0, z_0) representing the initial composition and (x_f, y_f, z_f) representing the desired final composition.**

If equations (1, 2) described the process behavior exactly, then one computation based on Equations (3a, b) and the specified boundary conditions would suffice to define the optimum control path, p(t). Thus, the p(t) schedule could be recorded on tape or other storage medium and played back through appropriate transducers and pressure controller to manipulate the process pressure according to the schedule. In the example under consideration, however, the model only approximates the process kinetics because of the neglect of such factors as variations in catalyst activity, other components in the reaction mixture, higher order terms in the kinetic equations, etc.

* See references 4 and 8 for derivations.
**In terms of the u, v coordinates, these boundary conditions are expressed as (u_0, O) and (u_f, v_f), respectively.

Open-loop control of the process would lead, therefore, to very significant deviations from the desired end-point.

The repetitive control concept was applied here. Equations (3a, b) are solved for the optimum control path leading from the current state of the process (based on the most recent composition measurement of the reaction mixture) to the specified final composition. Thus, each time a new composition measurement is made available to the computer, a new control path is computed. This technique was demonstrated to be very effective in forcing the process to the prescribed performance.

Application of the self-checking concept to this system is currently being implemented. It is assumed, in this approach, that the inadequacies of the model may be absorbed in the parameters of Equations (2a, b). Thus, the actual progress of the reaction is compared with that described by Equations (1, 2). The parameters A_1, A_2, N_1, and N_2 are adjusted periodically such as to minimize the discrepancy between system behavior and model prediction. Coupled with this self-checking is a statistical smoothing technique by which random errors in measurement are filtered out.

The combination of repetitive computer control and self-checking provides the basis for wide practical applicability of the model approach to optimizing control. In particular, the control effectiveness becomes very much less dependent on the accuracy or completeness of the model employed, providing there is an adequate information feedback to the control computer. Thus, the optimizing control for typically very complex systems may be realized with computer control facilities within the bounds of economic justification.

BIBLIOGRAPHY

1. C. S. Draper and Y. T. Li, "Principles of Optimalizing Control Systems", American Society of Mechanical Engineers, New York, 1951.

2. H. Ziebolz and H. M. Paynter, "Possibilities of a Two-Time Scale Computing System for Control and Simulation of Dynamic Systems", Askania Regulator Company, Chicago, 1953.

3. D. P. Eckman, T. J. Walsh, and F. E. Brammer, "Computer Control of Chemical Processing", Case Institute of Technology, Cleveland, 1953.

4. Process Automation Project: Report I, Case Institute of Technology, Cleveland, 1956.

5. G. Vasu, "Experiments with Optimalizing Controls Applied to Rapid Control of Engine Pressures with High Amplitude Noise Signals", ASME Trans. Vol. 79, No. 3, p. 481, April 1957.

6. M. Phister, Jr. and E. M. Grabbe, "Fitting the Digital Computer into Process Control", Control Engineering, Vol. 4, No. 6, p. 129, June 1957.

7. D. P. Eckman and I. Lefkowitz, "Optimizing Control of a Batch Chemical Process", Control Engineering, Vol. 4, No. 9, p. 197, September 1957.

8. I. Lefkowitz and D. P. Eckman, "Application and Analysis of a Computer Control System", paper presented at the ASME - IRD Conference, University of Delaware, April 1958.

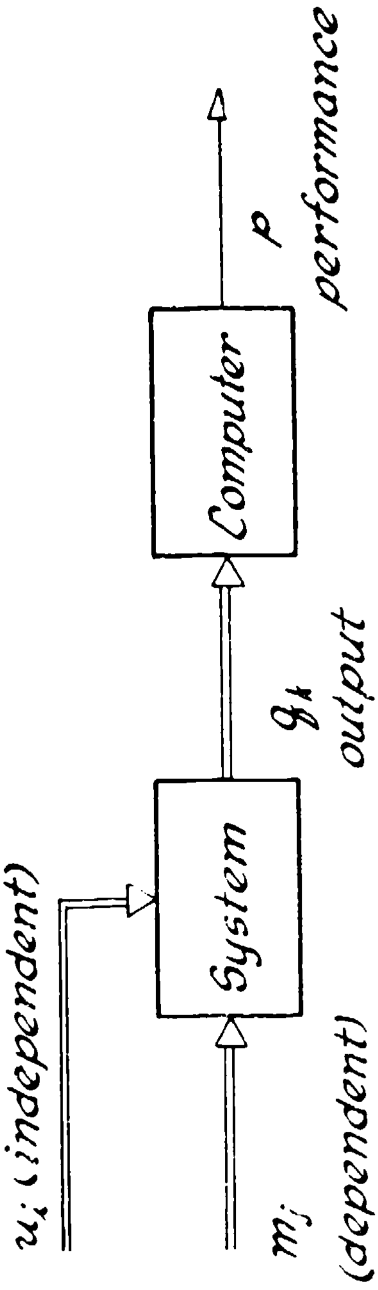


Fig.1 The System Performance

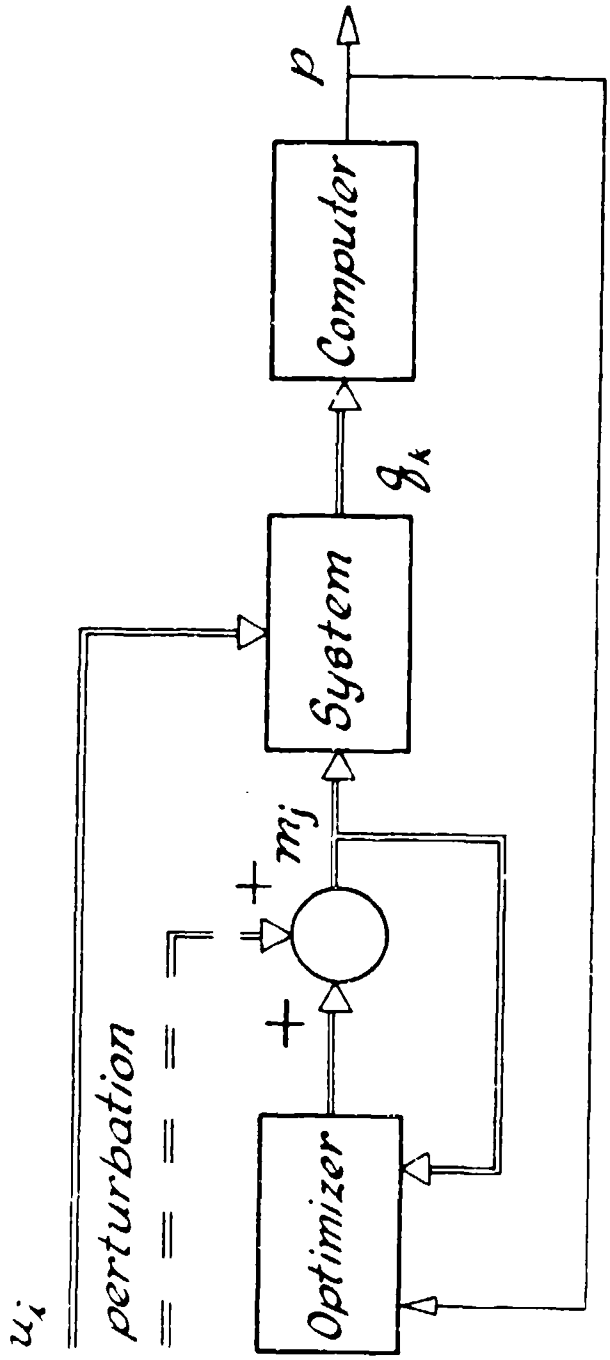


Fig. 2 Direct Optimization

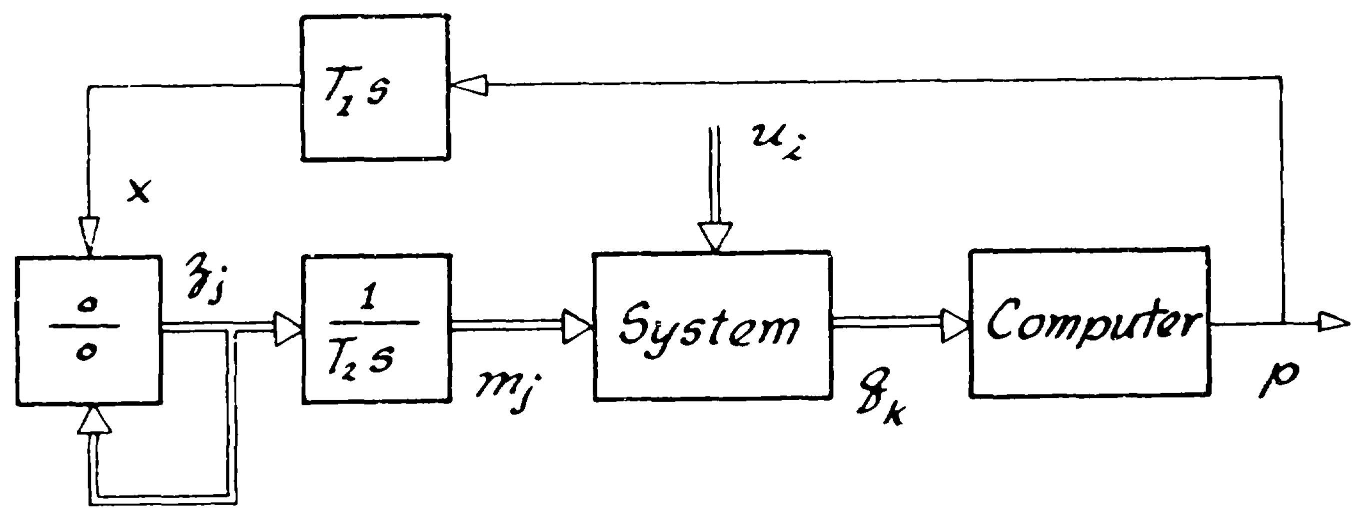


Fig. 3 Direct Optimization (without perturbation)

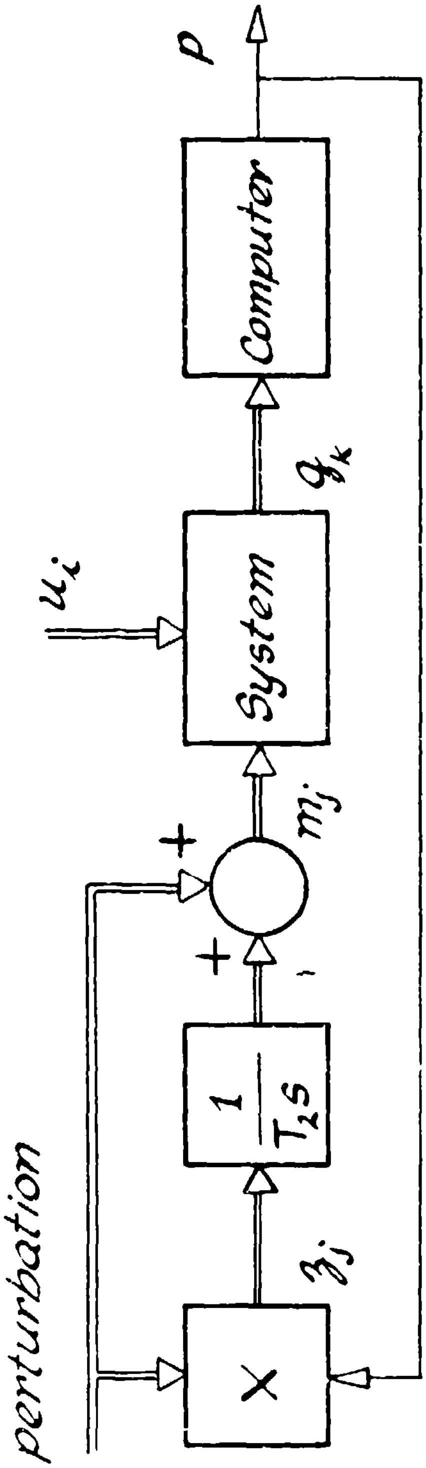


Fig. 4 Direct Optimization by Perturbation

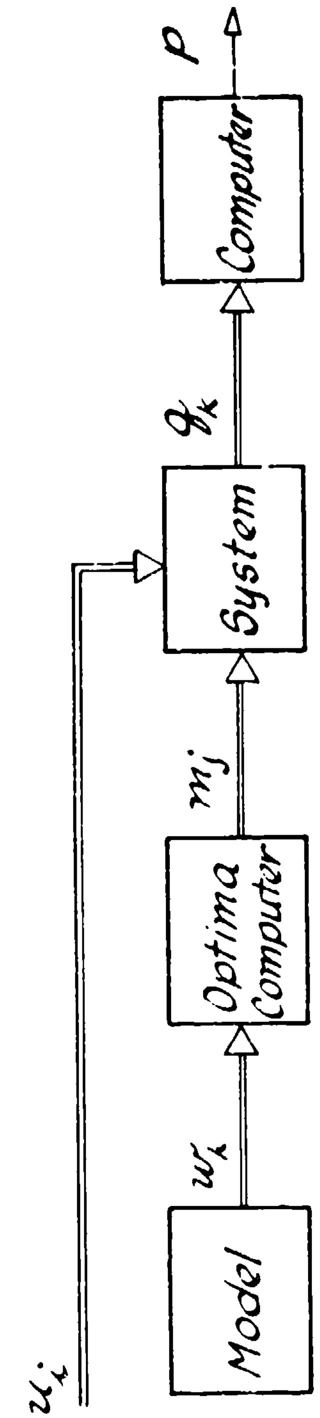


Fig. 5 Predetermined Optimization

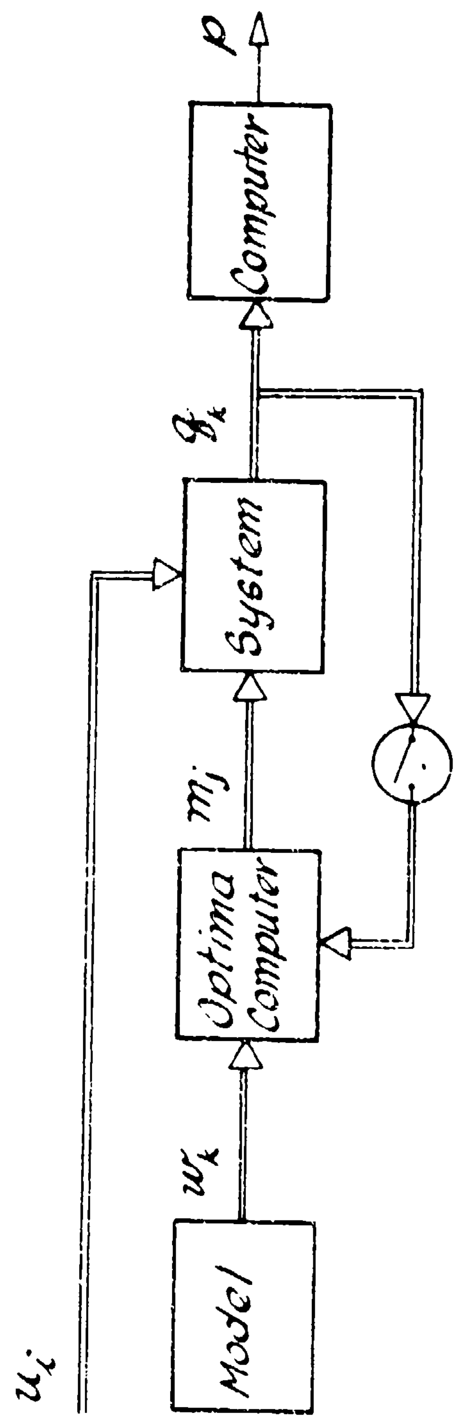


Fig. 6 Repetitive Computed Optimization

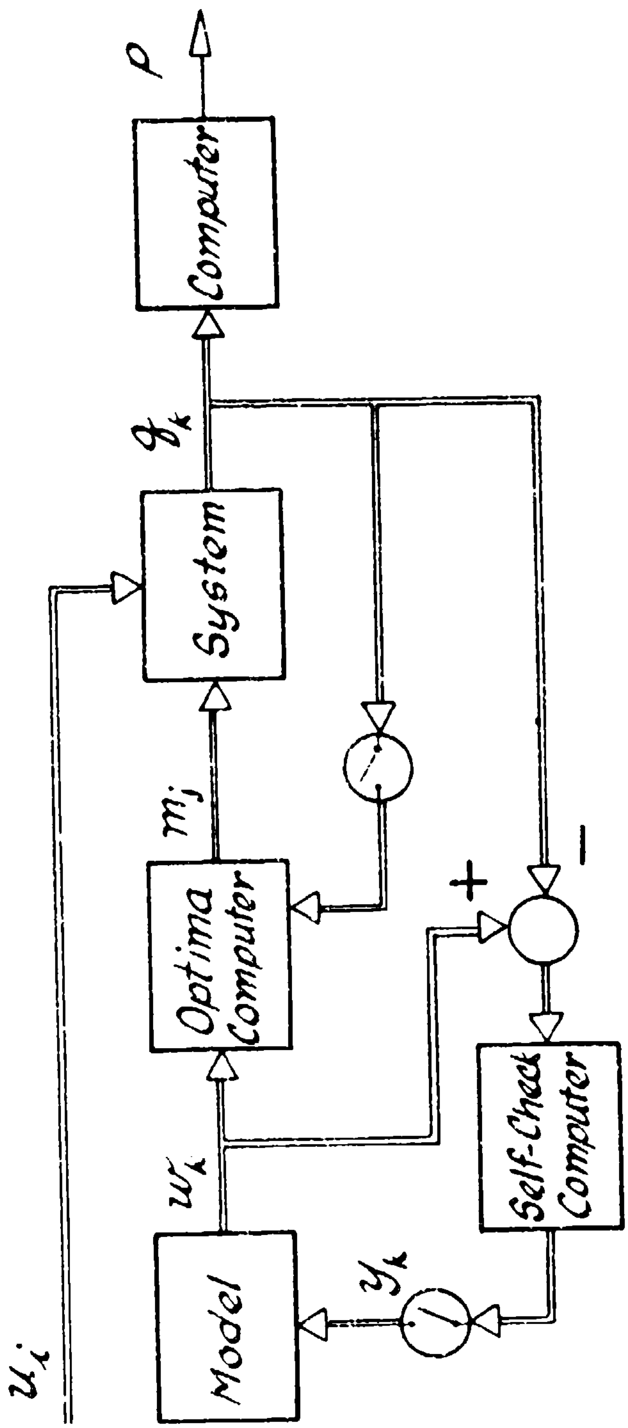


Fig. 7 Repetitive Computed Optimization with Self Checking

THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS SYMPOSIUM

PROGRAM

WADC AUDITORIUM

WEDNESDAY MORNING, 14 JANUARY 1959

SESSION V

Dr. John G. Truxal, Chairman
Brooklyn Polytechnic Institute

Dr. John G. Truxal
Head, Electrical Engineering Department
Brooklyn Polytechnic Institute

As Doctor Draper demonstrated yesterday, teachers' remarks and comments are always quantized in fifty minute sessions, so I would like to bow with just a few brief comments this morning. As Dr. Draper also promised, I won't talk very long because I know you are interested in hearing from the people who have actually done some work, and this morning we are going to have the privilege of hearing from five different organizations.

One job of the University ought to be to define problems and to orient the field and evaluate work in the field. It seems to me that if ever an area required or demanded a definition of the problem, the area of adaptive systems may fall in this category. Like many other organizations, we have done a lot of arguing about what constitutes an adaptive system. I came here without any real definition of an adaptive system! Yesterday I sat in the auditorium with Doctor Aseltine, who will be chairman of this afternoon's session, and we arrived at a definition of an adaptive system which I think has some merit. We think it does anyway. We were a little perturbed yesterday because, if you look at the MIT system and the Minneapolis-Honeywell system and some of the other very interesting pieces of work which we saw, you find that if you re-draw these systems a little differently they look like a conventional feedback control system, which we might analyze in our typical graduate course. For example, the MIT system which is startlingly adaptive as you look at it. If you re-draw it you find that this is really, if you have just one variable parameter for example, a two loop system, with a single non-linearity adjusted so that when you drive it with step functions as they do, it behaves in a linear fashion in the overall system. It looks like here we have an intentionally non-linear system or a non-linear system which was purposefully designed. The Honeywell system that we saw yesterday, if you look at it as an intentionally non-linear feedback control system, is perhaps a single loop with two forward paths.

I don't think this should be surprising because the general concept of feedback, as we all know, is completely arbitrary. We can talk about any system as having feedback or not having feedback, depending upon our personal wishes. We can look at it as having one loop, two loops or eight loops if we want so that there is a certain arbitrariness in a feedback configuration to begin with. It seems to us that this thing called adaptivity is an additional degree of arbitrariness. So we would like to define an adaptive feedback system as one which is designed with an adaptive viewpoint.

This sounds superficial when you first hear it but there really is considerable merit because nobody has any idea how to design a system with

an intentional non-linearity introduced into the system to obtain desirable performance. By this adaptive viewpoint one obtains a logical, simple, and straightforward technique toward the inclusion of a non-linear element within the system to obtain some reasonable performance specifications or meet some reasonable optimization criteria. We would say, particularly, that this was an intentionally non-linear feedback system of any number of loops you may wish designed with an adaptive viewpoint.

I don't know whether the systems you are going to hear about this morning will fall in this category or not. I don't think that this sort of facetious definition, as it may seem on the surface, takes anything away from the great importance of this subject of adaptive systems. What we want, above all, is a new viewpoint toward feedback control system design and it seems to me that the great importance of this subject of adaptive systems is that it gives us another way to get into the design problem. Essentially, it broadens the class of problems and the class of systems which we are now able to circumspect and design intelligently.

Doctor Aseltine, I am sure, will expand on these comments this afternoon and perhaps by that time you can demonstrate where we are wrong.

RECENT ADAPTIVE CONTROL WORK
AT THE GENERAL ELECTRIC COMPANY
BY M. F. Marx

The adaptive control work at the General Electric Company was initiated late in 1954 under a program known as the Pilot-Airplane Link System (PAL). Since this time, a flight test program on a B-25 airplane and extensive computer simulations were carried out. Some of the results of this work were made public at the AIEE computer symposium held at Atlantic City in the Fall of 1957. The present discussion summarizes the PAL program and some of the more recent work carried out.

PERFORMANCE CRITERIA

The basic criteria on performance were adopted quite early in the program. It is required that the system be nontailoring and provide for invariant response.

The first requirement of being nontailoring is quite evident. The system should be able to adjust its feedback parameters without the assistance of external information such as air data programs require. The system should be able to adapt itself to the particular situation on hand. It is implied here that the changes to which it must adapt itself can result from changes in configuration or flight condition.

The second requirement of being capable of invariant response stems from the fact that the desired response can be specified within rather narrow limits. Invariant inner loop response greatly simplifies any additional outer loop design.

In order to indicate the various adaptive control techniques considered, the airplane pitch rate control is examined. Figure 1 presents the airplane short period plus actuator configuration which is typical of most aircraft control applications.

The airplane gain, time constant, frequency and damping are assumed variable.

In any analysis of a pitch rate control, it is important to include the dynamics of both actuators since they are the limiting items in determining the system feedback gain. The systems were first analyzed on a linear basis. If feasibility existed, the analysis was extended to include such actuator nonlinearities as deadband, saturation and variable gain.

PILOT AIRPLANE LINK APPROACH

The PAL system, which was flight tested in a B-25 airplane, was primarily a normal acceleration command system (stick force per "g" constant). The flight test program demonstrated the adaptive control behavior and the feel and handling characteristics of the normal acceleration command system. Pilot opinion indicated the maneuver limiting features and maneuverability as excellent. Subsequent NASA work has substantiated this observation.

Basically the PAL System is a multiplier-divider arrangement as indicated in Figure 2.

The divider output, δ_D, is equal to $\epsilon / K_n n$. Also since $\epsilon = F_S \ \delta_D$, $F_S = K_n n$ or the stick force per "g" is constant.

Further insight as to the dynamic operation of this system can be obtained by considering the system mechanization. This is presented in Figure 3.

Examination of the integrator input indicates that $\epsilon = \delta_D K_n n$ from which $\delta_D = \epsilon / k_n n$. In short, the integrator output is the divider output.

Noting that the two inputs to the integrator (normal acceleration and stick force) are both multiplied by the divider output, δ_D, a simplication follows by omitting this multiplication. The resulting system is presented in Figure 4.

As it stands, the system shown in Figure 4 is not satisfactory due to conditions existing at zero stick force. Under these conditions there is no feedback whatsoever. In addition, the integrator can cause the multiplier to assume any position. The actuation can be remedied by adding the constant voltage, K_1, as shown in Figure 5.

As shown, under conditions of zero input force, the integrator will run so as to drive the acceleration, n, to zero. Since the airplane is now trimmed, it follows that the multiplier gain, K_X, times the voltage, K_1, commands trim elevator deflection. For the case of linear pitching moment curves, a value of $F_S = K_1$, will result in approximately a 2 "g" maneuver. Hence, the multiplier is positioned approximately in its correct position prior to maneuver entry. The quantity, c, produces a direct feed term around the integrator to help stabilize the system.

The block diagram of the system shown in Figure 6 indicates the form of the transfer function used for analysis. For simplicity, conditions existing at trim are not included.

The important things to notice here are the terms in the feedback. The feedback gain and lead are proportional to the command and response. Thus, the response is amplitude sensitive. Furthermore, negative commands of sufficient size can result in positive feedback and consequent instability.

Of these difficulties, the polarity sensitivity is felt most serious since it limits application to the pitch channel. The B-25 tests and computer simulation of high performance airplanes support this conclusion.

Since this approach did not satisfy fully the nontailoring and invariant response criteria, the system was replaced by the frequency sensitive servo technique.

FREQUENCY SENSITIVE SERVO

The use of pitch rate as the basic controlled variable results in better system integration than some of the other variables. The PAL program indicated the necessity of being able to vary the system dynamics suitably through the manipulation of a single parameter. Attainment of this objective, incidentally, considerably simplifies the gain changing problem by present air data programs.

The feedback configuration achieving this goal is shown in Figure 7.

This block diagram can be rewritten as Figure 8.

The root locus plot of this system has the characteristic form shown in Figure 9.

The two locii of interest are the ones starting at the airplane and parallel actuator poles. The blocks indicate the desired operating ranges. It can be shown that, for the feedback configuration shown, a value of open loop gain exists such that operation is within the desired blocks. Figures 10 and 11 present root locus plots for a high performance airplane operating at extreme flight conditions.

The system possesses sufficient gains to enable adequate cancellation of the low frequency root associated with the integrator by the airplane path time constant. For comparison purposes transient response data for the uncontrolled airplane, airplane with damper and integration, and the recommended system are presented in Figure 12. The same step pitch rate command and angle of attack disturbance were ap, ied in all instances.

Satisfactory invariant response is achieved for all conditions investigated. Invariant response results only when the frequency of the feedback term is greater than the highest airplane frequency. In this manner, the locus enters the zeros from the same direction.

The frequency sensitive servo is a device designed to control the system gain so that the closed loop poles lie within the regions indicated in Figure 9. The operation of the device depends on the fact that the frequency of the various modes depend upon the open loop gain. Hence, if frequency errors are used to control system gain, the operation within the blocks of Figure 9 should be achieved.

The frequency detector first investigated consisted of single lead and lag network combination as shown in Figure 13.

The time constant of the networks are set at the inverse of the desired operating frequency. If the system frequency is too high the lead network output will be greater than the lag network output. Polarity is set so that the integrator reduces gain. The reverse operation takes place if the gain is too low. The depolarizers are necessary in order to accept error signals of either polarity.

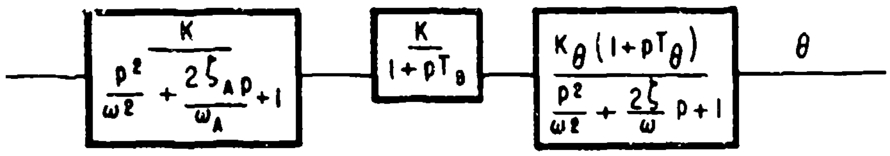
The frequency servo approach has been evaluated for the control of a simple quadratic device and found to be able to establish the correct feedback gain during one transient.

Figure 14 indicates the dynamic behavior of the frequency sensitive servo. The first traces present the response to a step input for the case where the multiplier had been positioned correctly prior to the transient. The results indicate a stable system and no sensitivity to input polarity. The third trace was prepared for the condition of zero gain at time zero. The large output from the lag network indicates that the gain increased transiently as desired. Thus it is seen that the system is capable of establishing the correct gain during a single transient.

Extension to the control of the airplane mode for the case on hand has led to difficulty due to the low frequency closed loop pole caused by the integrator. For cases where the required open loop gain is low, this pole results in low frequency components in the error which the frequency servo interprets as resulting from insufficient gain. Consequently, successive commands progressively increase the system gain until the actuator roots become oscillatory.

The frequency servo is presently being applied to control the frequency of the actuator loop directly. There are several benefits to be derived from this procedure. The low frequency components in the response due to the integrator can be filtered. This will alleviate the difficulty experienced in progressive gain increase following successive commands. The other advantage to be obtained by monitoring the actuator mode is that compensation for the effects of structural feedback is provided since the closed loop frequency is controlled in the presence of the structural feedback terms.

It has been shown that the system transient response can be made essentially invariant through the control of one variable. The only tailoring required is to ascertain that the feedback frequency is higher than the highest airplane natural frequency and that the range of the multiplier is adequate. The system is capable of self adjustment during transients without requiring special testing of input transients or steady state forcing. The system response is essentially linear with command inputs, i.e., it is not polarity or amplitude sensitive. The system techniques described can be applied to all channels without special provisions. The mechanizations are simple, employing continuous signal information thus eliminating switching or sampling requirements.



ω_A = actuator resonant frequency = 50 radians/second

ζ_A = actuator damping ratio = 0.7

T_B = power actuator time constant = 1/15 second

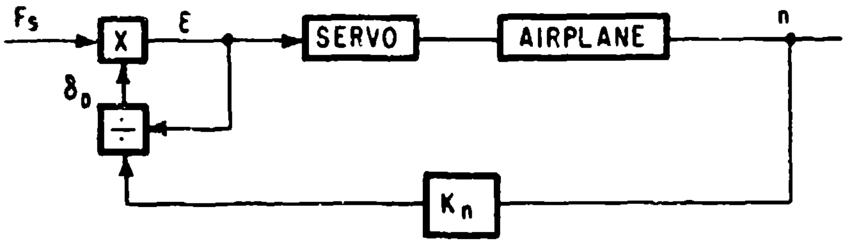
T_θ = airplane path time constant

ω = airplane short period resonant frequency

ζ = airplane short period damping ratio

K_θ = airplane short period gain.

Figure 1.



F_S = Force command

n = normal acceleration

δ_D = divider output

ϵ = servo input

K_n = accelerometer gradient

Figure 2.

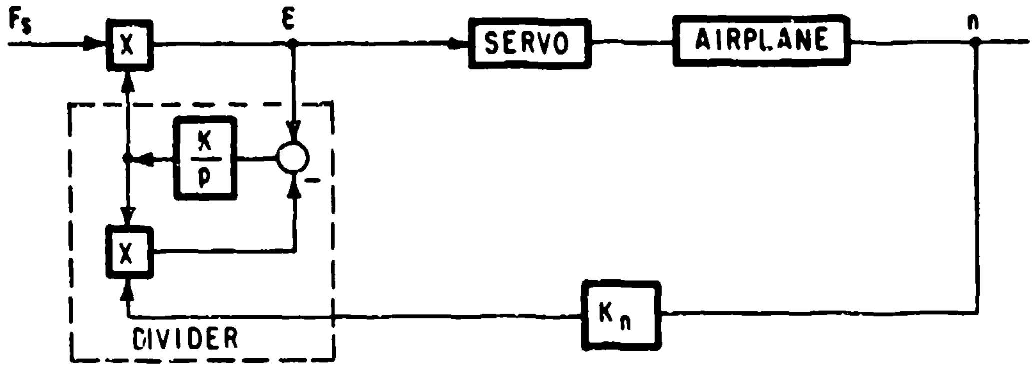


Figure 3.

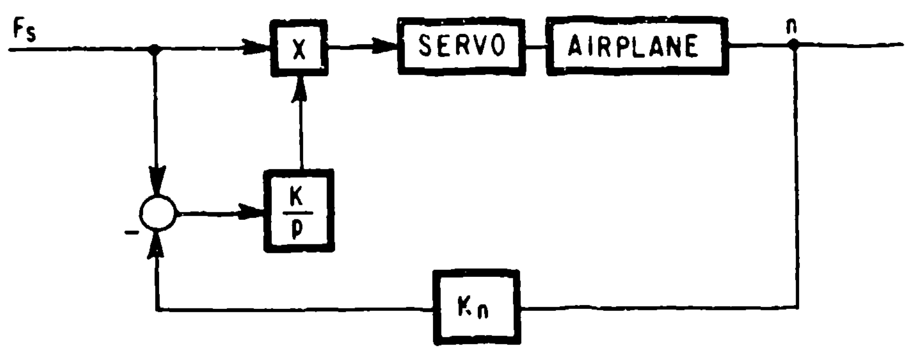


Figure 4.

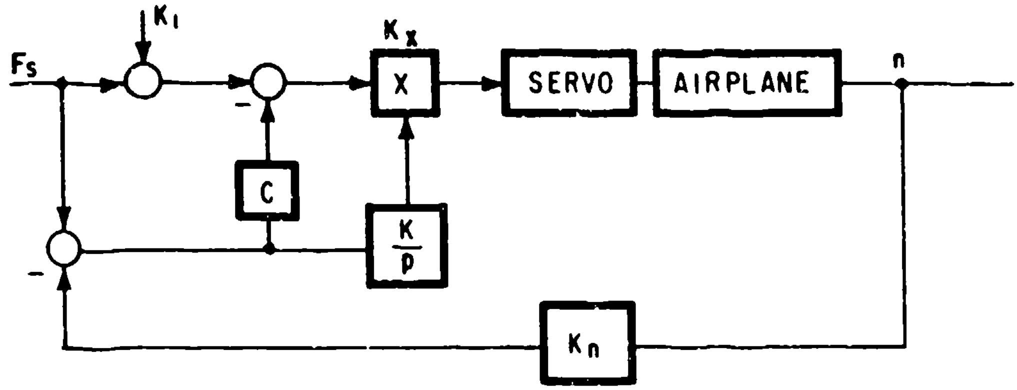


Figure 5.

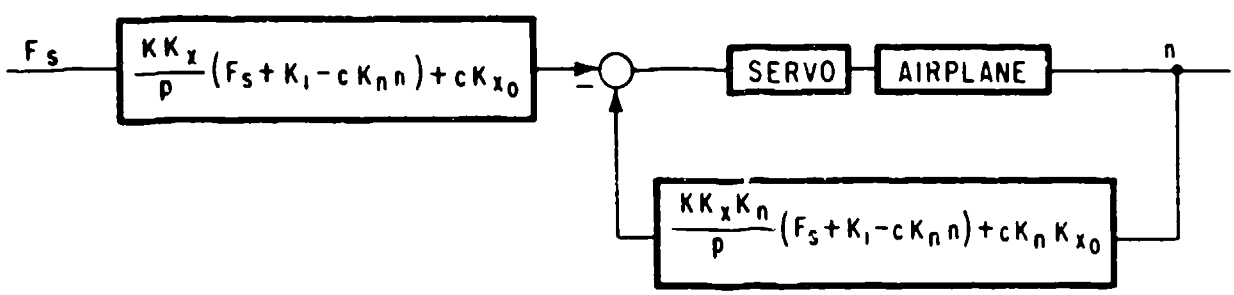
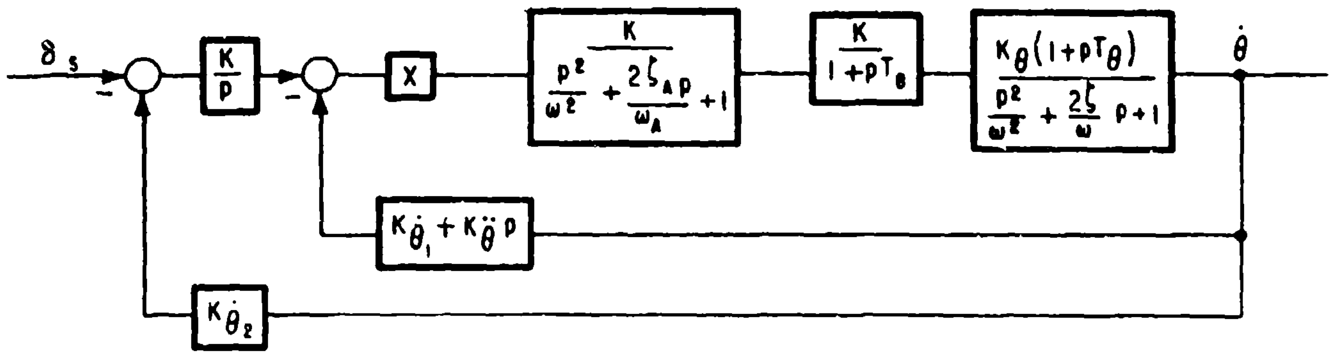


Figure 6.



δ_S = stick displacement

ω_A = parallel actuator resonant frequency

ζ_A = parallel actuator damping ratio

ω = airplane short period frequency

ζ = airplane short period damping ratio

K_θ = airplane short gain

$K_{\dot{\theta}_1}$ and $K_{\dot{\theta}_2}$ = rate gyro gradients

$K_{\ddot{\theta}}$ = angular accelerometer gradient.

Figure 7.

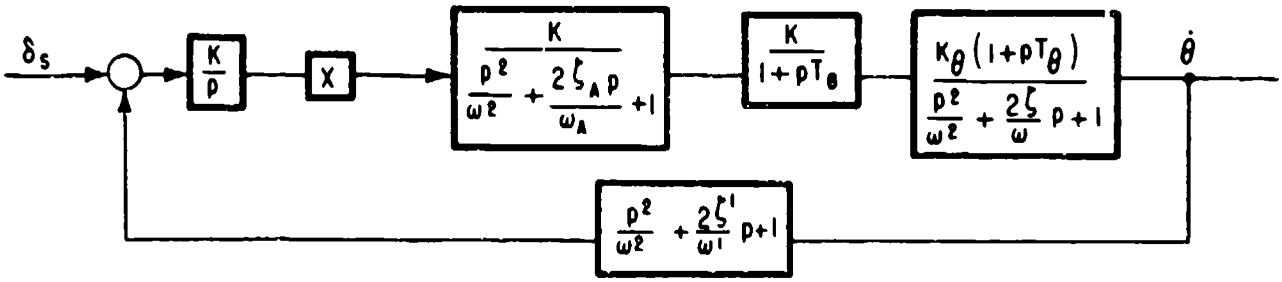


Figure 8.

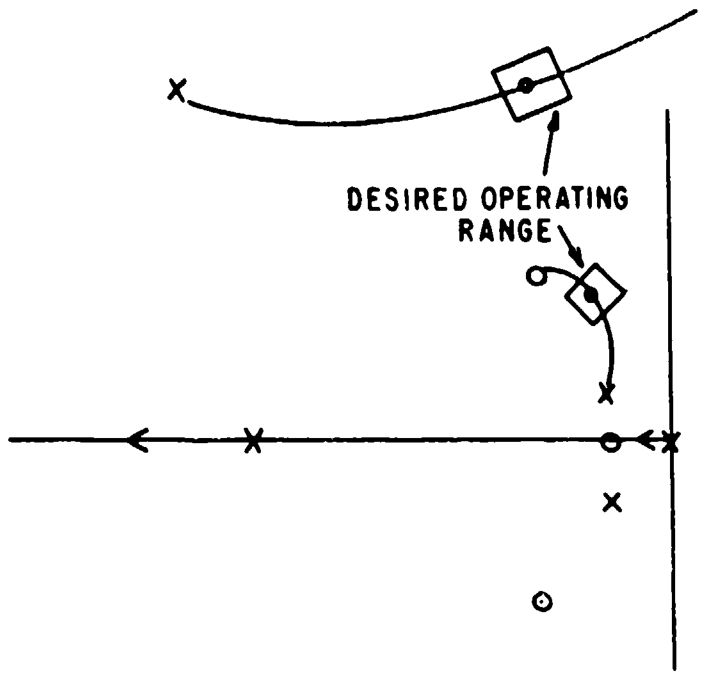


Figure 9.

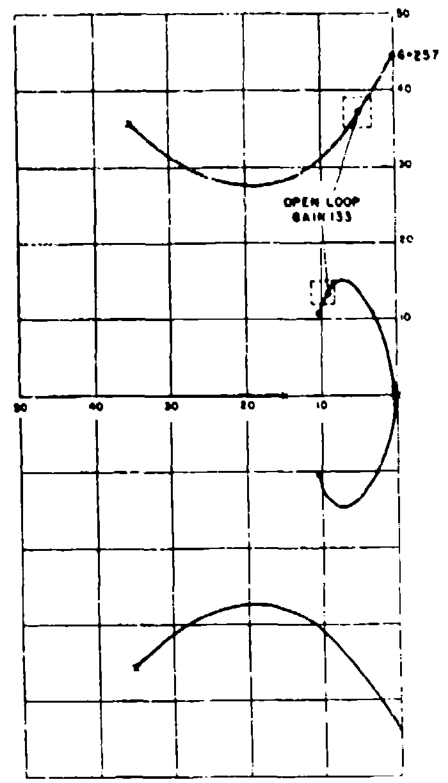


Figure 10. Linear System Having Complex Feedback

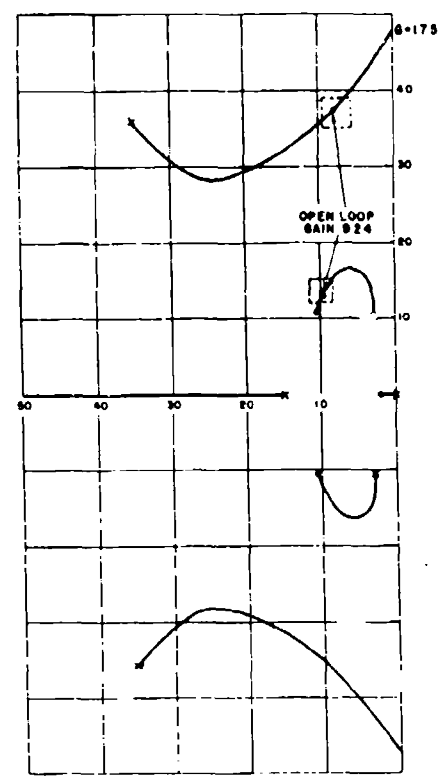


Figure 11. Linear System Having Complex Feedback

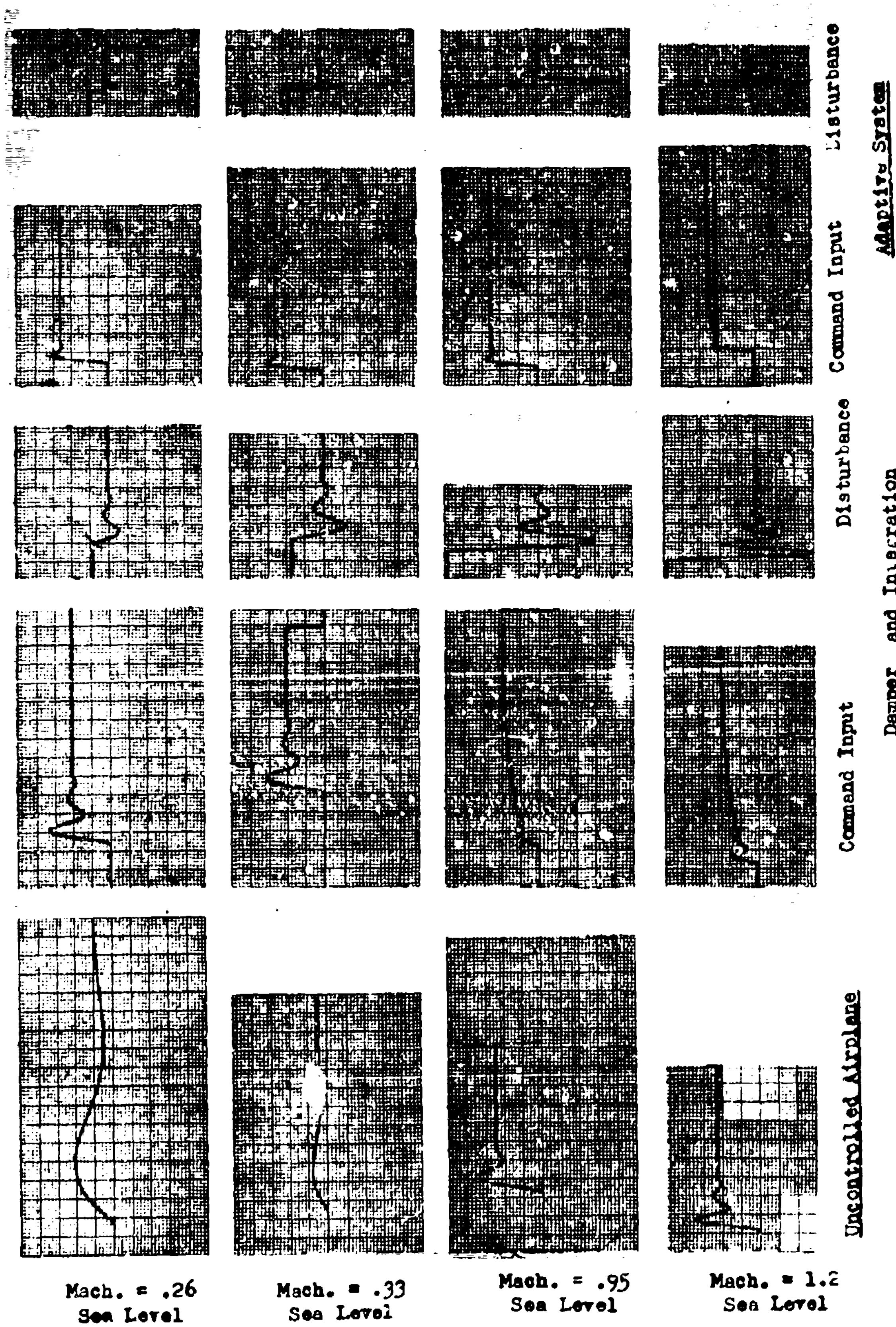
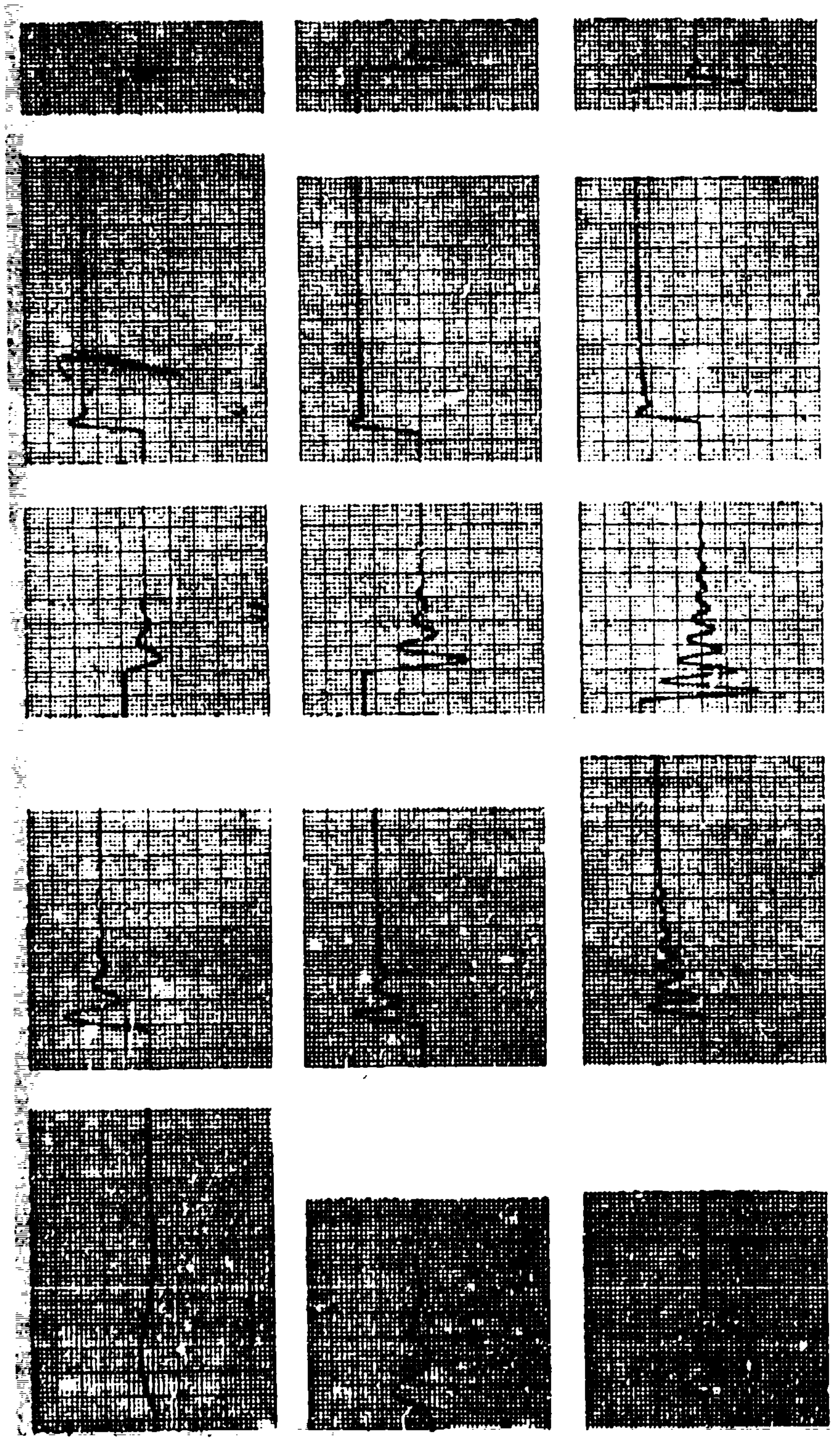


Figure 12. Comparison of Transient Response Data (Part 1)



Mach. = .4 20,000'

Mach. = .95 20,000'

Mach. = 1.5 20,000'

Figure 12. Comparison of Transient Response Data (Part 2)

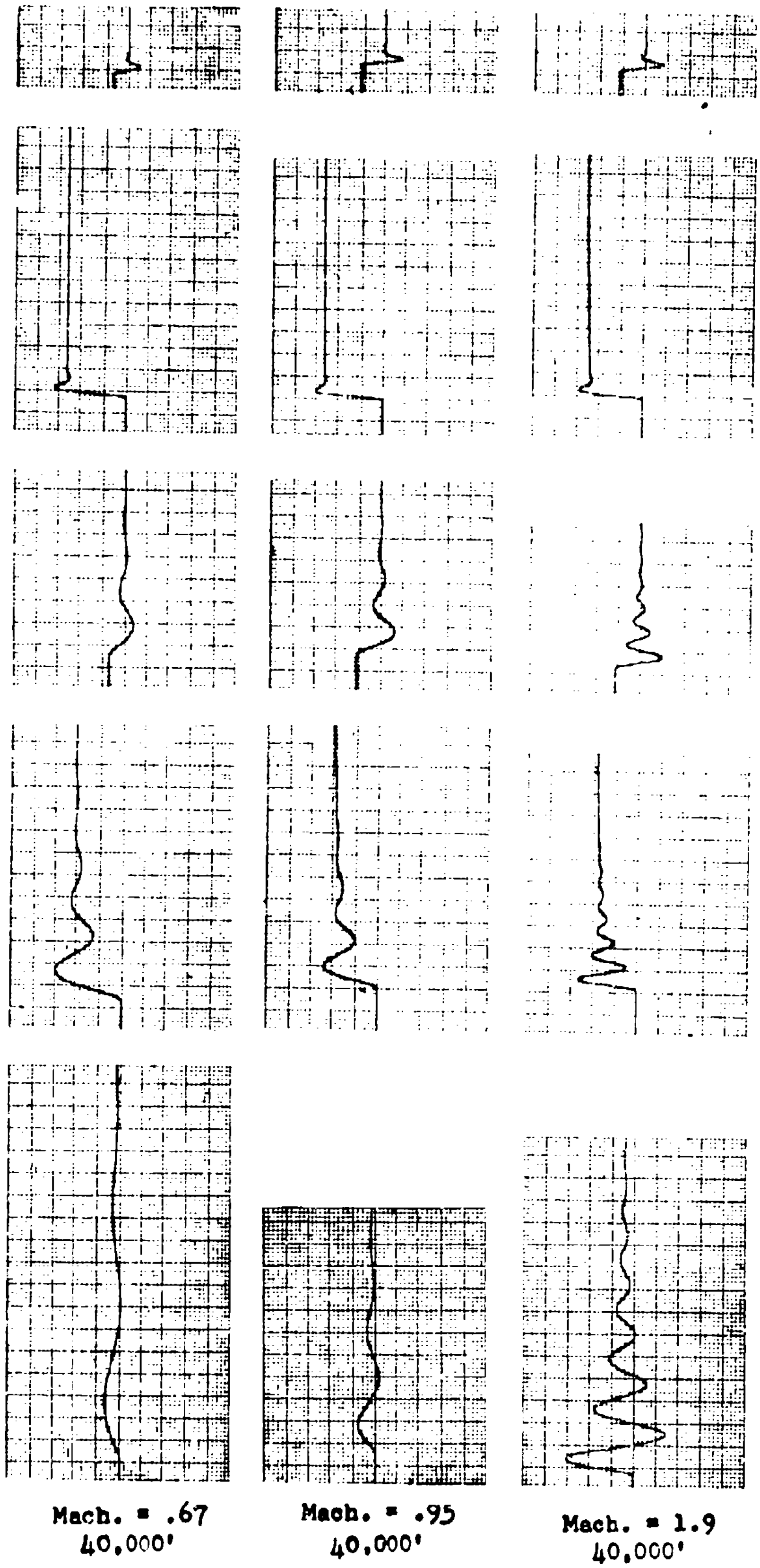


Figure 12. Comparison of Transient Response Data (Part 3)

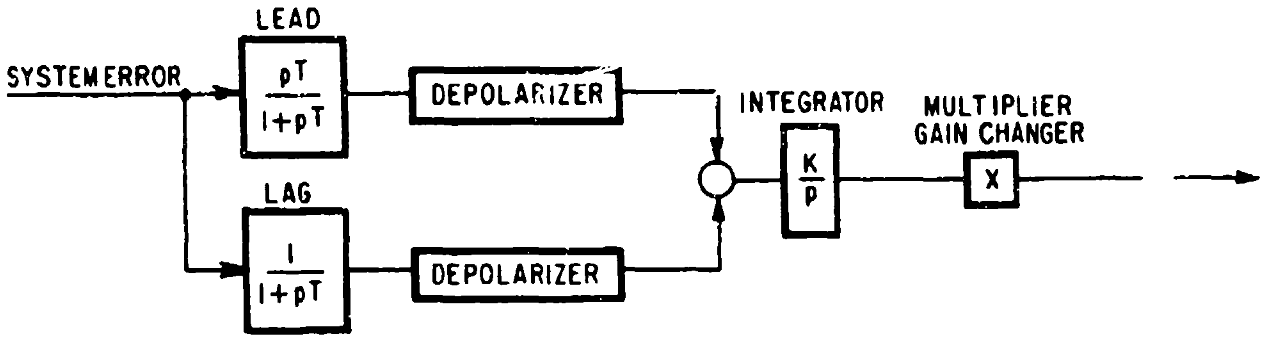


Figure 13.

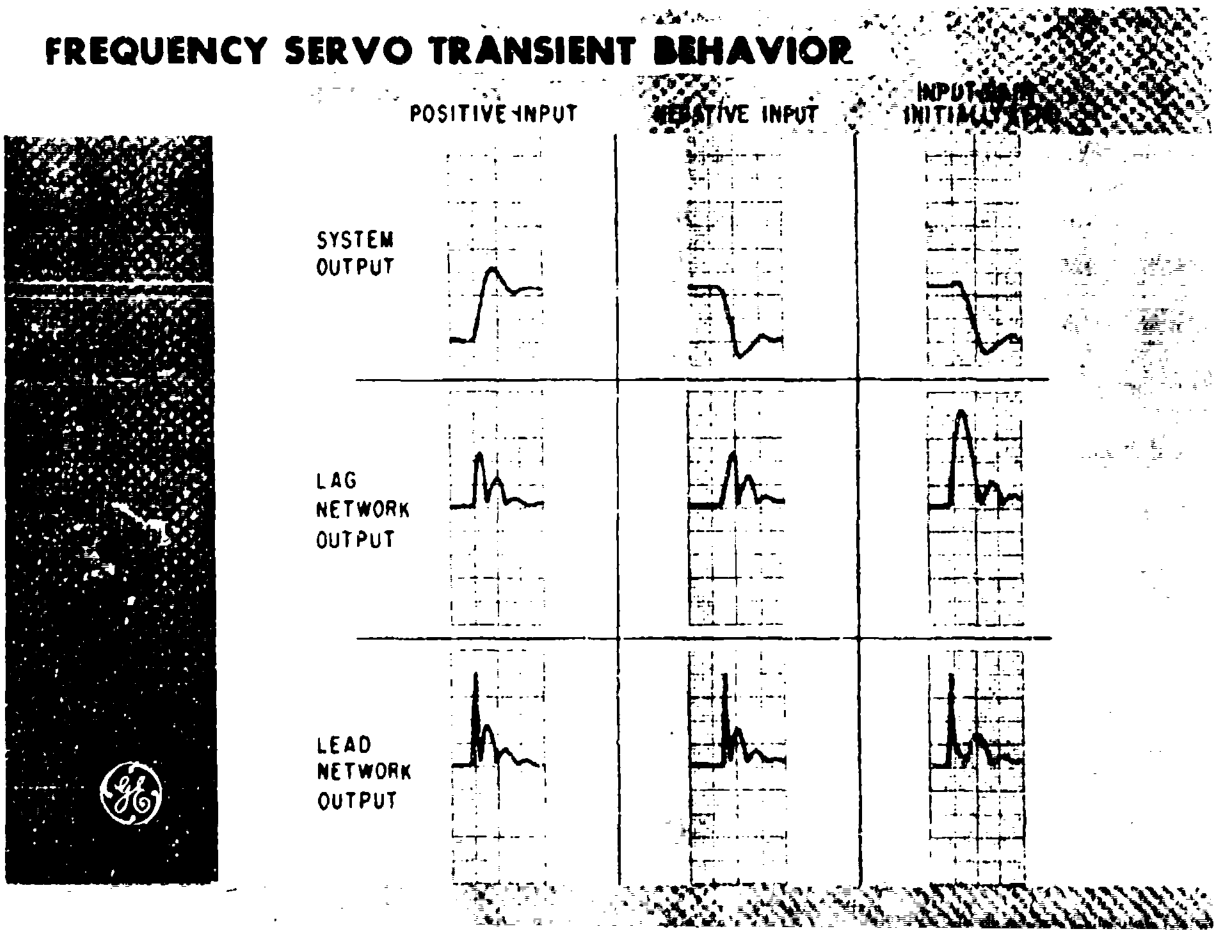


Figure 14.

DODCO, INC. RESEARCH IN OPTIMUM ADAPTIVE FLIGHT CONTROL

Roger L. Barron and Anthony J. Pennington

1.) BACKGROUND

DODCO, INC. is a small analytical research group located near Blawenburg, New Jersey in the countryside between Princeton and the Delaware River. The firm was founded in 1955 by Mr. Daniel O. Dommasch, then an Associate Professor of Aeronautical Engineering at Princeton University. In 1956 Mr. Dommasch left Princeton to devote full time to the rapidly expanding activities of the company. During that period, DODCO gained considerable initial momentum in the exhaustive study of advanced aircraft dynamic performance capabilities, and an energetic research team was created to focus attention on the exploitation and further development of certain dynamic performance concepts evolved by Mr. Dommasch while at Princeton.

As of the present time DODCO has published between 40 and 50 technical reports under contract with the U. S. Air Force, the Navy and private industry. This work has covered virtually the entire spectrum of aircraft dynamic performance and has branched off into a great variety of related problem areas. Primary emphasis has been placed on utilization of the Euler-Lagrange variational calculus and special techniques of dynamic performance (as perfected at DODCO) to predict optimum performance trajectories (paths in space) for high-speed aircraft and missiles. Some of the maneuvers analyzed are:

optimum climbs and descents

programmed-throttle optimum range

minimum-time turns

optimum flight under conditions of very low "q", including "zoom" maneuvers and turns occurring at less than one "g" normal loading

least radiation hazard (or escape) paths

optimum boost of semi-orbital and orbital vehicles

optimum re-entry for controlled orbital equipment

In addition, a number of non-optimum topics have been treated:

dynamic tilt-plane turns

effects of engine control acceleration time on aircraft performance

perturbations on satellite orbits due to earth oblateness and motion of the satellite relative to the sun, viewing the latter as center of inertial coordinates

dispersion of ballistic re-entry vehicles

achievement of optimum supersonic range through program control

On the basis of these methods (for which proven computer programs are available) it has become possible to push aircraft to the very limits of their dynamic performance capability. But, the operational attainment of such performance and successful flight on the performance envelope itself, have posed severe problems in flight-path stabilization and control. For example, an optimum "zoom" maneuver which carries an aircraft to altitudes far above its normal static ceiling poses unusual demands on the controller-airframe combination due to the extremely low dynamic pressure acting on the aerodynamic control surfaces. If we are to gain the significant tactical advantages implicit in operational utilization of optimum flight at altitudes in excess of the static ceiling, we must provide control systems having a high degree of anticipation and adaptability in order to surmount the inherent lag and sluggishness of control surfaces under these conditions.

2.) NATURE OF THE AIRCRAFT CONTROL PROBLEM

Before attempting discussion of optimum adaptive control configurations, it will be worthwhile to consider for a moment the nature of the aircraft control problem. If we restrict our thoughts to airplane motion within a vertical plane, and if we ignore the effects of structural elasticity, then the longitudinal equations of force and moment balance are (see Figure 2:1):

along the flight path

$$F_e \cos\alpha - D - m\dot{V} - mg\sin\gamma = 0 \qquad \text{2:1}$$

normal to the flight path

$$F_e \sin\alpha + L - mV\dot{\gamma} - mg\cos\gamma = 0 \qquad \text{2:2}$$

moments about the mass center

$$M_S + M_D + M_C - \frac{d}{dt}(J\dot{\theta}) = 0 \qquad \text{2:3}$$

where

F_e = effective engine thrust

α = angle of attack (between vehicle body axis and velocity vector)

D = drag = $qS(C_{D_e} + KC_L{}^2)$

m = mass = W/g

γ = inclination of velocity vector to local horizontal

L = lift = qSC_L = $qSa\sin\alpha$

M_S = static stability moment = $qSc\,\alpha C_{m_\alpha}$

M_D = damping moment = $qSc(\dot{\alpha}\,C_{m_{\dot{\alpha}}} + \dot{\theta}C_{m_{\dot{\theta}}})$

M_C = control moment = $qSc(\delta\,C_{m_\delta} + \dot{\delta}C_{m_{\dot{\delta}}})$

J = polar moment of inertia in pitch = $f(t)$

θ = angle between vehicle body axis and local horizontal = $\alpha + \gamma$

This, in simplest form, is the complete longitudinal dynamics picture. Should we, however, wish to consider flight at speeds in excess of about Mach 3.5, then additional terms must be incorporated to account for "orbital relief" effects, and at altitudes in excess of approximately 150,000 feet we must include the variation of gravity potential with altitude.

Using coefficient notation, we may write the moment equation (2:3) in the form

$$\ddot{\theta} = \frac{qSc}{J}\left[\alpha C_{m_{\alpha}} + \dot{\alpha} C_{m_{\dot{\alpha}}} + \dot{\theta}(C_{m_{\dot{\theta}}} - \dot{J}/qSc) + \delta C_{m_{\delta}} + \dot{\delta} C_{m_{\dot{\delta}}}\right] \quad \ldots\ldots\ldots\ldots 2{:}4$$

The $\dot{J}$ term in this expression is of considerable interest; it arose during total differentiation of the product $J\dot{\theta}$ in the basic moment equation, and its appearance in 2:4 indicates that time rate of change of polar moment of inertia may produce an alteration in the apparent damping due to $\dot{\theta}$. The $\dot{J}$ effects may become quite significant during periods of high fuel flow rate, such as occurs with afterburner use.

The control of an aircraft involves a surprisingly long sequence of events, which (essentially in the order of their occurrence) are:

1. The pilot reacts to a combination of stimuli.
2. A control-stick motion or force applied by the pilot is converted to a command signal by the controller.
3. Hydraulic fluid flows in the servo actuator.
4. The control surface undergoes acceleration.
5. Once sufficient time has elapsed, a significant change in control surface displacement has occurred, and thus the aerodynamic flow pattern about the elevator begins to change.
6. After another delay, the new aerodynamic circulation field is obtained, and a new resultant force is produced on the control surface.

7. This control force will, in general, alter $\ddot{\theta}$.

8. The angular acceleration integrates to a value of θ, which occurs, initially, primarily in the form of an α increment.

9. The new value of α changes (in the course of time) the circulation about the wings and the resulting lift.

10. The change in lift produces a change in $\dot{\gamma}$.

11. $\dot{\gamma}$ integrates to a new flight path inclination γ.

With unfavorable conditions, the cumulative effects of the various lags just mentioned may produce a very critical "dead time", which can (in some cases) render aerodynamic controls virtually useless. *

We mention these rather gloomy considerations because they have considerable bearing on the design of optimum configurations for adaptive controllers. Mathematically, we are interested in certain partial derivatives of θ, $\dot{\theta}$ and $\ddot{\theta}$ with respect to control displacement δ; it follows that:

$$\partial\theta/\partial\delta = 0 \quad \ldots\ldots 2{:}5$$

$$\partial\dot{\theta}/\partial\delta = 0 \quad \ldots\ldots 2{:}6$$

$$\partial\ddot{\theta}/\partial\delta = qScC_{m_\delta}/J \equiv \beta_0 \quad \ldots\ldots 2{:}7$$

for any instant of time t.

The problem of aircraft control is fundamentally one of generating appropriate command signals for the control-surface actuator. As we have seen, displacements of the control surface ultimately have an effect on aircraft angle of

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* Professor D. C. Hazen of Princeton University has reported a 180 degree phase difference between the angular position of an airfoil oscillating at 5 c.p.s. and the resultant lift direction. This data was obtained at a low airspeed (30-40 f.p.s.). It is probable that the frequency for 180 degree lag is roughly proportional to airspeed.

attack, which in turn must prescribe simultaneous values of $\dot{V}$ and $\dot{\gamma}$, obtainable (without ambiguity*) from the equations of motion (2:1, 2:2). In the course of its flight the aircraft "integrates" these differential relationships to give values of V and γ which are everywhere compatible with the "path function", expressed either as $\alpha(t)$, $\theta(t)$ or $\delta(t)$ with, of course, appropriate initial conditions where required. Inasmuch as α and θ are directly related through γ, they may be viewed as alternative variables; however, the specification of a $\delta(t)$ demands that we employ the moment balance equation (2:4).

From the adaptive control systems point of view, the aircraft equations of force and moment balance, along with such relations as might exist between servo-actuator command and output, constitute the fundamental system properties in mathematical form. It is important that we recognize the inseparable nature of these relationships, that is to say, that a process of continuous interaction is going on between them and that--therefore--no single equation can possibly express the total dynamic situation. This fact remains just as true in the limit (at a localized point in space and time) as it does for the entire integrated path.

The crux of the adaptive control problem is that we seldom have adequate foreknowledge of the various system coefficients (and there are many) within the pertinent set of equations. Handicapped as we are by this lack of vital information, we are forced to base control system adaptation on the instantaneous values of pilot command and existing system output, along with certain lower-ordered derivatives of these quantities.

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* If we specify $\dot{V}$ rather than α, there are at least two values of $\dot{\gamma}$ which will satisfy the equations of motion. This fact is readily established by eliminating α between equations 2:1 and 2:2.

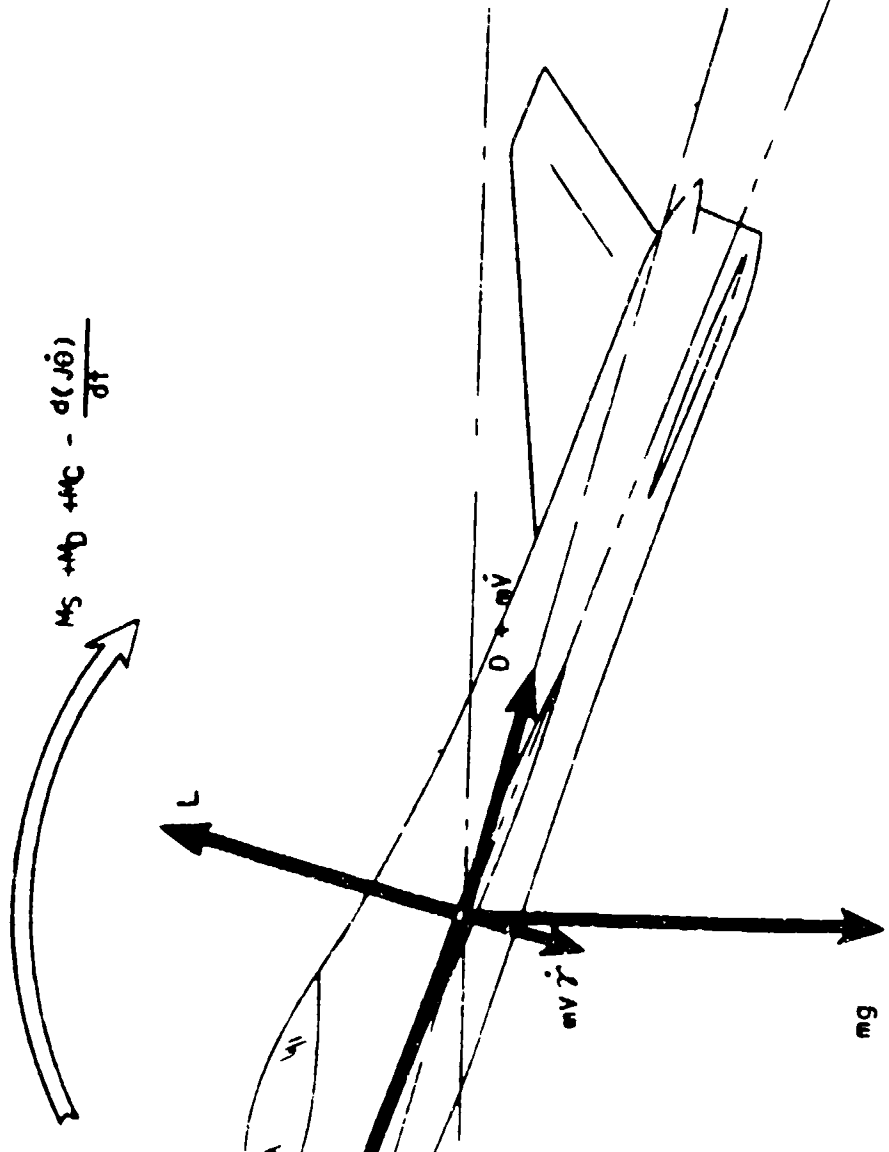


Figure 2:1

MOTION OF AN AIRCRAFT
WITHIN THE VERTICAL PLANE

3.) ADAPTIVE CRITERIA AND THEIR VARIATIONAL INTERPRETATION

For the problem we have just defined, namely, control with limited information about the dynamic elements involved, one may postulate a number of plausible adaptive criteria. Much of the adaptive control work undertaken by DODCO has been concerned with the relative merits of various criteria expressible in integral form, and particularly, with the variational calculus interpretation of these integrals.

In writing integral criteria, we observe that, although various coordinate transformations may be possible, time remains the fundamental independent variable for all dynamic systems. Thus it is appropriate to write generalized integral criteria in the form

$$\psi = \int_0^t G \, dt \quad \text{.......................................} 3{:}1$$

where G might be referred to as the "variational integrand" and the specification of the upper limit t signifies that this is a <u>definite</u> integral with which we are dealing (in practice, the value of t need not be known in advance).

For purposes of direct comparison with existing (arbitrary) adaptive flight controllers, we may define system "error" as the difference between the output of a linear model (which is continuously responding to the pilot's commands) and a measure of the aircraft pitch rate, $\dot{\theta}$. Hence

$$e = \dot{\theta}_m - \dot{\theta} \quad \text{.......................................} 3{:}2$$

where $\dot{\theta}_m$ is the model value. In terms of this definition, several variational integrands suggest themselves immediately, for example

$$G = e^2$$

$$G = t/e/$$

These functions (and a number of other related forms) have received considerable attention in linear analyses, and we might therefore expect them to yield useful results when subjected to variational treatment (by which we mean other than the statistical approach). If we set aside for the moment any considerations of fundamental system restraints (which may require introduction of one or more Lagrange multipliers) we may rapidly examine integrands of the type just given to ascertain the nature of their variational extremums, if any.

Since we are dealing with an aircraft control problem, the elevator angular displacement δ represents (to a large extent) the basic Euler "freedom" within the system, that is, the primary dependent variable in the Euler sense. The secondary dependent quantities (governed by δ acting within the framework of the system equations) are, presumably, θ, $\dot{\theta}$ and $\ddot{\theta}$.

Now if we recall the conclusions summarized in the partial derivative relations 2:5, 2:6 and 2:7 it is (perhaps rather painfully) evident that $\partial\ddot{\theta}/\partial\delta$ is the only such parameter which does not vanish explicitly from any formulation. The blunt interpretation of this fact is that δ cannot in any way alter the value of e (equation 3:2) in less than a finite time interval, and hence that no valid "time now" variational formulation exists which can handle a $G = e$ type integrand .

This may seem somewhat disappointing, and in fact, we have considered going to a small-interval prediction scheme so as to forcefully introduce θ and $\dot{\theta}$ dependence on elevator deflection. But this, however, proves unnecessary, for the real fault may be shown to lie within the postulated criterion.
In the course of our study of the problem at DODCO, we have arrived at the following general approach to the specification of adaptive criteria:

1. It is the task of the control system used to "control with authority" and therefore the unrestrained integrand must be based on a term which demands this accomplishment.

2. To prevent excessive actuation levels, and most particularly, to provide constraints on the error and error derivatives built up along the path, a subsidiary condition must be introduced which demands simultaneous satisfaction of the error requirements peculiar to the given system.

To be specific, the first item mentioned leads to the following term within the integrand G

$$\frac{1}{\beta_o \Delta\delta}$$

where β_o is defined as the value of $\partial\ddot{\theta}/\partial\delta$ (see 2:7) and $\Delta\delta$ is defined as the hypothetical instantaneous jump in the displacement of the elevator which we should like to have occur at the moment of time in question.

The second item discussed may lead variously to restraints of the form

$$\frac{\lambda}{e + K\dot{e}} \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{A}$$

$$\frac{\lambda}{e + K_1\dot{e} + K_2\ddot{e}} \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{B}$$

$$\frac{\lambda}{(1+t/\tau)e + K\dot{e}} \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{C}$$

$$\frac{\lambda}{e_c + e + K\dot{e}} \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{D}$$

and

$$\frac{\lambda}{\dfrac{e}{\dot{e}^2} + \dfrac{K_1\dot{e}}{e^2} + A_1 e + A_2\dot{e}} \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{E}$$

in which λ is the Lagrange multiplier (a constant due to the definite integral form of the subsidiary condition); A_1, A_2, K, K_1, K_2 and τ are positive constants (other than zero) and e_c is what we might best term the

"feed-forward error", given by

$$e_c = \dot{\theta}_c - \dot{\theta} \quad \text{3:3}$$

where the subscript "c" denotes the value of the pilot's command.

Let us consider restraint "A", the simplest of those listed. Using it, the complete integrand becomes

$$G = 1/\beta_0 \Delta\delta + \lambda/(e + K\dot{e}) \quad \text{3:4}$$

It should be emphasized that variational integrands of the type given in 3:4 are not necessarily the most appropriate when almost everything is known in advance about system dynamic properties, or when reasonably accurate measurements may be readily made which will yield the needed information. Functions of this type do, however, appear to work out very well in terms of the strict <u>adaptive</u> control problem, for which there exists a definite paucity of either advance or in-flight measured data regarding characteristics of the aircraft.

Variationally, we seek to find extremums of the integral of G as expressed in equation 3:4. It is possible to show that the extremums obtained by an application of the pertinent Euler relation are those which involve maximums of the product $\beta_0 \Delta\delta$, subject of course to the stipulated condition on e and $\dot{e}$. Since the quantity β_0 is precisely the instantaneous control effectiveness, it is evident that the extremums will produce greater elevator deflection changes $\Delta\delta$ when the control effectiveness is small, yet will be "satisfied" with lesser deflections when the effectiveness is large.

As for the subsidiary condition, it follows from 3:4 that we are demanding that the integral values of e and $\dot{e}$ (between the limits 0 and t) possess the particular values fore-ordained by our choice of λ. Thus we are, mathematically,

insisting that the system produce a certain amount of

$$\int_0^t (e + K\dot{e})\, dt$$

No system can ever be perfectly free of error in its response to commands and disturbances, so from the practical point of view this restraint is tantamount to a stringent limitation on the build-up of error. Furthermore, the restraint prevents the variationally-correct but unwanted and trivial solution which says, "set $\Delta\delta$ to infinity."

4.) APPLICATION OF THE VARIATIONAL METHODS

The integrand given in equation 3:4 may be readily analyzed to obtain an expression for the optimum $\Delta\delta$ as a function of time. Treating $\Delta\delta$ as the aircraft system freedom, we ask the question: "What is the optimum way in which to vary $\Delta\delta$ with time so that the adaptive performance index (integral criterion) is rendered a minimum?"

The pertinent Euler variational relation is

$$\frac{d}{dt}(\partial G/\partial \Delta\dot{\delta}) - \partial G/\Delta\delta = 0 \quad \text{.......} 4:1$$

which constitutes the necessary condition which the controller must satisfy. Evaluating the various derivative terms, we obtain:

$$\partial G/\Delta\dot{\delta} = 0 \quad \text{(ignoring the very small } C_{m_{\dot{\delta}}} \text{ moment contribution)} \ \text{...} 4:2$$

$$\partial G/\Delta\delta = -1/\beta_o(\Delta\delta)^2 \ -\lambda(K\partial\dot{e}/\partial\Delta\delta)/(e + K\dot{e})^2 \quad \text{.......} 4:3$$

But

$$\partial\dot{e}/\partial\Delta\delta = -\partial\ddot{\theta}/\partial\Delta\delta = -\beta_o \quad \text{.......} 4:4$$

We note in this connection that β_o is independent of $\Delta\delta$, although saturation (control-surface stall) effects cause β_o to vary with δ. Combining the four equations just set forth, and solving for $\beta_o\Delta\delta$ gives

$$\beta_o\Delta\delta = (e + K\dot{e})/\sqrt{\lambda K} \quad \text{.......} 4:5$$

Were the instantaneous value of β_o known as a function of time, equation 4:5 would yield an immediate answer in terms of the optimum $\Delta\delta$. Although measurement of the control effectiveness does not present an impossible air-data problem, it is desirable to free ourselves as much as we can from the necessity of measuring or computing such a parameter during flight.

To circumvent the need for measurement or calculation of β_o, it is only necessary that we recognize the relationship between increments in angular acceleration and increments in elevator position; thus

$$\beta_o \Delta\delta = \Delta\ddot{\theta} \quad \text{.......................................} 4:6$$

This expression is especially useful because it takes into account all the various lags which are involved in producing a $\ddot{\theta}$ response to elevator deflection.

Presuming the existence of a small airborne analog or digital computer capable of generating desired values of $\Delta\ddot{\theta}$, all that remains for us to accomplish within the system is the algebraic addition of this desired increment to the actual angular acceleration existing at the point in question, and then the excitation of an acceleration control loop which causes the aircraft to follow the desired total $\ddot{\theta}$ with a reasonable degree of precision. This inner acceleration loop (which encompasses the servo-actuator and the aircraft) might also contain some form of optimum adaptive compensation, but for our present purposes it is sufficient to think of it as simply a fairly high-gain conventional loop driven by an error signal defined as

$$\epsilon = \ddot{\theta}_d - \ddot{\theta} \quad \text{.......................................} 4:7$$

where $\ddot{\theta}_d$ is the desired total $\ddot{\theta}$ obtained on a sampled basis from the control computer, in accordance with the relation

$$\ddot{\theta}_d = \ddot{\theta}_{actual} + \Delta\ddot{\theta} \quad \text{.......................................} 4:8$$

The sampling interval should be of the order of 0.005 to 0.010 second.

Perhaps a few words regarding the general character of equation 4:5 (the optimum governing expression) are in order. It might have been anticipated that the results applicable to the limited-information control problem would not be especially complex, yet it is refreshing to find the variational methods leading to so straightforward a solution.

It is, further, of great interest that the variational calculus has led to a configuration which (from past experience) we know to be highly workable. Thus, the use of proportional-error and proportional error-rate control is known to produce very good response, with the possible exception being the steady-state condition, in which some integral error feedback may be of value.

It is clearly possible to specify alternate criteria which place more exacting demands on the control system and therefore, in general, exhibit superior response characteristics. One such restraint is that labeled "E" in the list above. In addition to the usual restriction on e and $\dot{e}$ we have introduced the terms $e/\dot{e}^2$ and $K_1\dot{e}/e^2$. The first of these places an exceptionally strong limitation on the build-up of steady-state-error; when $\dot{e}$ becomes small (as in the steady state), e is greatly emphasized and the controller is forced to reduce it vigorously. The term $K_1\dot{e}/e^2$ limits transient overshoot type errors, since when e becomes small in the presence of a large $\dot{e}$ (overshoot conditions) the $\dot{e}$ is strongly emphasized and therefore restrained. The use of this restraint condition leads to the relations

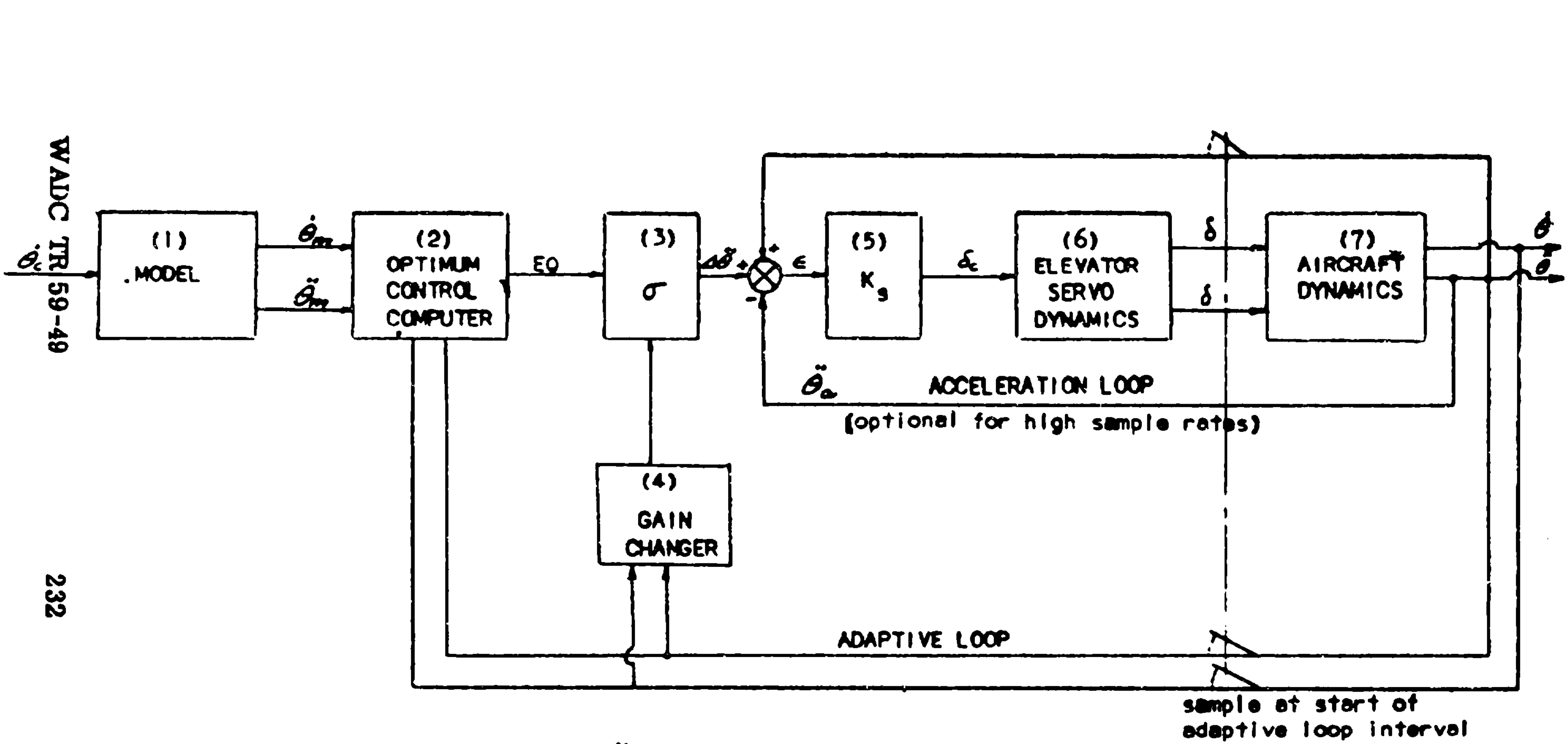
$$\Delta\ddot{\theta} = \beta_0 \Delta\delta = \sigma \left[\frac{e}{\dot{e}^2} + K_1 \frac{\dot{e}}{e^2} + A_1 e + A_2 \dot{e} \right] \qquad \ldots\ldots 4{:}9$$

$$\sigma = 1 \Big/ \sqrt{\lambda \left[A_2 + \frac{K_1}{e^2} - \frac{2e}{\dot{e}^3} \right]} \qquad \ldots\ldots 4{:}10$$

The factor σ may be considered as a variable optimum gain, since it multiplies the error quantity of the restraint "E" to produce the actuation signal, $\Delta\ddot{\theta}$. This is in accord with conventional servomechanism terminology regarding gain. A preliminary analysis of numerical results obtained using this controller indicates that the optimum gain plays a very important role.

Perhaps the greatest significance of these results lies in the clear-cut connection between them and the relevant limited-information integral criteria. For instance, it has been demonstrated that proportional-error plus proportional-error-rate control is actually the best that can be done in terms of the criterion of equations 3:1 and 3:4. Likewise, a configuration more sensitive to transient overshoot and steady state error conditions is derived from an integrand involving the restraint "E".

Figure 4:1 presents the optimum adaptive control configuration described by equations 4:9 and 4:10. In keeping with the previous discussion an inner acceleration control loop has been incorporated in the system.



(1) $\dddot{\theta}_m = \omega_{n_m}{}^2 (\dot\theta_c - \dot\theta_m) - 2\zeta_m \omega_{n_m} \ddot\theta_m$

(2) $(EQ) = e/\dot e^2 + K_1 \dot e/e^2 + A_1 e + A_2 \dot e$

(3) $\Delta\ddot\theta_d = \sigma \cdot (EQ)$

(4) $\sigma = 1\Big/\sqrt{(\lambda)(M/e^2 + A_2 - 2e/\dot e^3)}$

(5) $\delta_c = K_s \quad \epsilon$

(6) $\ddot\delta = \omega_{n_s}{}^2 \delta_c - 2\zeta_s \omega_{n_s} \dot\delta - \omega_{n_s}{}^2 \delta$

(7) $\ddot\theta = q\frac{Sc}{J} (\alpha C_{m_\alpha} + \dot\alpha C_{m_{\dot\alpha}} + \dot\theta C_{m_{\dot\theta}}$

$+ \dot\delta C_{m_{\dot\delta}} + \delta C_{m_\delta}) - \dot\theta (\dot J/J)$

Figure 4:1
OPTIMUM ADAPTIVE CONTROL SYSTEM BLOCK DIAGRAM

5.) DIGITAL COMPUTER RESULTS AND CONCLUDING REMARKS

Figures 2:2 through 2:9 on the next three pages present curves of the various aircraft coefficients and engine thrust values as functions of Mach number and (in some cases) Mach number plus altitude or elevator deflection. These coefficient values have been arranged in tabular form suitable for rapid table-lookup routines prepared especially for the LGP-30 stored program computer. The actual numbers assigned to the coefficients represent a purely hypothetical aircraft, and no aircraft with these exact characteristics is known to exist.

These data have been utilized in the ABLE computer program (Adaptive Behavior in Longitudinal Environment); the equation flow within this program follows essentially the pattern given in the Appendix to this paper.

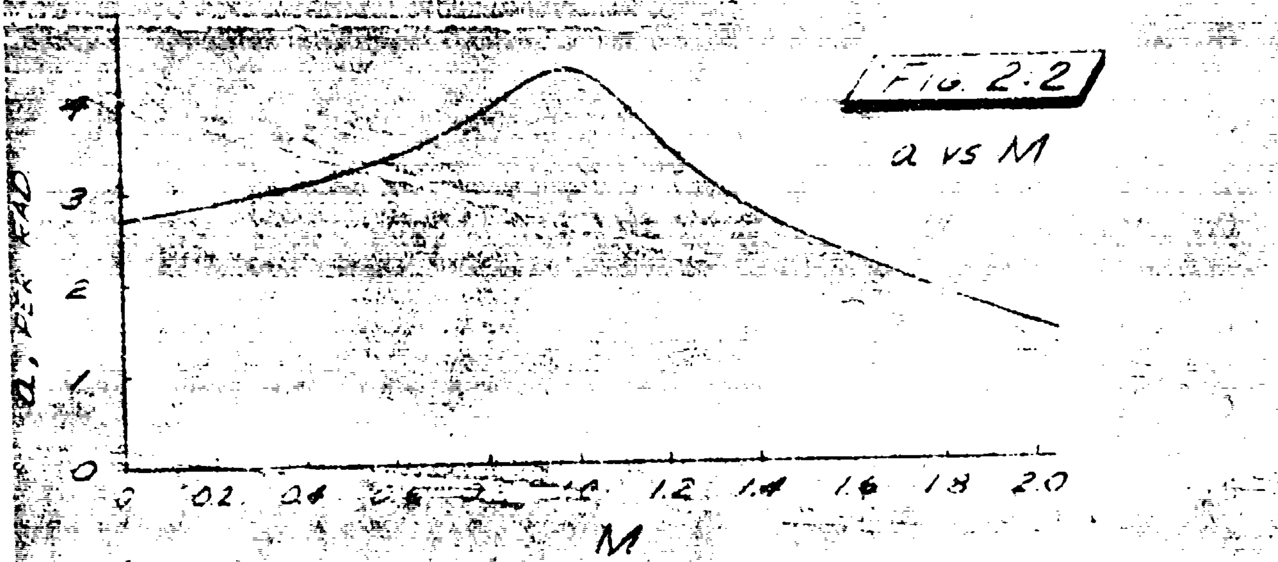
The figures on the succeeding pages of this section present preliminary numerical results obtained with the ABLE routine, employing two different optimum control relations and several different initial conditions, as well as acceleration-loop gain constants. Figure 5:1 is devoted exclusively to the response characteristics of the elevator-servo plus aircraft combination, and illustrates the acceleration-control servo dynamic properties as well as the nature of the aircraft $\ddot{\theta}$ buildup (assuming no circulation buildup lag, which is essentially true at these altitudes and Mach numbers). Figures 5:2 -- 5:4 are detailed plots of the aircraft response, elevator displacement, and parameter variations for a ramp input command from the pilot. The controller used was a linear optimum configuration derived from a combination of restraints C and D, and thus involved both ti -weighted and feed-forward errors. The final four figures in this section represent data obtained using the optimum variable-gain controller described by equations 4:9 and 4:10, as compared with an arbitrary configuration developed from empirical considerations.

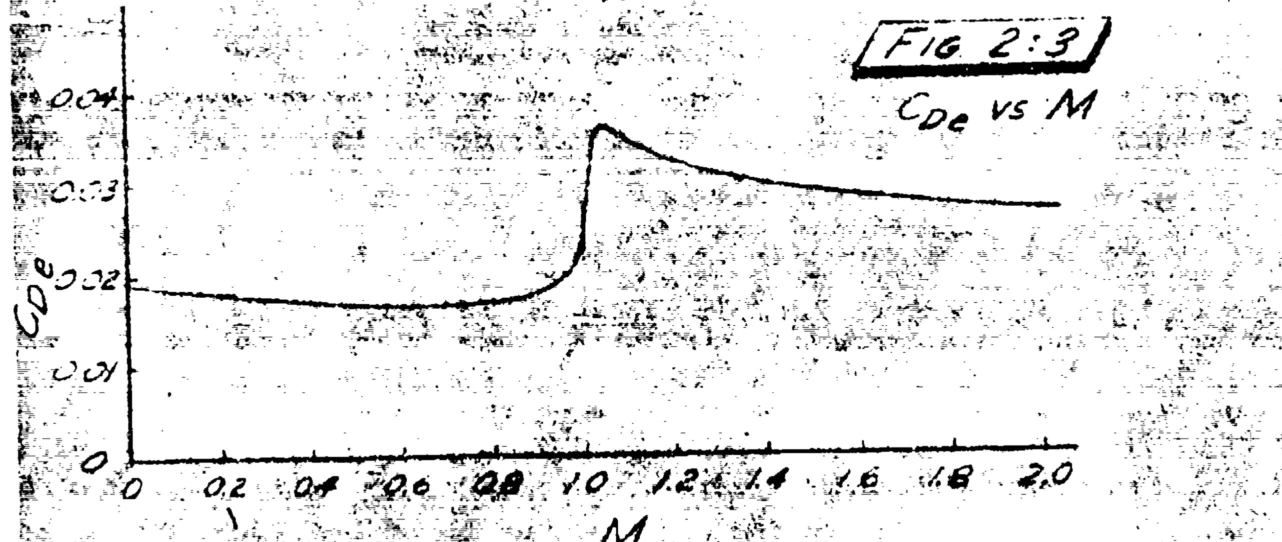
The first two figures (5:5 and 5:6) show the response to a step input command; it can be seen that the optimum variable-gain configuration response follows the second order model characteristics almost identically. It is doubtful that following errors below this level could be sensed. Figures 5:7 and 5:8 show the response to a large amplitude, high frequency sinusoidal input obtained with the variable gain and arbitrary controllers, respectively.

A great number of problem areas still remain to be investigated as a part of the contractual effort in which DODCO is engaged, however, we feel that we have gained some degree of initial orientation in the adaptive control subject. The power of the variational calculus in the design of high performance control configurations for the case of limited system information has been demonstrated, and there is every reason to believe that even higher levels of sophistication may be attained, both as understanding of integral criteria increases and as the methods are applied to problems for which we possess somewhat more advance or measurable data regarding the system dynamic properties. The preliminary results of a complete digital-computer simulation of aircraft response would seem to indicate that therein exists a very fruitful avenue for future exploitation.

ABLE - VEHICLE CHARACTERISTICS

(TRIMMED FLIGHT)





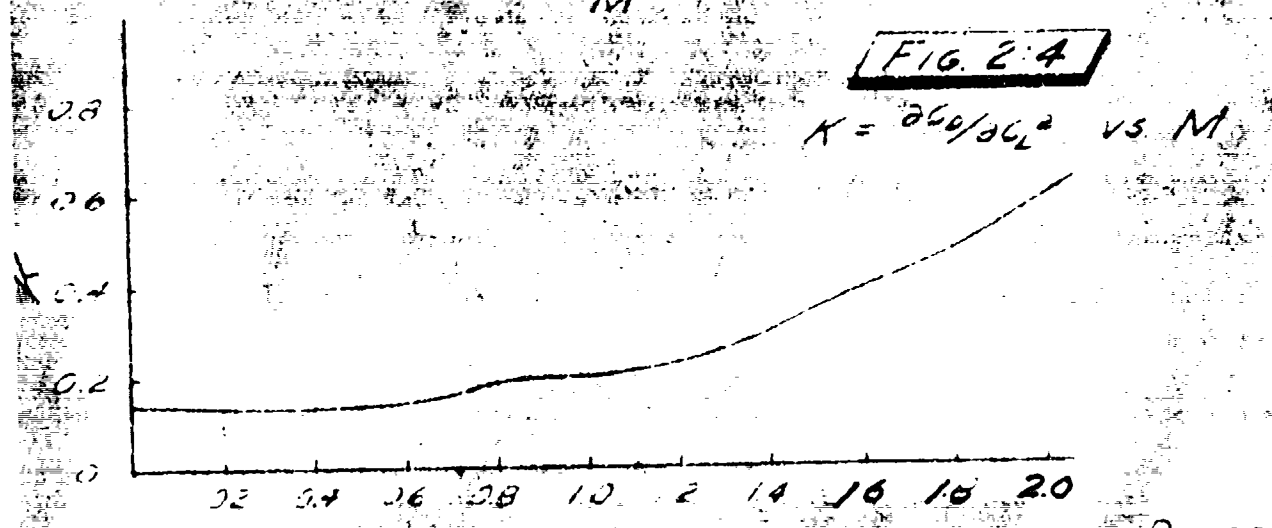


Fig 25

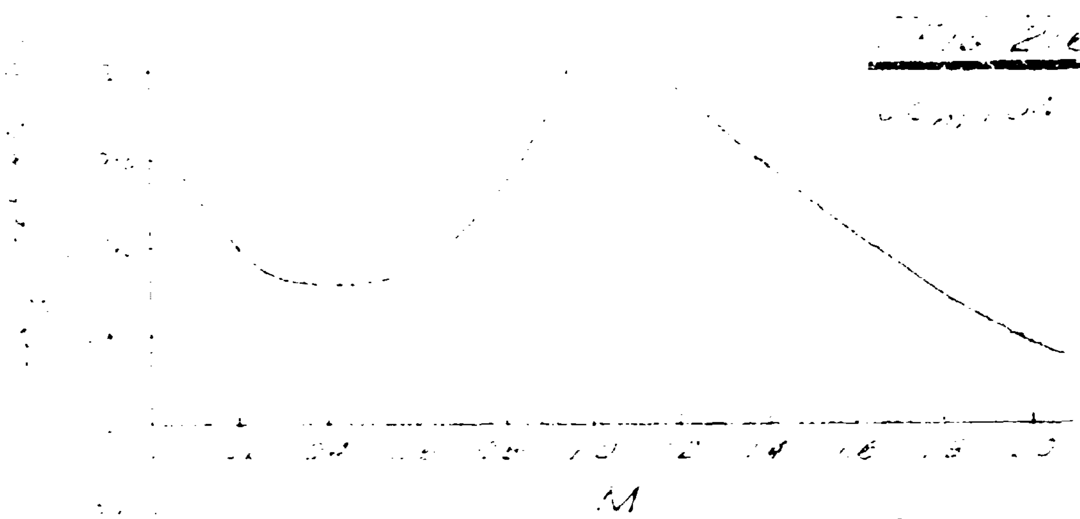


Fig 26

Fig 27

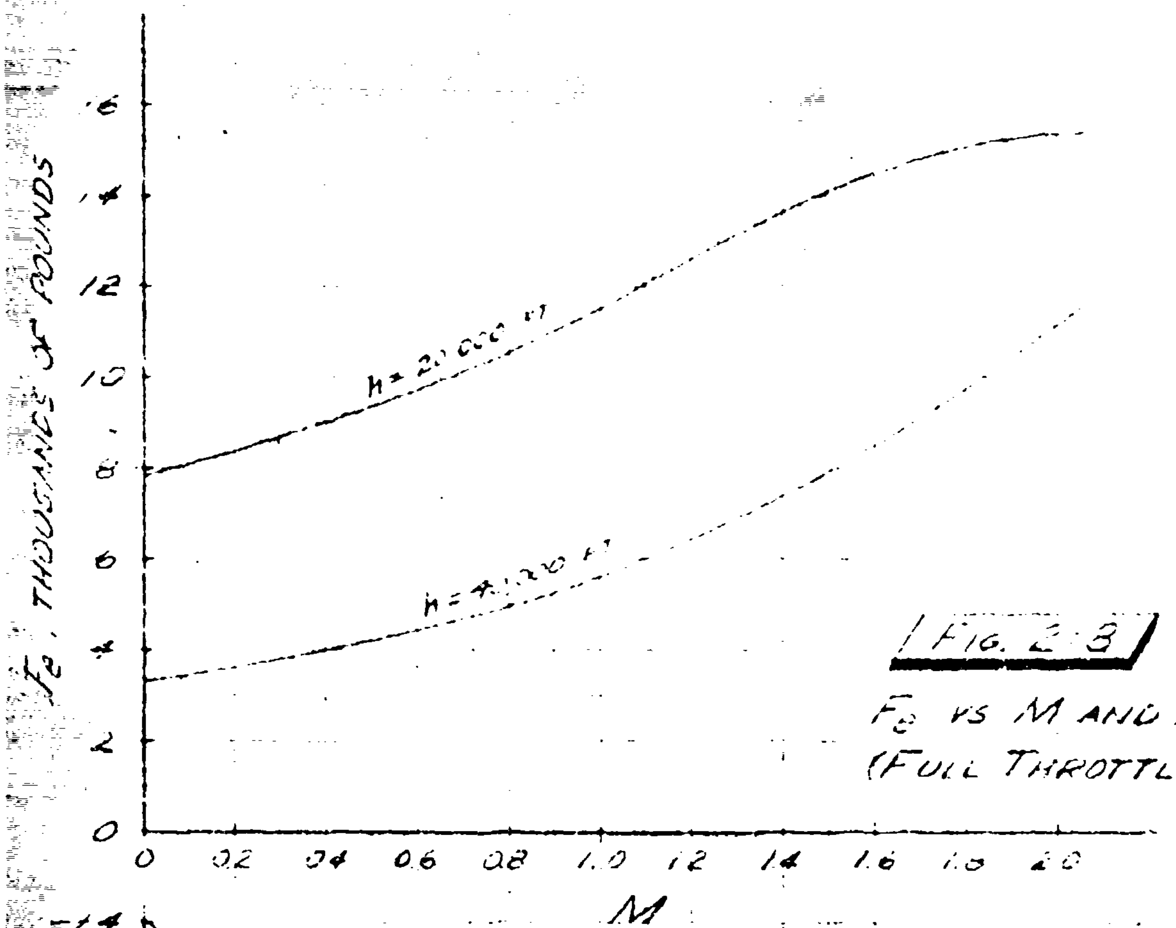


FIG. 2.8

F_e VS M AND h
(FULL THROTTLE)

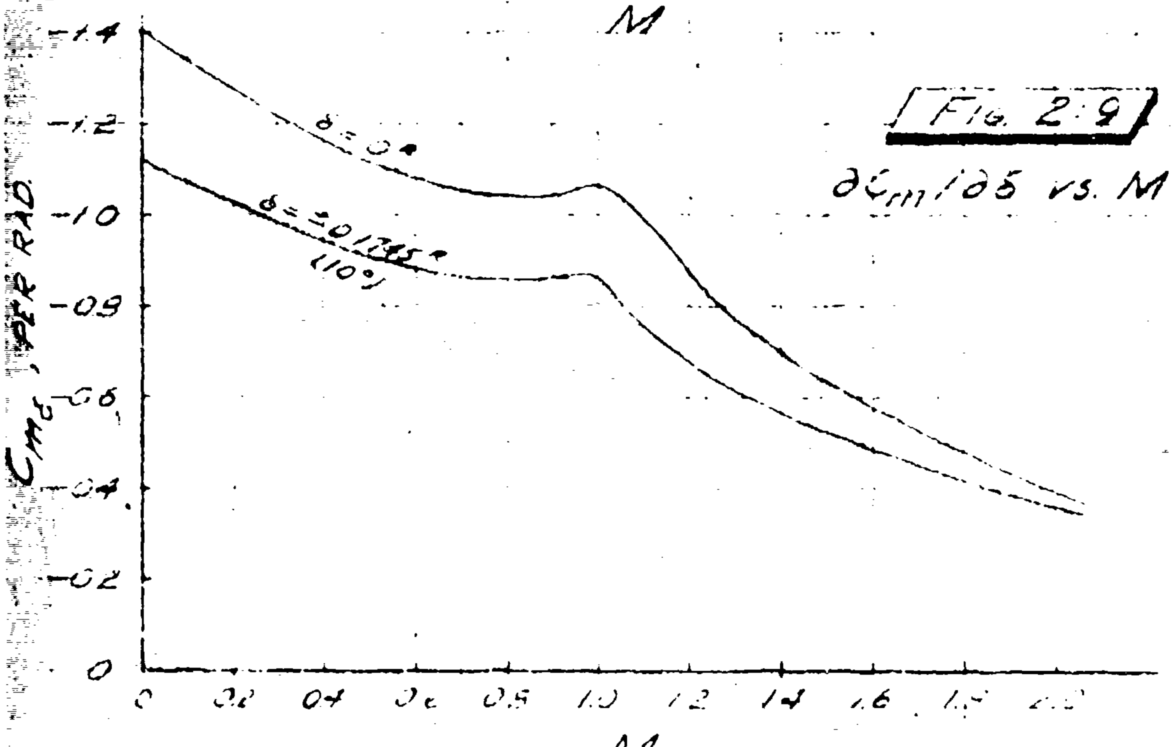
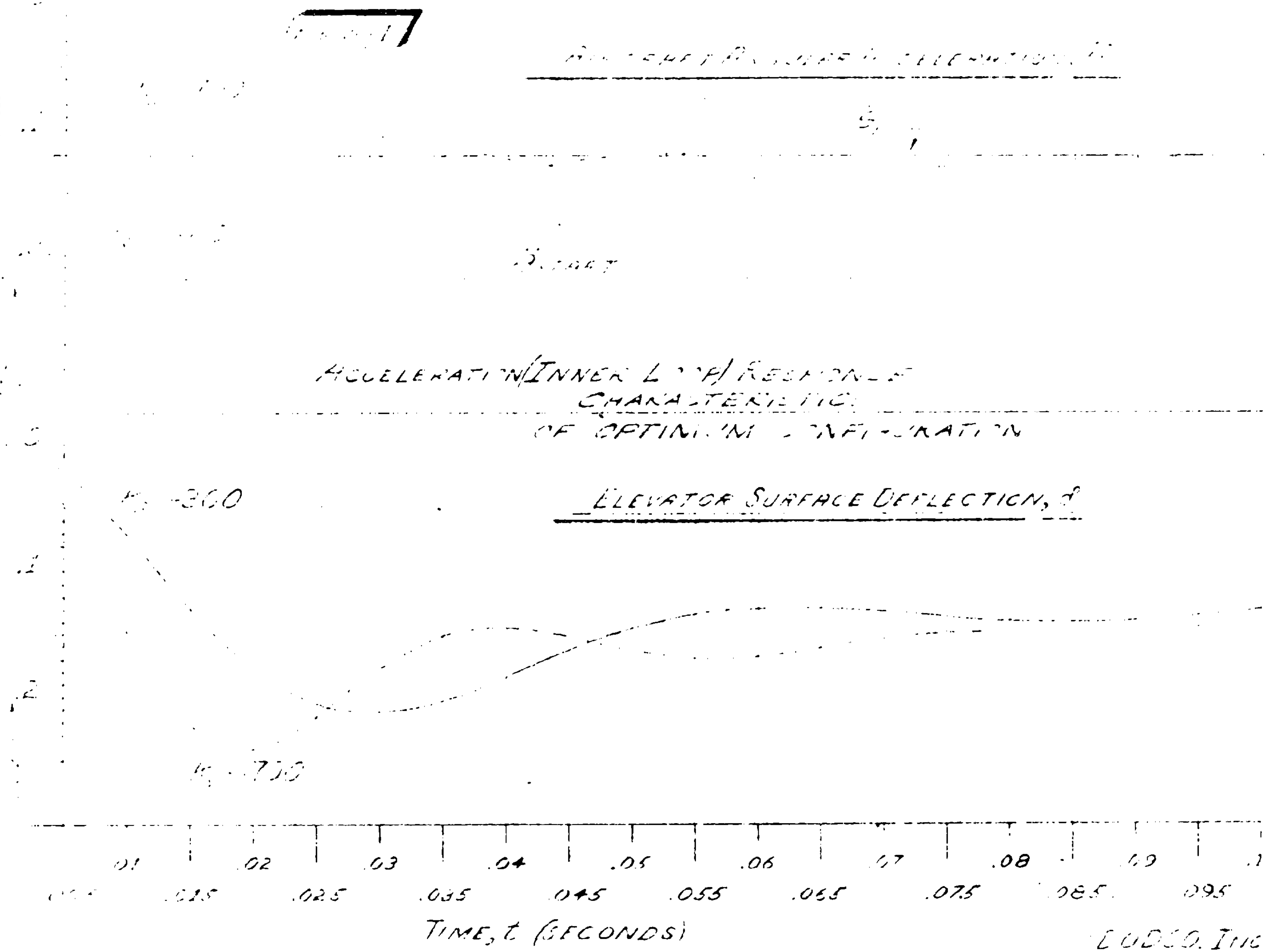
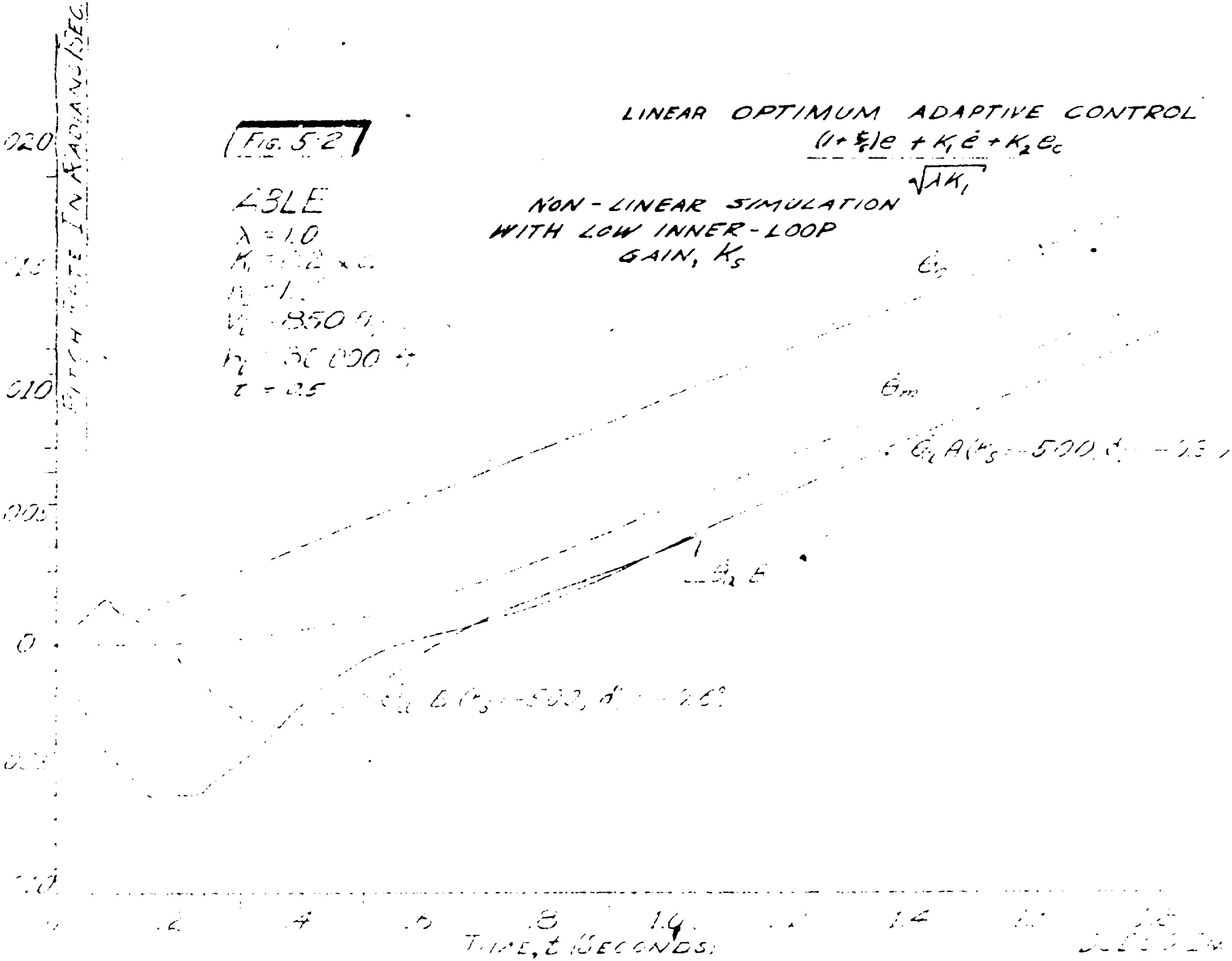
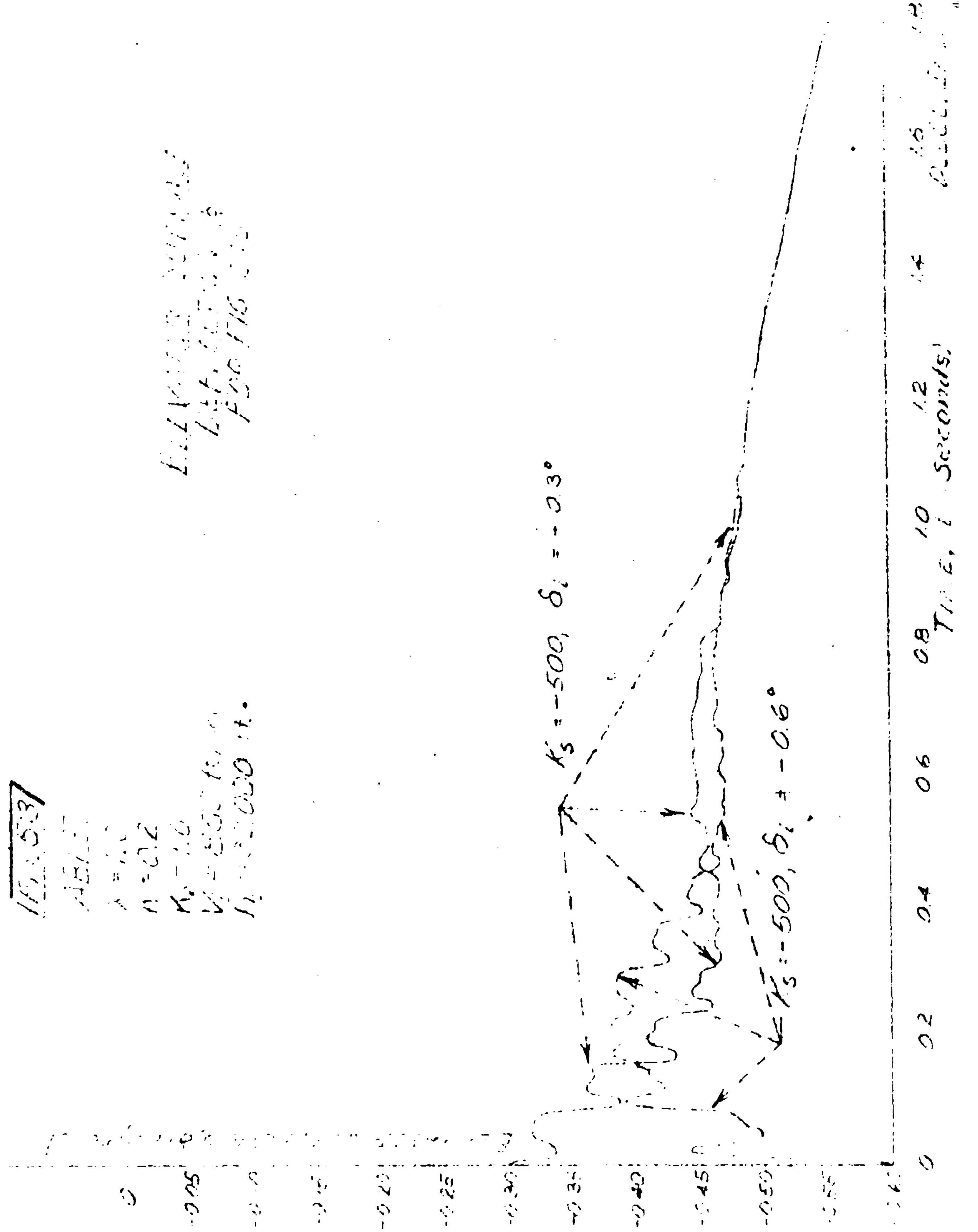


FIG. 2.9

$\partial C_m / \partial \delta$ VS. M AND δ


ACCELERATION (INNER LOOP) RESPONSE
CHARACTERISTIC
OF OPTIMUM CONFIGURATION
ELEVATOR SURFACE DEFLECTION, δ
.01
.02
.03
.04
.05
.06
.07
.08
.09
.1
.025
.035
.045
.055
.065
.075
.085
.095
TIME, t (SECONDS)


LINEAR OPTIMUM ADAPTIVE CONTROL
$\frac{(1+\frac{\tau}{\lambda})e + K_1\dot{e} + K_2 e_c}{\sqrt{\lambda K_1}}$
NON-LINEAR SIMULATION
WITH LOW INNER-LOOP
GAIN, K_S
Fig. 5.2
λ = 1.0
τ = .25
θ_m
.020
.010
.005
0
TIME, t (SECONDS)
.2
.4
.6
.8
1.0
1.4



K_s = -500, δ_i = -0.3°
K_s = -500, δ_i = -0.6°
Seconds

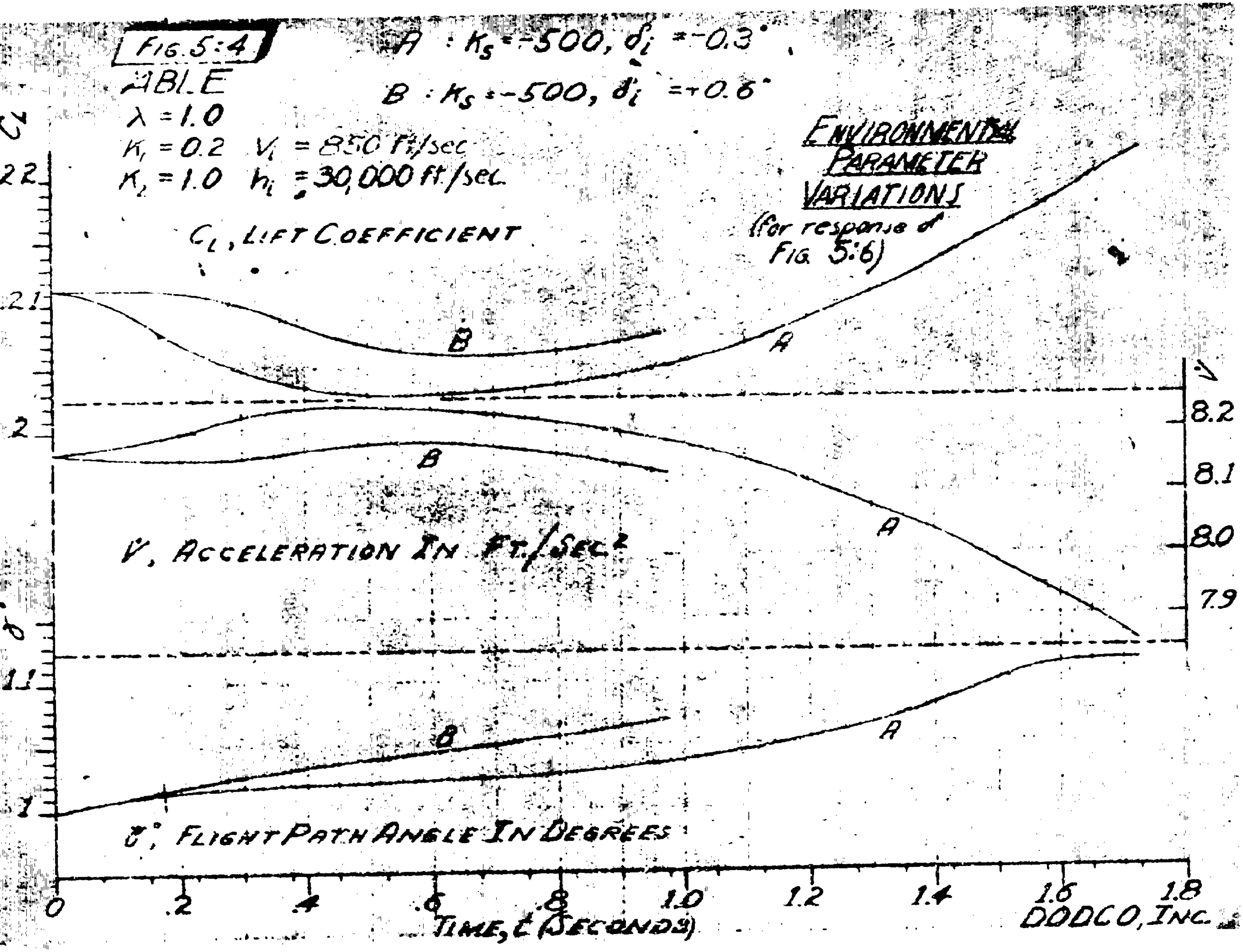
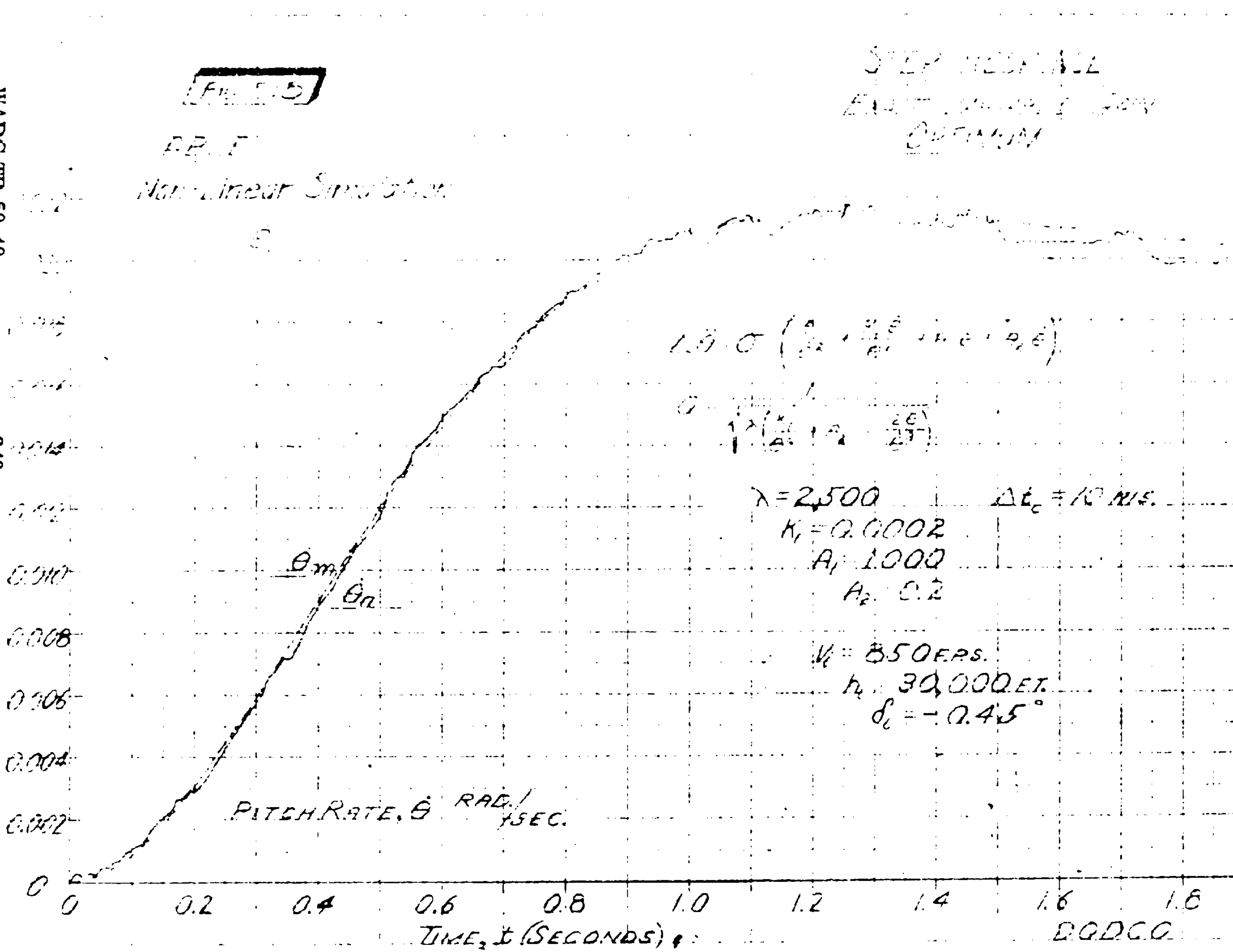
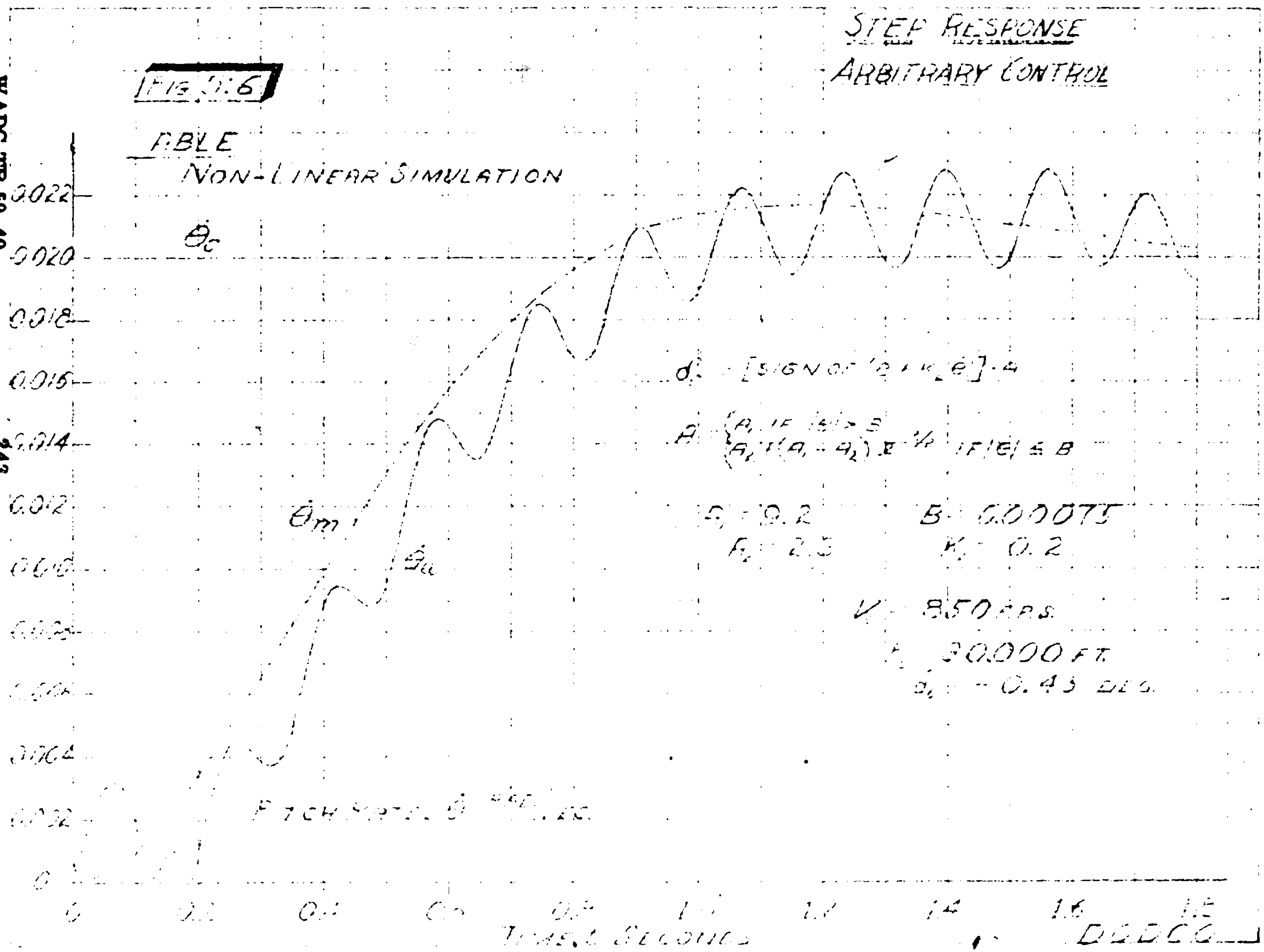
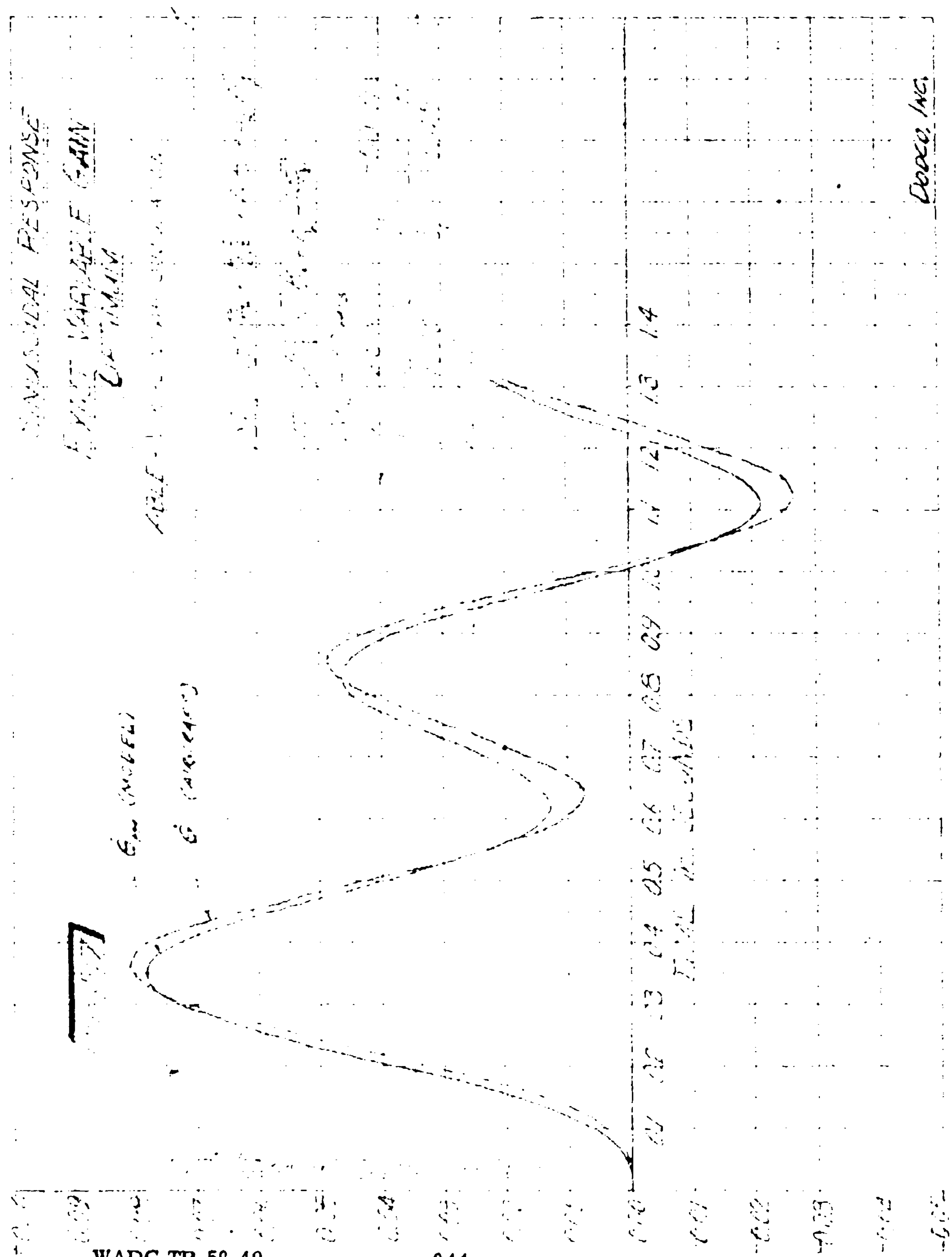
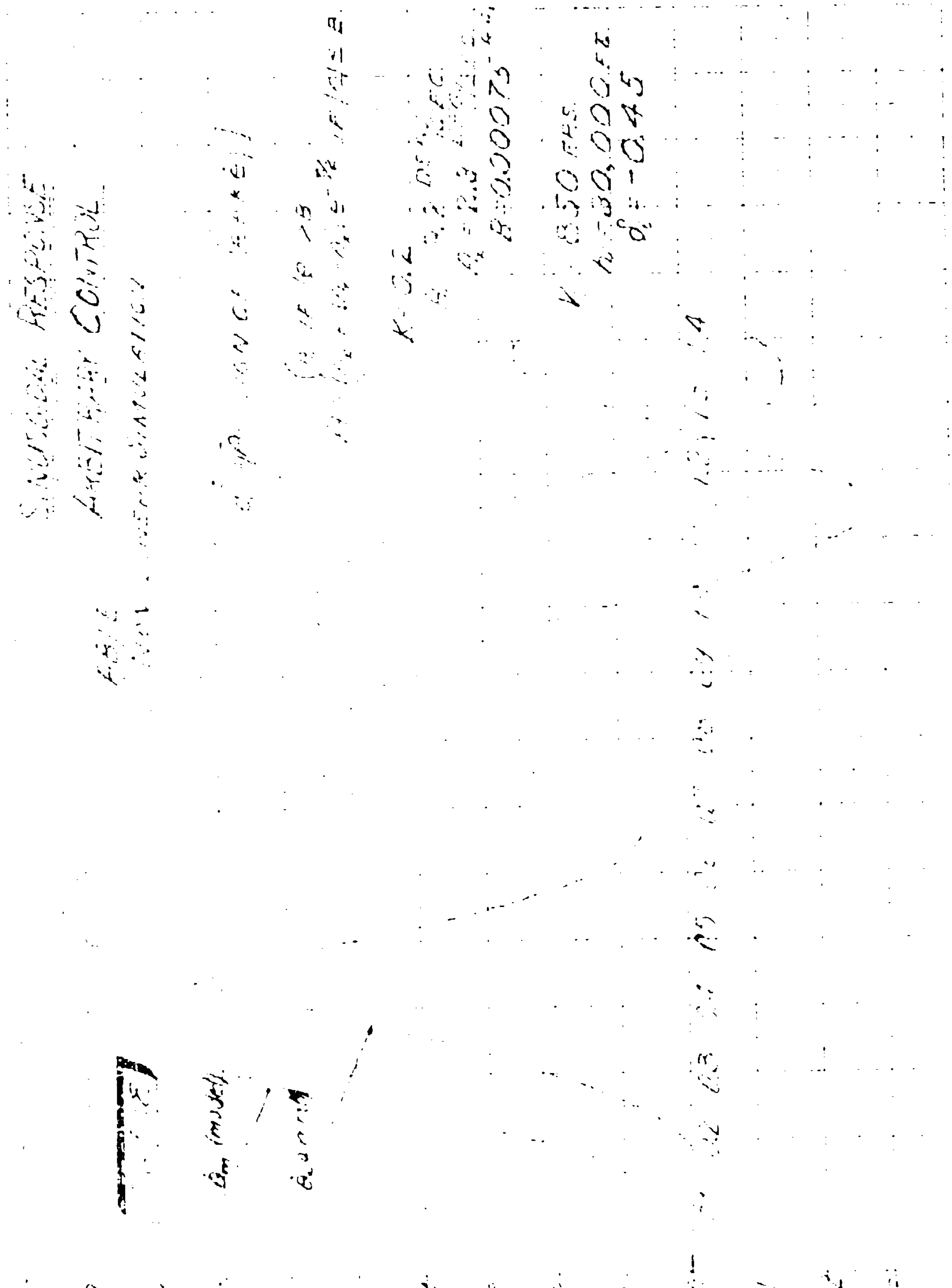


FIG. 5:4
ABLE
$\lambda = 1.0$
$K_1 = 0.2$ $V_i = 850$ ft/sec
$K_2 = 1.0$ $h_i = 30,000$ ft/sec.
A : $K_s = -500$, $\delta_i = -0.3°$
B : $K_s = -500$, $\delta_i = +0.6°$
ENVIRONMENTAL PARAMETER VARIATIONS
(for response of FIG. 5:6)
C_L, LIFT COEFFICIENT
$\dot{V}$, ACCELERATION IN FT./SEC^2
$\gamma°$, FLIGHT PATH ANGLE IN DEGREES
TIME, t (SECONDS)
DODCO, INC.
C_L
.22
.21
.2
$\gamma°$
1.1
1
$\dot{V}$
8.2
8.1
8.0
7.9
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
A
B


Non-Linear Simulation
$\lambda = 2{,}500$
$K_1 = 0.0002$
$A_1\ 1000$
$A_2\ 0.2$
$\Delta t_c = 10$ MS.
V_i = 850 FPS.
h_i 30,000 FT.
$\delta_i = -0.45°$
$\dot{\theta}_m$
$\dot{\theta}_a$
PITCH RATE, $\dot{\theta}$ RAD./SEC.
0.010
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
TIME, t (SECONDS)


STEP RESPONSE
ARBITRARY CONTROL
ABLE
NON-LINEAR SIMULATION
0.022
0.020
0.018
0.016
0.014
0.012
0.004
0.002
0
0.4
0.6
0.8
1.2
1.4
1.6
B = 0.00075
K = 0.2
V = 850 FPS
30,000 FT
-0.43 DEG


DODCO, INC.


ARBITRARY CONTROL
K = 3.2
B = 0.00075
V = 850 FPS
h = 30,000 FT
-0.45

APPENDIX

THE EQUATIONS FOR DIGITAL COMPUTER SIMULATION OF AIRCRAFT DYNAMIC RESPONSE AND OPTIMUM ADAPTIVE CONTROL

[illegible] OF EQUATIONS

Integration Block II:

$$t_n = t_{n-1} + \Delta t$$

$$V_n = V_{n-1} + \dot{V}_{n-1}\,\Delta t + \ddot{V}_{n-1}\,\overline{\Delta t}^2/2$$

$$\gamma_n = \gamma_{n-1} + \dot{\gamma}_{n-1}\,\Delta t + \ddot{\gamma}_{n-1}\,\overline{\Delta t}^2/2$$

$$h_n = h_{n-1} + \dot{h}_{n-1}\,\Delta t + \ddot{h}_{n-1}\,\overline{\Delta t}^2/2$$

$$W_n = W_{n-1} + \dot{W}_{n-1}\,\Delta t + \ddot{W}_{n-1}\,\overline{\Delta t}^2/2$$

$$m = W/g$$

$$J = K_g m$$

Atmosphere Block:

(a) Gradient Regions:

$$T = T_0 + a_j h$$

$$\eta = 1/a_j R$$

$$\partial\rho/\partial h/\rho = -\,a_j(\eta+1)/T$$

$$\partial T/\partial h/T = a_j/T$$

$$\epsilon = -\,\eta \mathrm{LOG}_e\,(T/T_1)$$

$$c_a = 49.04135\sqrt{T}$$

(b) Isothermal Regions:

$$T = T_1$$

$$\partial\rho/\partial h/\rho = -1/RT$$

$$\partial T/\partial h/T = 0$$

$$\epsilon = -\frac{1}{RT}(h-h_1)$$

$$c_a = c_{a_1}$$

(c) All Regions:

$$P/P_0 = \delta_1 e^{\epsilon}$$

$$\rho = \rho_0 \frac{T_0}{T} \cdot \frac{P}{P_0}$$

Parameter Block 1:

$$M = V/c_a$$

$$q = \rho V^2/2$$

$$\dot{h} = V \sin\gamma$$

$$\alpha = \theta - \gamma$$

$$C_L = a \sin\alpha$$

$$C_D = C_{D_e} + KC_L^2$$

$$D = qSC_D$$

$$\dot{\gamma} = \frac{1}{V}\left[\frac{(F_e + aqS)\sin\alpha}{m} - g\cos\gamma\right]$$

$\dot{\ } = $ [illegible]

$\dot{\ } = $ [illegible]

$\dot{q}/q = 2\left[\dfrac{\dot{V}}{V} + (\partial\rho/\partial h)\ [illegible]\ \dfrac{\dot{h}}{\rho}\right]$

$\dfrac{\dot{M}}{M} = \dfrac{\dot{V}}{V} - (\partial T/\partial h/T)\ \dfrac{\dot{h}}{2}$

$\dot{a} = (\partial a/\partial M)\dot{M}$

$\dot{C}_{D_e} = (\partial C_{D_e}/\partial M)\dot{M}$

$\dot{K} = (\partial K/\partial M)\dot{M}$

$\dot{F}_e = (\partial F_e/\partial M)\dot{M} + (\partial F_e/\partial h)\dot{h}$

$\ddot{V}$, $\ddot{\gamma}$ Block:

$\dot{\alpha} = \dot{\theta} - \dot{\gamma}$

$\dot{D} = D\left(\dfrac{\dot{q}}{q}\right) + qS\left[\dot{C}_{D_e} + KC_L{}^2\left(\dfrac{\dot{K}}{K} + \text{[illegible]}\right)\right]$

$\ddot{V} = \dfrac{F_e\cos\alpha}{m}\left(\dfrac{\dot{F}_e}{F_e} - \dot{\alpha}\tan\alpha - \dfrac{\dot{m}}{m}\right) - \dot{\gamma}g\cos\gamma - \dfrac{D}{m}\ [illegible]$

$\ddot{\gamma} = \Big\{ g\cos\gamma\ (\dot{\gamma}\tan\gamma + \dfrac{\dot{V}}{V})$

$\quad + \dfrac{\sin\alpha}{m}\left[(F_e + aqS)\left(\dfrac{\dot{\alpha}}{\tan\alpha} - \dfrac{\dot{m}}{m} - [illegible]\right) + \dot{F}_e + aqS\left(\dfrac{\dot{a}}{a} + \dfrac{\dot{q}}{q}\right)\right]\Big\}/V$

$\ddot{h} = \dot{h}\left(\dfrac{\dot{\gamma}}{\tan\gamma} + \dfrac{\dot{V}}{V}\right)$

$\ddot{W} = c_T\,\dot{F}_e$

[illegible]

$$\dot{\theta}_{m_n} = \dot{\theta}_{m_{n-1}} + \ddot{\theta}_{m_{n-1}} \Delta t + \dddot{\theta}_{m_{n-1}} \overline{\Delta t}^2/2$$

$$\ddot{\theta}_{m_n} = \ddot{\theta}_{m_{n-1}} + \dddot{\theta}_{m_{n-1}} \Delta t$$

$$\dot{\theta}_c = a_0 + a_1 t$$

$$\dddot{\theta}_m = \omega_{n_m}^2 (\dot{\theta}_c - \dot{\theta}_m) - 2 \zeta_m \omega_{n_m} \ddot{\theta}_m$$

[illegible]

$$\ddot{\theta} = [illegible] \left[[illegible] + \dot{\theta} C_{m_{\dot{\theta}}} + \delta C_{m_{\dot{\delta}}} + \delta C_{m_{\delta}} \right] - [illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] $\omega_{n_s}^2$ [illegible]

[illegible] $= \ddot{\theta} +$ [illegible]

[illegible]

(2) [illegible] Block II:

$t_j = t_{j-1} + \Delta t$

$\theta_j = \theta_{j-1} + \dot{\theta}_{j-1} \Delta t + \ddot{\theta}_{j-1} \overline{\Delta t}^2/2$

$\dot{\theta}_j = \dot{\theta}_{j-1} + \ddot{\theta}_{j-1} \Delta t$

[illegible] on Block 1

[illegible] Loop $\ddot{\theta}$, $\ddot{\delta}$

[illegible] Inner Loop [illegible]

[illegible]

[illegible]

$\ddot{\theta} = \frac{[illegible]}{I}$ [illegible]

$\Delta\ddot{\theta} = \ddot{\theta}_d - \ddot{\theta}$

[illegible]

[illegible]

THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS SYMPOSIUM

SESSION VI

Dr. John G. Truxal, Chairman
Brooklyn Polytechnic Institute

Dr. John G. Truxal
Head, Electrical Engineering Department
Brooklyn Polytechnic Institute

I think there is one thing from this morning's program that certainly strikes a teacher. Control engineers, in general, are starting to utilize and exploit to a rather remarkable extent the theories that have been developed by the feedback theory people and which we have occasionally tried to teach to our students. This is a very remarkable situation. This is not true in most of the other fields of electrical engineering. It certainly is going to call for us in the Universities to change our approach. One of my colleagues has summed up the attitude of the teaching profession very appropriately in a comment which some of you have heard. When a student asks him an embarrassing practical question like " why doesn't a hi fi work" , he always replies with great disdain that his knowledge is completely unbesmirched by any practical application. I think the remarkable thing about the symposium so far has been the extent to which some of the feedback theories, concepts of multi-loop systems, and non-linear system theories have been put to work in the evaluation and design of control systems.

I think it might be well to point out that there is one aspect of adaptive systems which we seem to be overlooking entirely. I'm not sure anything can be done about this. There has been a great deal of interest among communications engineers in adaptive systems, or what they call adaptive systems. If you look at the proceedings of the last London conference on information theory, you find a great deal of talk about adaptive systems. What the communications engineer generally means by an adaptive system is one which involves an element of learning. We might design upon probablistic models and changing the character of the non-linearities or the probablistic model parameters, on the basis of experience we acquire so that we get a system which does some anticipation and is susceptible to learning on the basis of past experience. Perhaps this is another realm beyond where we are now with adaptive systems but it seems to me that the communications engineer's adaptive systems and the control engineer's adaptive system are probably too far apart and an important area is the merging of these two.

ANALYSIS AND SYNTHESIS OF A LINEAR, SELF ADAPTIVE, STABILITY AUGMENTATION SYSTEM

Marcel Dandois Convair

In the operational concepts for many new high-speed high-altitude aircraft and missiles, extreme variations in flight environment are required. As a result of these requirements, designers of aircraft control systems have had difficulties in arriving at system concepts through whose implementation uniform response and stability characteristics could be obtained at all flight conditions. In recent higher performance airplanes, uniform stability has been obtained by scheduling control parameters as functions of air data measurements. Use of this approach has led to the requirement that the aircraft carry a reliable, accurate air data computer. It has also led to the requirement that the dynamic stability of the aircraft be predictable at all flight conditions so that the proper control compensations may be scheduled.

Difficulties in providing these capabilities have led to the use of a different approach. In this approach uniform stability is obtained by compensating for changes in aircraft stability directly. This may be accomplished by adjusting control parameters as functions of the airframe response or by other special arrangements (with fixed parameters) that maintain a uniform response over a wide range of conditions. Control systems falling in this category have been called "Self-Adaptive Controls."

The need for self-adaptive flight control systems for applications to military aircraft was brought to the attention of the aircraft industry at WADC in the fall of 1955. As a result of activities undertaken at WADC and sponsored by this agency and as a result of independent research work in advanced automatic control systems already under way at other agencies, Convair embarked on a company-sponsored study and survey of self-adaptive methods of control. From a review of the available literature related to this subject and from original studies made at Convair, there emerged several methods of control which merited preliminary investigation. These techniques have been described in Reference 1.

Upon completion of this preliminary survey, this corporate program was extended in the form of a more detailed study of one of these techniques. It was felt that, in order to be worthwhile, the control method selected should provide a sizeable improvement in stability augmentation over presently employed control systems and should allow the incorporation of the method in present systems with little redesign. For these reasons, a method was selected which

comprised a control system made up of three linear feedback loops with fixed parameters. These consisted of position, rate, and acceleration feedback whose combined dynamic effects represent the inverse of the desired system dynamics. With sufficient control gain this arrangement results in a system which maintains desired aircraft characteristics over a wide range of conditions. The higher the gain, the more insensitive the system becomes to changes in flight environment. Although well known in servo theory, this method has not been generally applied to flight control systems because of the likelihood of encountered instability as a result of the high feedback gains. It can be shown that this tendency is due essentially to the dynamic effects of the sensors and servo actuator characteristics. With the advent of control instrumentation with superior response rates, higher gains will be allowable, and this method may become practical in certain applications. Because of its simplicity, it appears worthwhile to investigate the advantages and limitations of this method in a typical application. Results of this investigation are presented in the present report.

There are some methods of self-adaptive control which, by means of simplifying assumptions, may be linearized to a special arrangement of high gain feedback. Simplifications such as these entail a loss in the representation of the original system and are justifiable only as methods of obtaining the distinguishing characteristics to be expected.

The particular method which is discussed in this report may be viewed in fact as a simplification of a self-adaptive control in which a measurement of damping ratio is employed to adjust the feedback parameters. Therefore, the results herein also show the approximate performance to be expected, within certain restrictions, with a type of nonlinear self-adaptive control.

NOTATION

$A(s)$	Term in the denominator of closed-loop transfer function of airplane-control system
$B(s)$	Term in the denominator of closed-loop transfer function of airplane-control system
C	Selected value of measured short period undamped natural frequency
C_1	Steady state gain of hydraulic actuator
C_2	Gain of feedback loop around hydraulic actuator
C_3	Steady state gain of rate gyro
f_r	Viscous damping of rate gyro output axis
$F_o(s)$	Numerator of closed-loop transfer function of typical controlled system
$G_o(s)$	Desired transfer function of typical system
H	Spin momentum of rate gyro
$H(s)$	Transfer function of feedback elements in typical control system
I	Moment of inertia of accelerometer mass
J_r	Moment of inertia of rate gyro about the free axis
k	Torsional spring constant of angular accelerometer
K	Arbitrary multiplying factor
K_i	Elastic constant of rate gyro input axis
K_r	Elastic constant of rate gyro output axis
K_o	Airframe steady state gain
K_1	Design closed-loop steady state gain of airframe-control system
K_2	Variable loop gain in airframe-control system

NOTATION (Cont'd)

K_{2a}	Portion of variable loop gain in forward loop of airframe-control system
K_{2b}	Portion of variable loop gain in feedback loop of airframe-control system
K_3	Actual closed-loop steady state gain of airframe-control system
K_4	Adjustable feedback gain which determines amount of damping ratio control
$P(s)$	Term in open-loop transform of typical system
$Q(s)$	Term in open-loop transform of typical system
s	Laplace transform complex variable
$x(s)$	Transform of input to servo actuator
$\delta_e(s)$	Laplace transform of elevator deflection
ζ_A	Accelerometer damping ratio
ζ_m	Measured airplane short period damping ratio
ζ_q	Damping ratio of second order lead term in approximation of B(s)
ζ_x	Damping ratio of hydraulic actuator second order lag factor
ζ_0	Free airframe short period damping ratio
ζ_1	Design airframe short period damping ratio
ζ_3	Actual closed-loop damping ratio of airframe-control system
ζ'	Quasi damping ratio of closed-loop airframe-control system
$\theta_C(s)$	Input command to airframe pitch control system
$\theta_0(s)$	Laplace transform of airframe output pitch deflection

NOTATION (Cont'd)

$\dot{\theta}_{0_m}$	Angular deflection of rate gyro output
$\ddot{\theta}_{0_m}$	Angular deflection of accelerometer mass with respect to accelerometer case
ω_A	Accelerometer undamped natural frequency
ω_{HS}	Hydraulic servo characteristic frequency
ω_i	Frequency at intersection of dual Nyquist diagram
ω_m	Measured airplane short period undamped natural frequency
ω_p	Characteristic frequency of first order lead term in approximation of B(s)
ω_q	Undamped natural frequency of second order lead term in approximation of B(s)
ω_{RG_1}	Characteristic frequency of rate gyro first order factor
ω_{RG_2}	Characteristic frequency of rate gyro first order factor
ω_{RG}	Rate gyro characteristic frequency
ω_x	Undamped natural frequency of hydraulic actuator second order lag factor
ω_0	Free airframe short period undamped natural frequency
ω_1	Design airframe short period undamped natural frequency
ω_3	Actual closed-loop undamped natural frequency of airframe-control system
ω_I, ω_{II}, ω_{III}, ω_{IV}	frequencies on dual Nyquist diagram.

DESCRIPTION OF METHOD

The control system discussed in this report consists of a feedback configuration whose transfer function is equivalent to the reciprocal of the desired aircraft characteristics. The method also may be regarded as a simplified version of a self-adaptive control in which a measurement of damping ratio is employed to adjust its feedback parameters. These two concepts will be explained in detail.

Reciprocal Model Feedback Concept

The first concept is not new. It stems from an elementary principle of feedback control which may be illustrated as follows. Let $G_O(s)$ be the transfer function of a system whose response characteristics are to be controlled. Assume that this function varies with environmental operating conditions. Let $G_1(s)$ be a fixed transfer function representing desired dynamic characteristics of $G_O(s)$. In the following block diagram a feedback configuration is shown which forces the dynamic characteristics of the system to approach the desired values regardless of changes in $G_O(s)$.

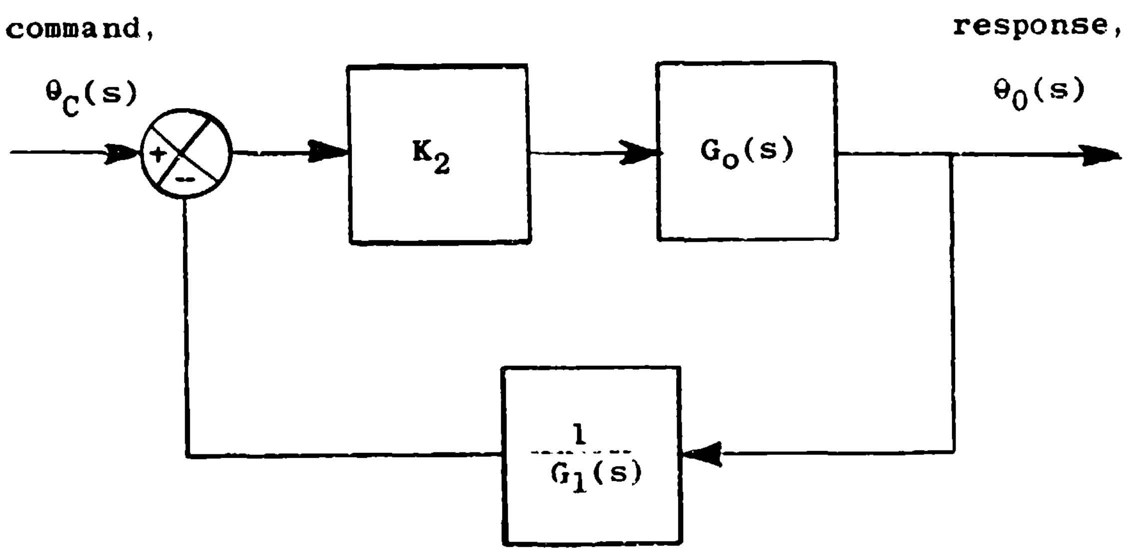


Fig. 1 -- ILLUSTRATION OF FEEDBACK COMPENSATION

The function $\frac{1}{G_1(s)}$ is the transfer function of elements of the feedback path, $\theta_0(s)$ is the output response transform, and $\theta_C(s)$ is the input command transform. The factor K_2 represents a variable gain term which is introduced to allow the proper degree of feedback correction to be applied.

The magnitude which K must assume in order to force the transfer function of the closed system to approach $G_1(s)$ are dictated by the formula given below, which expresses the closed loop transfer function:

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_2G(s)}{1 + \frac{K_2G(s)}{G_1(s)}} \tag{1}$$

It is evident from Equation 1 that if the gain K_2 is made sufficiently large so that

$$\left|\frac{K_2G_o(s)}{G_1(s)}\right| \gg 1 \tag{2}$$

then the transfer function is reduced to

$$\frac{\theta_0(s)}{\theta_C(s)} \approx G_1(s). \tag{3}$$

The condition which must be satisfied is

$$K_2 \gg \left|\frac{G_1(s)}{G_o(s)}\right| \quad . \tag{4}$$

It is to be noted that $G_1(s)$ is independent of the airframe transfer function. The magnitude of K_2 is dictated by Equation 4; it is affected by variations in the airframe characteristics $|G_0(s)|$. The larger K_2 becomes, the smaller is the effect of $|G_0(s)|$ and the larger are the tolerable variations in $|G_o(s)|$. The terms $|G_0(s)|$ and $|G_1(s)|$ are frequency dependent functions; therefore, the choice of K_2 is affected by the range of frequency within which best control is required.

The method described above may be used in airplane and missile flight control systems to improve the stability of their natural modes of motion and to maintain a specified stability regardless of flight condition. An example is illustrated in Figure 2a in which the longitudinal short period motion is augmented by means of a feedback loop having characteristics equal to the reciprocal of the desired short period motion. The transfer function expressing this motion in response to a commanded elevator deflection (or any suitable longitudinal control surface) may be expressed as follows:

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_0}{\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1} \cdot \quad (5)$$

The term K_0 represents the steady state pitch sensitivity to elevator commands. The natural frequency and damping ratio is specified by ω_0 and ζ_0, respectively. These terms vary with flight conditions.

The feedback terms shown in Figure 2a are equivalent to the reciprocal of the transfer function in Equation 5 except that fixed optimum values are chosen for the steady state gain, the natural frequency, and the damping ratio.

Synthesis of the feedback function may be accomplished by sensing pitch attitude, pitch rate, and pitch acceleration, multiplying each by the appropriate gain, and by summing the results. This is shown in Figure 2b.

Damping Ratio Feedback Concept

The feedback system illustrated in Figure 2b may be synthesized on the basis of a different control concept. The basic method of synthesis is illustrated in Figure 3a. The system shown consists of rate feedback divided into two parts, one with a fixed gain and one with a variable gain adjusted as a function of measured damping ratio, ζ_m. Since an increase in pitch rate negative feedback causes an increase in pitch damping, it is obvious that the gain multiplying factor ζ_m must be introduced in such a manner that a decrease in the over-all negative feedback is attained when damping increases. In Figure 3a the factor ζ_m multiplies a portion of the rate feedback signal determined by K_4, and this portion is then applied as the positive feedback shown by the minus sign at the summing point. Since positive feedback has an effect opposite to

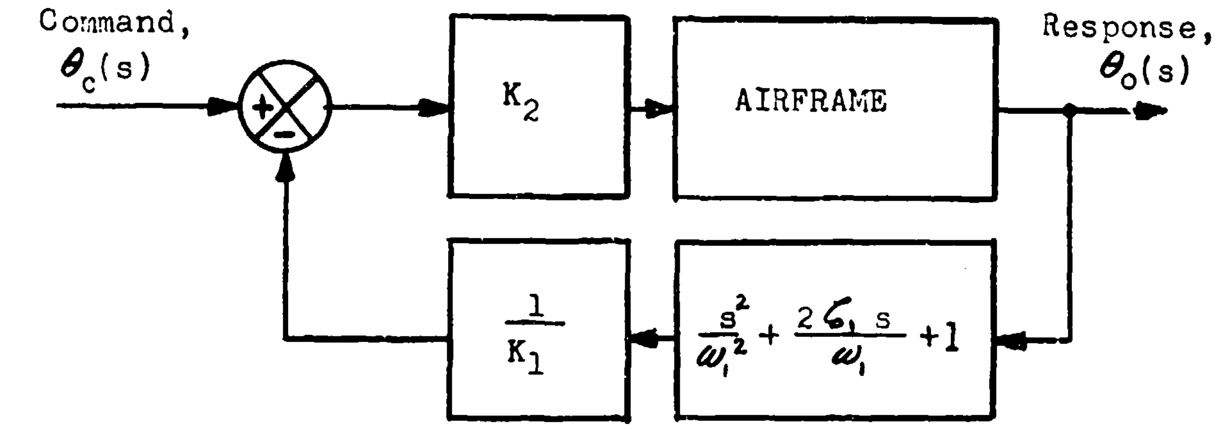


Fig. 2a SHORT PERIOD CONTROL CONFIGURATION

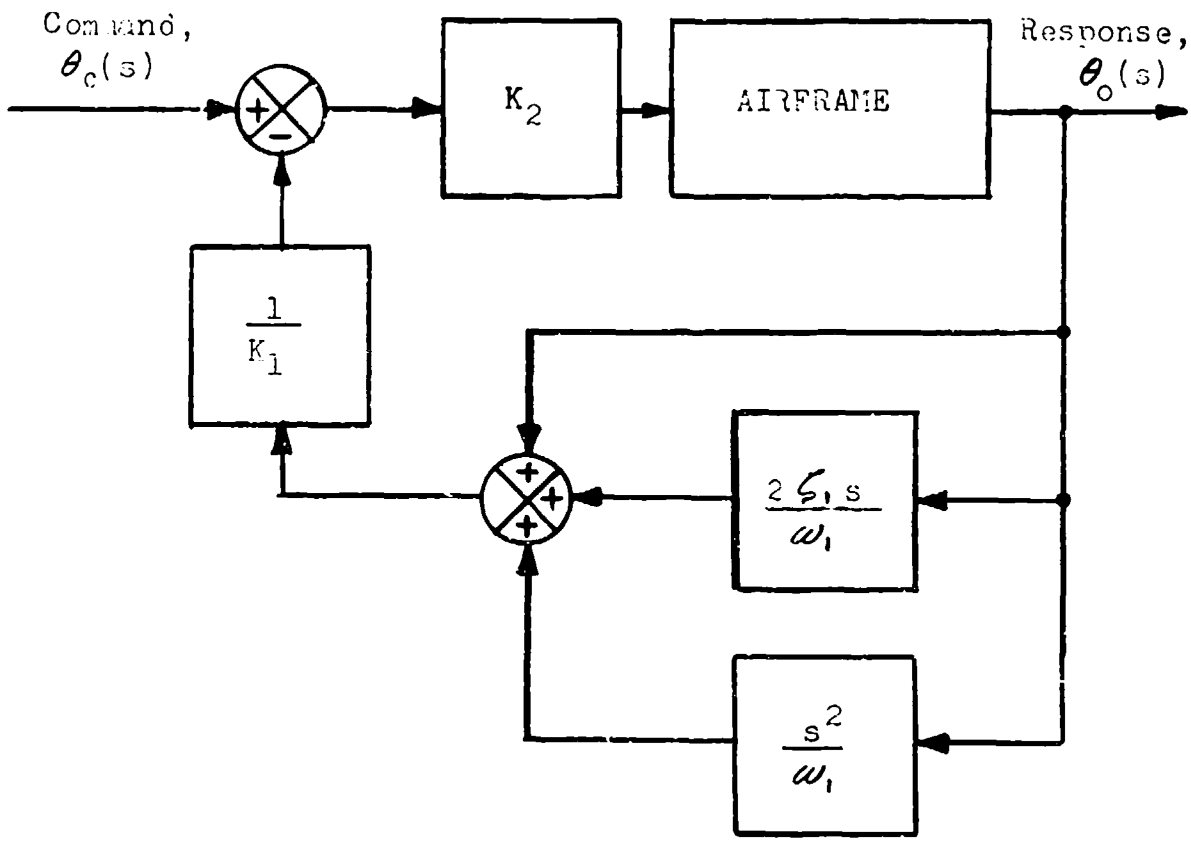


Fig. 2b SHORT PERIOD CONTROL CONFIGURATION

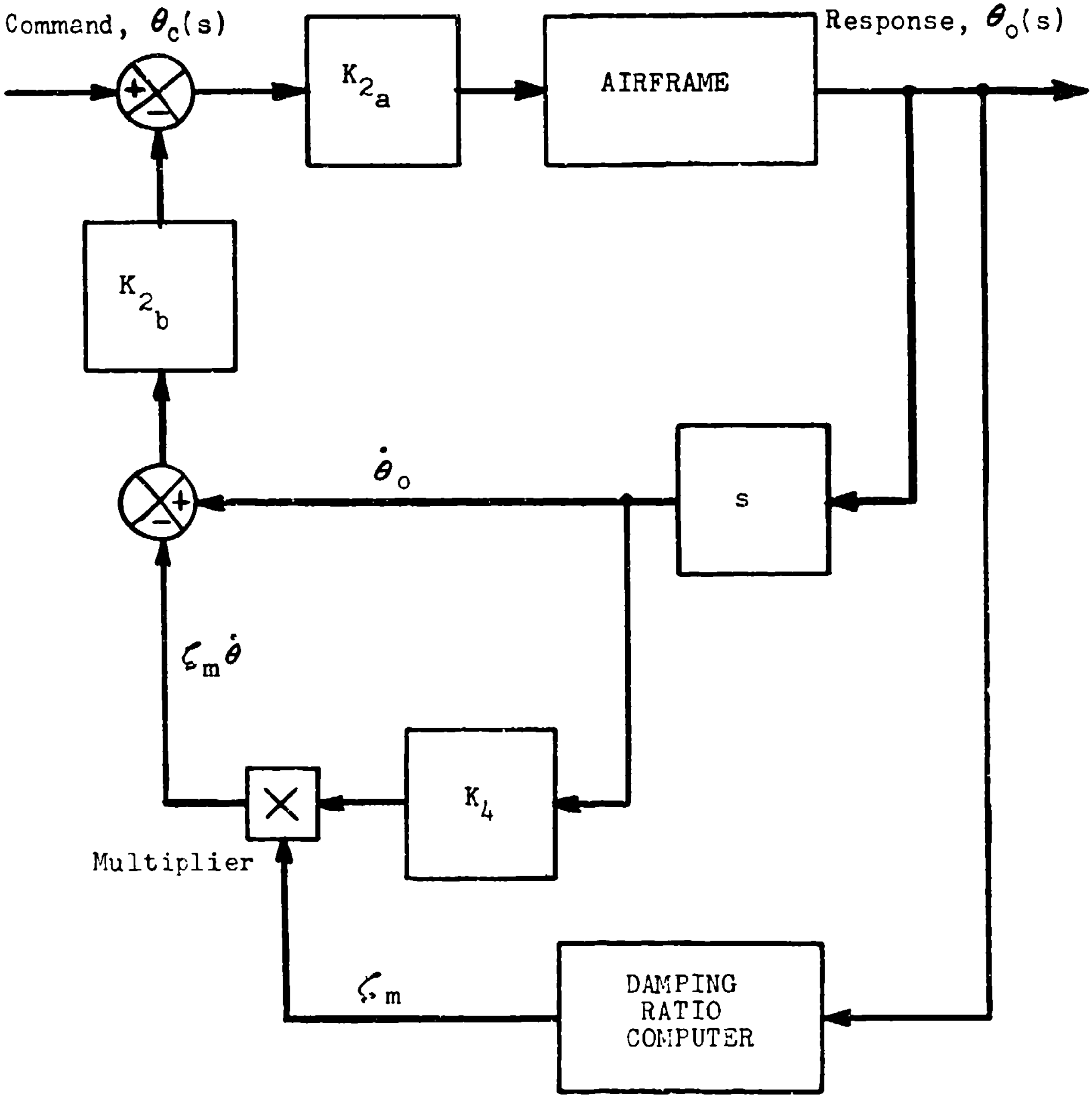


Fig. 3a DAMPING RATIO FEEDBACK CONCEPT

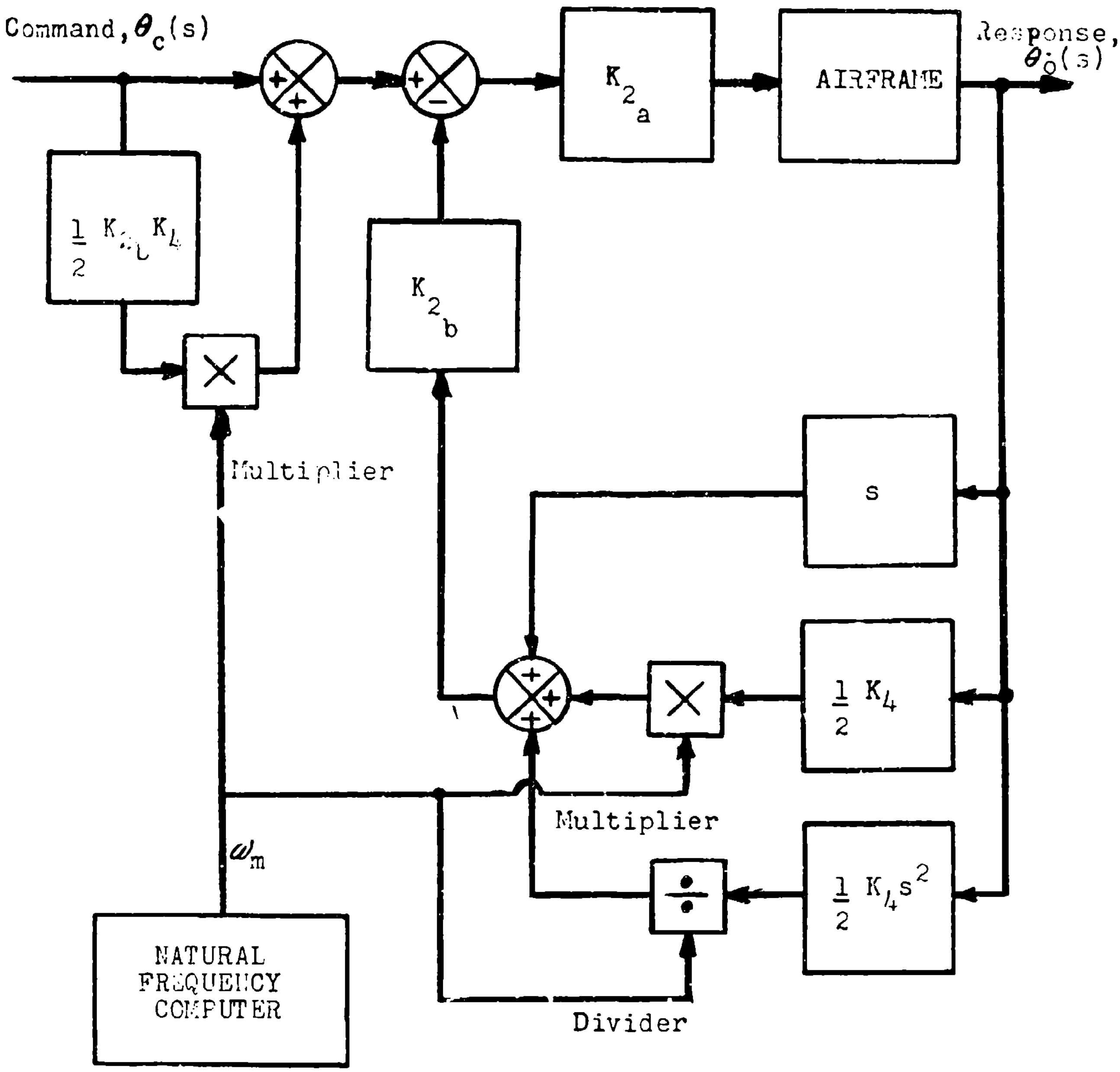


Fig. 3b DAMPING RATIO FEEDBACK CONCEPT

that of negative feedback, it is seen that the proper compensation is achieved.

The over-all amount of rate feedback is determined by the over-all loop gain. Two adjustments of loop gain are provided by K_{2_a} and K_{2_b}. One is in the forward loop and one is in the feedback loop. This arrangement allows the closed loop steady state gain to be adjusted independently. Steady state gain, defined as the steady state value of the ratio $\frac{|\theta_0|}{|\theta_C|}$, is the equal to the forward loop gain divided by the product of the forward and feedback loop gains plus one. Therefore, the steady state gain is adjustable by fixing the value of K_{2_a} in relation to the value of the product of K_{2_a} and K_{2_b}. The over-all amount of rate feedback on the other hand is adjustable by fixing the product of K_{2_a} and K_{2_b} only.

Zero output damping reduces the gain of the auxiliary rate feedback loop, K_4, to zero (Fig. 3a). Therefore, at flight conditions where the airframe is neutrally stable (zero output damping) the gain K_2 (the product of K_{2_a} and K_{2_b}) alone determines the stability augmentation. This observation provides a convenient method of determining the proper adjustment of K_2. On the other hand, when the airframe response is damped, then the gain K_4 determines the amount of negative feedback cancellation. At flight conditions when the free airframe is satisfactorily damped, positive and negative feedback should cancel one another since no compensation is necessary. This condition provides a method of adjusting K_4. Should the airframe become unstable, the computed damping ratio signal must change sign in order to increase the over-all negative feedback.

Damping ratio is a second order characteristic; its computation must thus be based on the assumption that the pitch response resembles a second order response in the short period mode. Experience has shown that this assumption is justified. A second order relationship may be represented in general by an equation of the form

$$\frac{1}{\omega_0^2}\ddot{\theta}_0 + \frac{2\zeta_0}{\omega_0}\dot{\theta}_0 + \theta_0 = \theta_C \qquad (6)$$

By rearranging terms, Equation 6 may be written

$$\zeta_0 \dot{\theta}_0 = \frac{\omega_0}{2} \theta_C - \frac{\omega_0}{2} \theta_0 - \frac{1}{2\omega_0} \ddot{\theta}_0 . \quad (7)$$

From Equation 7 it is deduced that positive pitch rate feedback multiplied by measured damping ratio, ζ_m, (Fig. 3a) may be replaced by negative feedback of pitch attitude and pitch acceleration plus additive input command compensation and appropriately measured natural frequency correction, ω_m. This substitution is illustrated in Figure 3b.

The mechanization of the relation expressed by Equation 7 as shown in Figure 3b, would be complex since the quantity ω_m must be injected as a multiplying factor in two of the loops and as a dividing factor in a third loop. The exact short period natural frequency of the airframe, ω_0, is itself not readily measurable nor computable from the output response. In the system discussed herein, this difficulty was avoided by ignoring variation in ω_0 and by setting ω_m, in Figure 3b, equal to a predetermined constant C. This simplification allows a linear analysis of the system since all three feedback loops have constant gains. As another result of this assumption, the additive compensation becomes a fixed gain compensation and may be included into the gain terms inside the control loop. Thus a quasi damping ratio ζ' may be defined by the following equation in terms of the airframe response in pitch.

$$\zeta' \dot{\theta}_0 = -\frac{C}{2} \theta_0 - \frac{1}{2C} \ddot{\theta}_0 . \quad (8)$$

Incorporation of this relationship into the block diagram leads to the system illustrated in Figure 4.

There is no essential difference between the systems shown in Figures 2b and 4. An analysis of the character of the performance of the control represented by these systems would be identical in basic treatment and the results would be the same.

The investigation discussed in this report was performed as part of a study and evaluation of the nonlinear damping ratio feedback system illustrated in Figure 3a. The particular simplifying assumptions which were made in the linearization of the control system in order to facilitate its mathematical analysis has led to the system shown in Figure 4. Because another simpler and well-known method of control synthesis led to the same configuration (Fig. 2b), it was

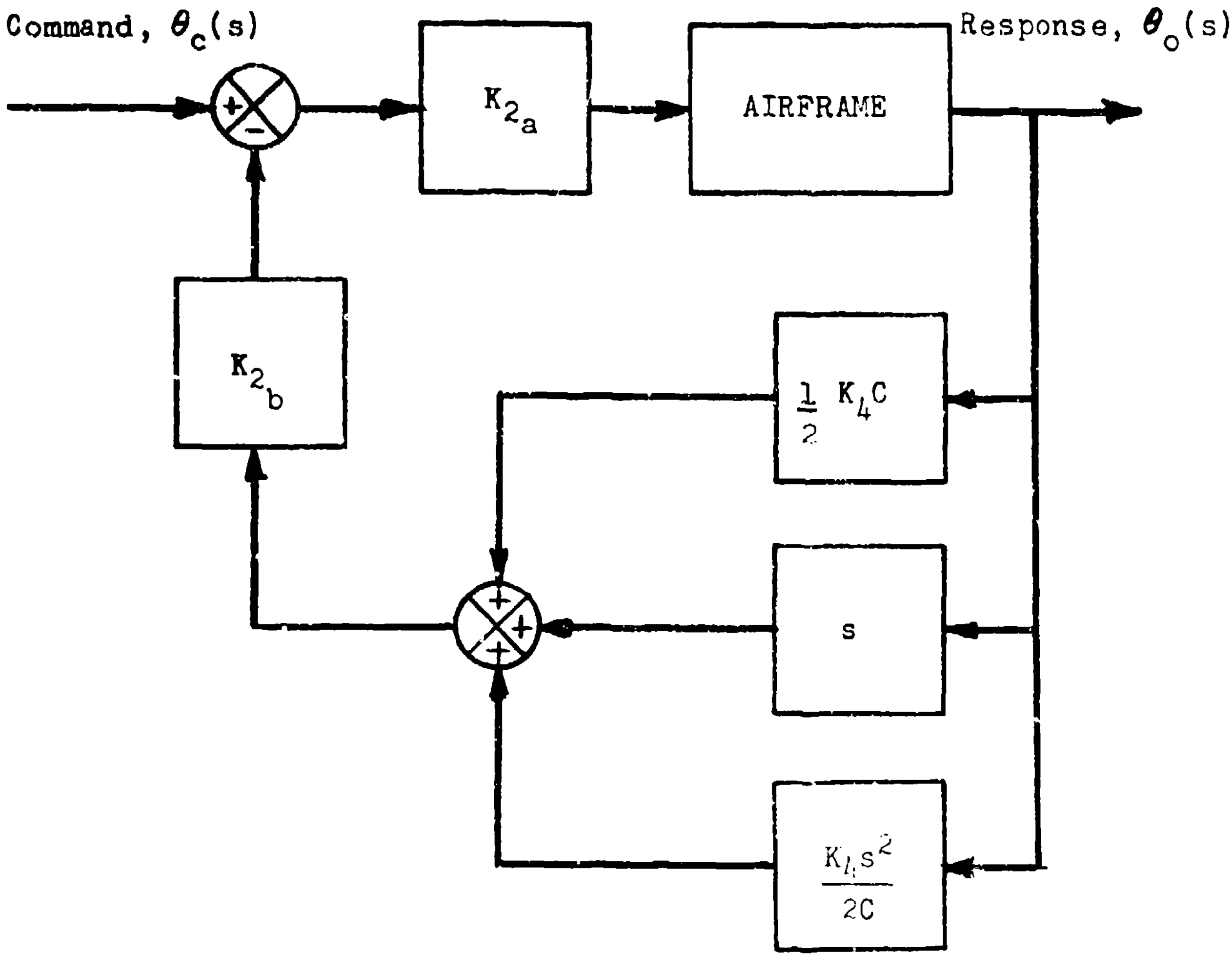


Fig. 4 SIMPLIFIED DAMPING RATIO FEEDBACK CONTROL

proper to introduce both approaches independently. However, the analysis of the final control form (Fig. 2b or 4) will be presented independently of the original approach.

Further details on each of the approaches mentioned and other aspects which emphasize the difference between them may be found in Reference 1.

ANALYSIS OF SELF-ADAPTIVE METHOD IN TYPICAL IDEALIZED FLIGHT CONTROL SYSTEM

The method of self-adaptive control discussed earlier may be incorporated within several control channels of an aircraft stability augmentation system. The feedback quantities may differ in each channel; the amount of stability correction needed may vary; but the basic operations of the control system will remain the same. In general, the longitudinal short period mode motion is the most critical in stability control. For this reason the method will be analyzed as it would be applied in a typical short period mode control. The short period motion will be assumed to be independent of other airframe modes; hence it is sufficient to represent the airframe by its short period dynamics.

In this portion of the analysis the control system will be assumed to have ideal dynamic characteristics (i.e., the dynamics of the instrumentation and of the hydraulic servo are neglected).

Typical of airframe short period dynamics are the second order characteristics represented by Equation 5. The complete airframe-control system configuration to be analyzed is illustrated in Figure 5.

By employing the formula given in Equation 1, the transfer function of the over-all control loop illustrated in Figure 5 is obtained as follows:

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{\dfrac{K_oK_2}{1 + \dfrac{K_oK_2}{K_1}}}{\dfrac{1}{\omega_0^2}\left[\dfrac{1 + \dfrac{\omega_0^2 K_oK_2}{\omega_1^2 K_1}}{1 + \dfrac{K_oK_2}{K_1}}\right]s^2 + \left[\dfrac{\dfrac{2\zeta_o}{\omega_o} + \dfrac{2\zeta_1 K_oK_2}{\omega_1 K_1}}{1 + \dfrac{K_oK_2}{K_1}}\, s+1\right]} \tag{9}$$

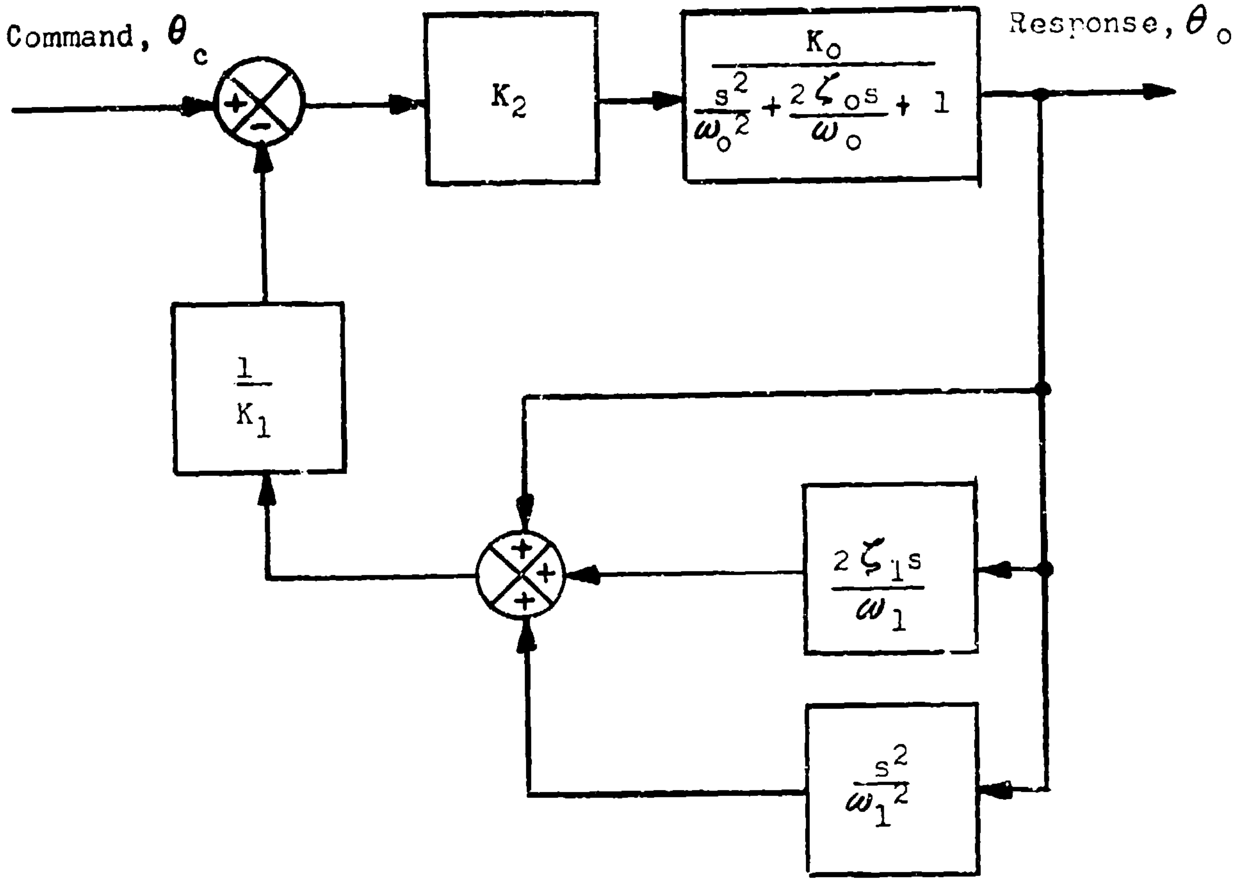


Fig. 5 DIAGRAM OF CONTROL APPLICATION WITHOUT INSTRUMENTATION

Equation 9 is of the general form

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_3}{\dfrac{s^2}{\omega_3^2} + \dfrac{2\zeta_3 s}{\omega_3} + 1}, \tag{10}$$

where

$$K_3 = \frac{K_o K_2}{1 + \dfrac{K_o K_2}{K_1}}, \tag{11}$$

$$\omega_3 = \omega_o \sqrt{\frac{1 + \dfrac{K_o K_2}{K_1}}{1 + \dfrac{\omega_0^2 K_c K_2}{\omega_1^2 K_1}}}, \tag{12}$$

and

$$\zeta_3 = \frac{\zeta_0 + \dfrac{\omega_0 \zeta_1 K_o K_2}{\omega_1 K_1}}{\sqrt{1 + \dfrac{K_o K_2}{K_1}}\sqrt{1 + \dfrac{\omega_0^2 K_o K_2}{\omega_1^2 K_1}}} \tag{13}$$

The quantity K_2 represents the adjustable gain in the control loop. It can be seen that Equations 11, 12, and 13 can be reduced, respectively, to

$$K_3 \approx K_1, \tag{14}$$

$$\omega_3 \approx \omega_1, \tag{15}$$

and

$$\zeta_3 \approx \zeta_1 \tag{16}$$

when K_2 is sufficiently large to satisfy the following inequalities

$$\frac{K_o K_2}{K_1} \gg 1, \tag{17}$$

$$\frac{\omega_0^2 K_o K_2}{\omega_1^2 K_1} \gg 1, \tag{18}$$

and

$$\frac{\omega_0 \zeta_1 K_o K_2}{\omega_1 \zeta_0 K_1} \gg 1 \ . \tag{19}$$

When K_2 is not sufficiently large to satisfy the above inequalities, the parameters K_3, ω_3, and ζ_3 lie between their ideal values, K_1, ω_1, and ζ_1 and their respective free airplane values K_o, ζ_0, and ζ_0. The degree of compensation achieved is shown graphically in Figures 6 and 7 as a function of the value of $\frac{K_o K_2}{K_1}$. These graphs represent the behavior of the system illustrated in Figure 5. The curves shown were obtained by calculating the roots of the equation

$$\frac{1}{\omega_0^2}\left[\frac{1+\dfrac{\omega_0^2 K_o K_2}{\omega_1^2 K_1}}{1+\dfrac{K_o K_2}{K_1}}\right]s^2+\left[\frac{\dfrac{2\zeta_0}{\omega_0}+\dfrac{2\zeta_1 K_o K_2}{\omega_1 K_1}}{1+\dfrac{K_o K_2}{K_1}}\right]s+1=0 \tag{20}$$

for all combinations of the following values

ω_0 = 1, 2, 4, 6 and 10

ζ_0 = 0, 0.2, 0.707 and 1.0

$\dfrac{K_o K_2}{K_1}$ = 0.5, 1.0, 2, 4, 10, 20 , 40 and 100

ω_1 = 2

ζ_1 = 0.707.

Each of the dashed curves in Figure 6 shows the locus of the roots of Equation 20 as the loop gain $\frac{K_o K_2}{K_1}$ varies from zero to infinity. Each curve represents an airframe at a particular flight condition. As $\frac{K_o K_2}{K_1}$ increases, all the curves approach the design point (ω_1, ζ_1). The solid lines in this figure describe the boundaries of the short period response of the closed-loop system for an airframe whose open-loop response lies within the region bounded by

$$1 \text{ radian per second} \leq \omega_0 \leq 10 \text{ radians per second}$$

$$0 \leq \zeta_0 \leq 1.0$$

As the loop gain is increased, the boundary encloses a smaller and smaller area about the design point.

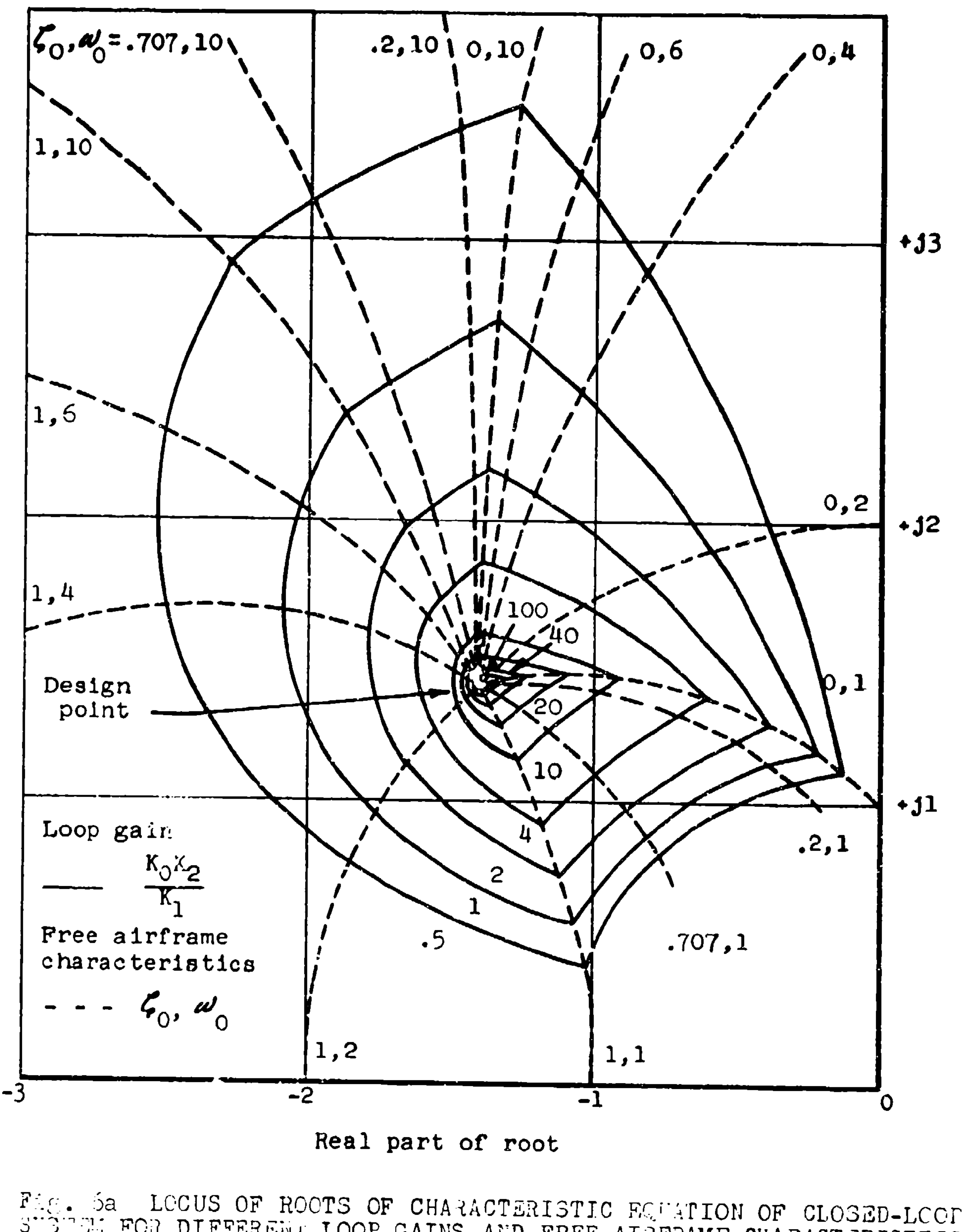


Fig. 5a LOCUS OF ROOTS OF CHARACTERISTIC EQUATION OF CLOSED-LOOP SYSTEM FOR DIFFERENT LOOP GAINS AND FREE AIRFRAME CHARACTERISTICS

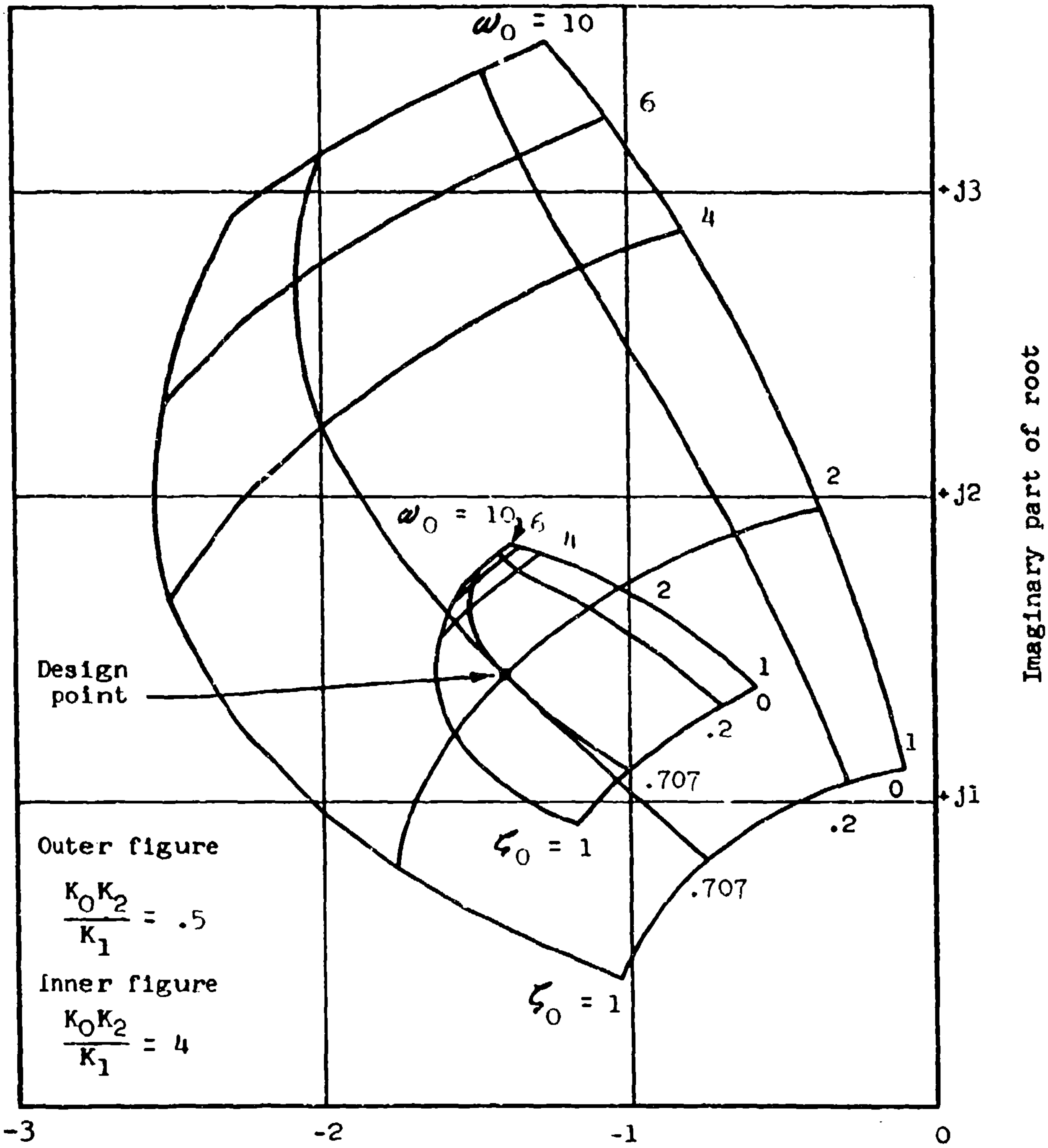


Fig. 6b LOCUS OF ROOTS OF CHARACTERISTIC EQUATION OF CLOSED-LOOP SYSTEM FOR DIFFERENT LOOP GAINS AND FREE AIRFRAME CHARACTERISTICS

In Figure 7 two particular values of loop gain have been selected to illustrate the location of the roots of the closed loop for all possible combinations of ω_0 and ζ_0 listed on the preceding page. The curves which are somewhat radial from the origin are loci of constant ζ_0; those which are approximately circular about the origin represent loci of constant ω_0.

The preceding discussion of the self-adaptive control method indicates that ideal performance characteristics are attainable simply by providing a sufficiently large control loop gain. From Equation 10 it can be seen that the closed-loop system used for illustration is of the second order. Since free airframe damping was assumed to be greater than or equal to zero and negative feedback was chosen, it follows that the system shown in Figure 5 cannot become unstable at any loop gain.

Actual aircraft-flight control system combinations possess physical characteristics which must be described by differential equations of higher orders than second. In such an instance Equations 11, 12, and 13 are no longer exactly correct so that inequalities given in Equations 17, 18, and 19 are not sufficient to produce the approximations given in Equations 14, 15, and 16. This problem is investigated in the next section.

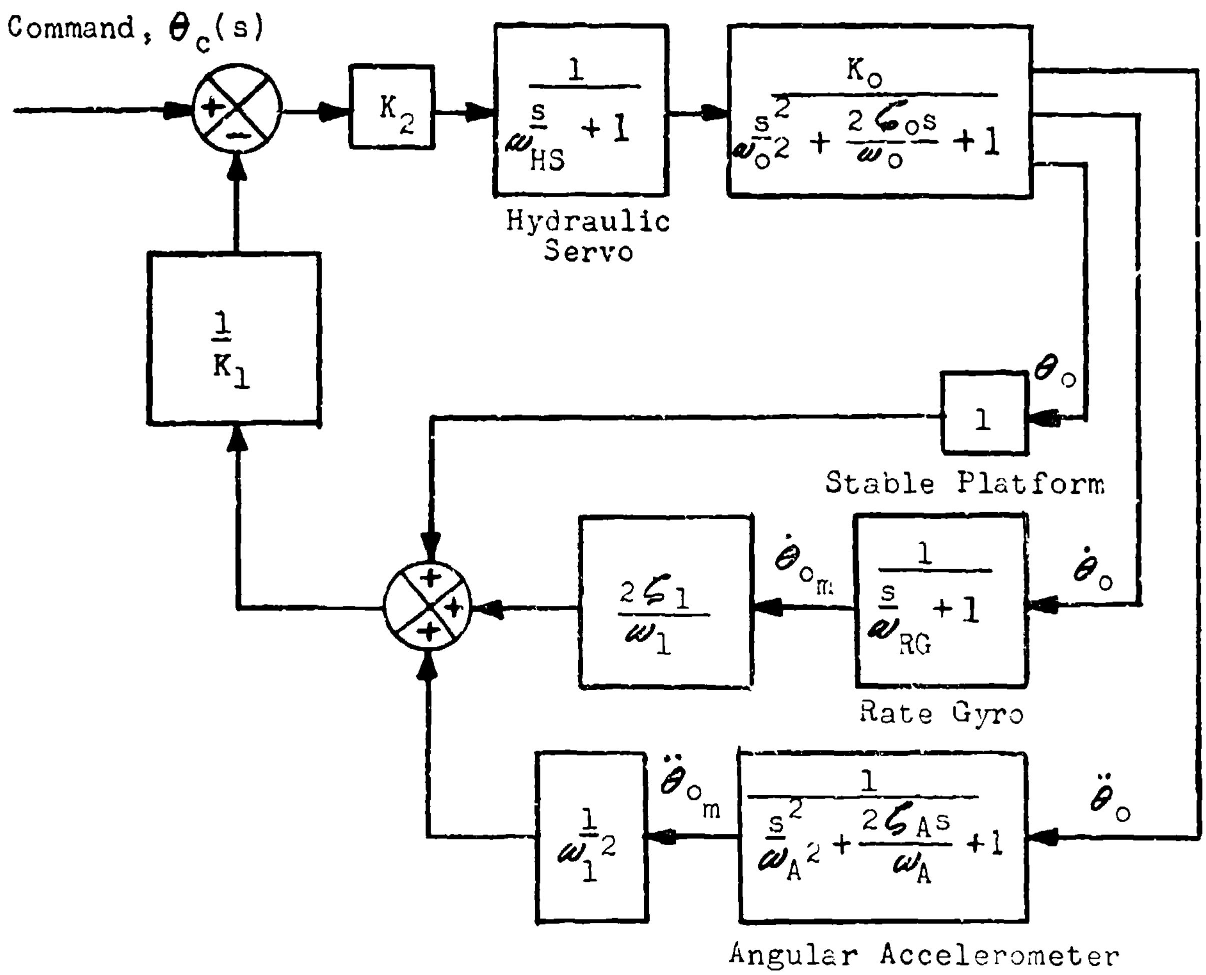


Fig. 7 DIAGRAM OF CONTROL APPLICATION INCLUDING INSTRUMENTATION

ANALYSIS AND SYNTHESIS OF TYPICAL FLIGHT CONTROL SYSTEM WITH SENSING INSTRUMENTS AND HYDRAULIC SERVO

Description of Additional Components

Components which contribute chiefly to higher order characteristics in aircraft control systems are those which convert electrical command signals into motion of the airframe control surfaces and those which measure the airframe motion and transduce this motion into electrical signals. In order to illustrate the effects of these components on the self-adaptive system described previously, the following will be required: a stable platform to measure the pitch angle of the airframe, a rate gyro to measure pitch angular velocity, an angular accelerometer to measure pitch angular acceleration, and a hydraulic servo to convert command signals into elevator deflections.

In the block diagram in Figure 7 the location of each of these components in the loop and the form of their transfer functions are shown. Only the major dynamic effects were included. The performance of the self-adaptive control was then calculated for the expected ranges of values of these dynamic effects. The inclusion of these components into the system introduces additional phase lag which may produce system instability at sufficiently large loop gains. The ranges which were selected were considered to be sufficient to show the trends in system stability and accuracy.

The Hydraulic Servo Actuator

Flow type hydraulic actuators produce actuator piston rates of displacements that are essentially proportional to their respective control valve positions. Transient dynamics appear as a result of actuator piston inertia, fluid compressibility, fluid leakage, line lags, and valve inertia. Some of these factors are often neglected; however, when actuator and valve position are linked by mechanical feedback, the complete diagram of a typical hydraulic servo system may be represented as in Figure 8 below.

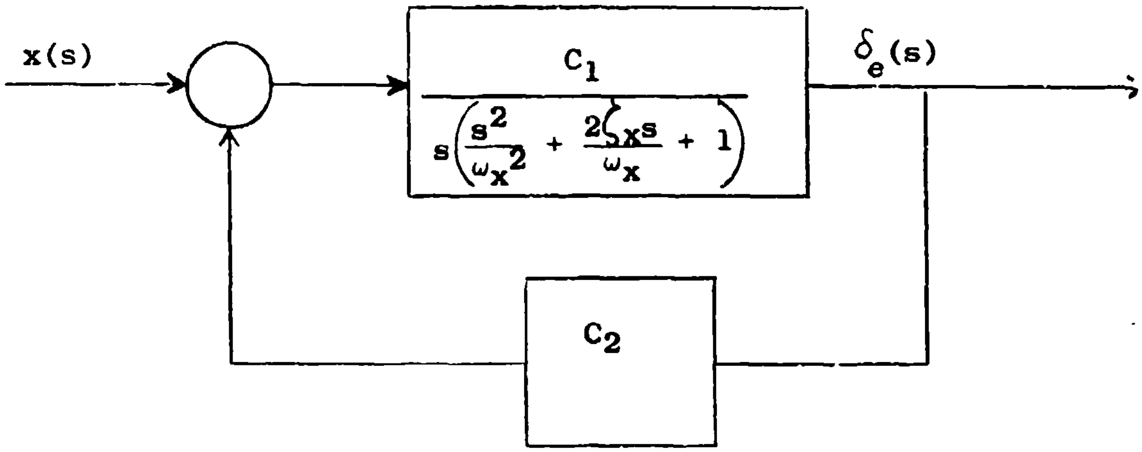


Fig. 8 - DIAGRAM OF TYPICAL HYDRAULIC SERVO SYSTEM

The second order lag factor characterized by the term ω_x and ζ_x represent the inertia and compressibility effects mentioned previously. For representative hydraulic systems, typical values of ω_x are in the neighborhood of 600 radians per second and typical values of ζ_x are near zero. In the discussions which follow, these dynamics will be neglected since their frequency characteristics are outside the frequency range of the airplane. The closed-loop transfer function of the hydraulic system illustrated in Figure 9 is then reduced to the function

$$\frac{\delta_e(s)}{x(s)} = \frac{\frac{1}{C_2}}{\frac{s}{C_1 C_2} + 1} .$$

The hydraulic servo characteristic frequency ω_{HS}, represented in Figure 8, is adjustable by changing the servo loop gain C_1C_2. The following values were selected in this analysis

$$20 \text{ radins per second} \leq \omega_{HS} \leq 50 \text{ radins per second.}$$

The gain of the hydraulic servo was assumed to be unity.

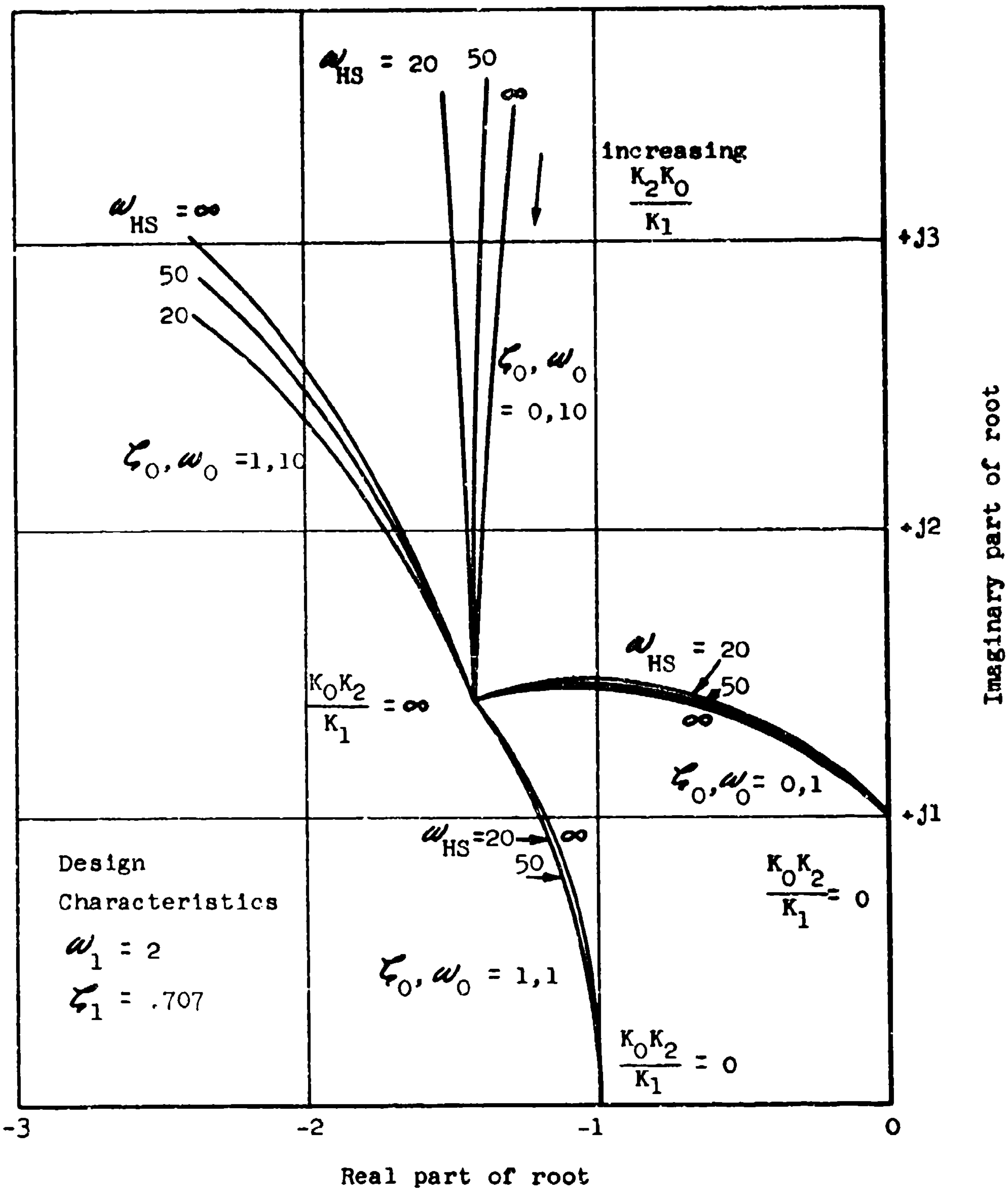


Fig. 9 EFFECT OF HYDRAULIC SERVO ON CLOSED-LOOP, SHORT PERIOD CHARACTERISTICS FOR DIFFERENT LOOP GAINS AND FREE AIRFRAME CHARACTERISTICS

The Rate Gyro

A rate gyro is a single-degree-of-freedom gyro with an elastic restraint on its movement about the free axis so that the angular deflection of this axis is proportional to the angular velocity about the input axis fixed to the airplane. Factors which have an effect on the dynamic response of a rate gyro are essentially the compliance of the elastic restraint, the moment of inertia about the free axis, and the viscous damping introduced to damp the free gimbal deflection.

In most practical cases the transient behavior of rate gyros may be represented mathematically by a second order system. The corresponding transfer function may be written in the form

$$\frac{\dot{\theta}_{0_m}(s)}{\dot{\theta}_0(s)} = \frac{\frac{H}{K_r}}{\left(\frac{H^2}{K_i K_r} + \frac{J_r}{K_r}\right) s^2 + \frac{f_r}{K_r} s + 1} .$$

Typical rate gyros have natural frequencies ranging approximately from 40 to 500 radians per second. The usually desired value of damping ratio ranges from 0.6 to 0.8. Damping ratio is known to vary with ambient temperature. A temperature change spanning 250°F, for example, will vary damping ratio from approximately 0.3 to 2.0 in a typical case. Furthermore, an increase in viscosity is known to increase the gyro moment of inertia. This increase in moment results in a decrease in natural frequency. With a damping ratio less than unity, the rate gyro characteristics are represented by a second order transfer function with two complex poles. With a damping ratio greater than unity, two real poles are present which from two first order terms, one of which will have a dominant effect on phase lag.

For these reasons two representations of the rate gyro characteristics were considered:

$$\frac{\dot{\theta}_{0_m}(s)}{\dot{\theta}_0(s)} = \frac{C_3}{\frac{s^2}{\omega_{RG}} + \frac{2\zeta_{RG} s}{\omega_{RG}} + 1}$$

and
$$\frac{\dot{\theta}_{0_m}(s)}{\dot{\theta}_0(s)} = \frac{C_3}{\left(\frac{s}{\omega_{RG_1}} + 1\right)\left(\frac{s}{\omega_{RG_2}} + 1\right)}$$

These representations were compared by considering two typical cases, one involving a damping ratio of 0.7 and an undamped natural frequency of 100 radians per second and the other a damping ratio of 1.45 and the first order term characteristics $\omega_{RG_1} = 40$ radians per second and ω_{RG_2} is sufficiently large to be negligible in its effect on the system as compared to ω_{RG_1}. It is shown in a later section that there is no significant change in the effects of rate gyro dynamics on system stability whether one or the other of the above representations is used. For the above reasons, the simplest of the two representation appears justifiable; hence in this analysis the rate gyro characteristics were represented by

$$\frac{\dot{\theta}_{0_m}(s)}{\dot{\theta}_0(s)} = \frac{C_3}{\frac{s}{\omega_{RG}} + 1} .$$

The following values of ω_{RG} were assumed in this analysis.

40 radians per second $\leq \omega_{RG} \leq$ 60 radians per second

The gain of the rate gyro was assumed to be unity.

The Stable Platform

Three-axis, stabilized platform, normally used in aircraft, comprise three gyros for the three orthogonal axes and a gimbal system driven by servomotors controlled by the gyro outputs. The platform is usually large enough to mount instruments or devices which are required to be in a stabilized position. The angular position of the various gimbals referred to the platform is an indication of the attitude of the airframe about the various axes of inertial space.

The servomotors which stabilize the platform usually have a sufficiently rapid response so that their dynamics do not affect the outputs within the frequency range of interest. The dynamics of the platform suspension system may likewise be neglected for practical purposes. For these reasons the stable platform will be considered to be a perfect transducer.

The Angular Accelerometer

Typical angular accelerometers include a dynamically balanced mass suspended so that it has only one degree of freedom. The mass rotates against a restraining spring and damper; this movement is sensed by a transducer. The angular acceleration of the instrument case is proportional to the angular displacement of the mass with respect to the case. The transfer function of the instrument may be shown to be of the form

$$\frac{\ddot{\theta}_{0_m}(s)}{\dot{\theta}_0(s)} = \frac{\frac{I}{k}}{\frac{I}{k} s^2 + \frac{f}{k} s + 1} \, .$$

Typical accelerometers have natural frequencies ranging from 50 to 600 radians per second. The usually desired value of damping ratio is 0.6 since this value gives a good steady state sinusoidal response and a reasonable overshoot for transient inputs. In this analysis the following ranges of natural frequency, ω_A, and damping ratio ζ_A were chosen

$$50 \text{ radians per second} \leq \omega_A \leq 250 \text{ radians per second}$$

$$0.25 \leq \zeta_A \leq 2.0$$

The gain of the accelerometer was assumed to be unity.

Effects of Added Components on Short-Period Characteristics at Various Loop Gains

The effect of added components on the short period correction may be found by calculating the closed-loop transfer function of the over-all system depicted in Figure 7. By using the formula given in Equation 1, the following result is easily obtained.

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{\dfrac{K_oK_2}{\left(\dfrac{s}{\omega_{HS}} + 1\right)\left(\dfrac{s^2}{\omega_0^2} + \dfrac{2\zeta_0 s}{\omega_0} + 1\right)}}{1 + \dfrac{K_oK_2\left[\dfrac{s^2}{\omega_1^2\left(\dfrac{s^2}{\omega_A} + \dfrac{2\zeta_A s}{\omega_A} + 1\right)} + \dfrac{2\zeta_1 s}{\omega_1\left(\dfrac{s}{\omega_{RG}} + 1\right)} + 1\right]}{K_1\left(\dfrac{s}{\omega_{HS}} + 1\right)\left(\dfrac{s^2}{\omega_0^2} + \dfrac{2\zeta_0 s}{\omega_0} + 1\right)}} . \quad (23)$$

When K_2 is made sufficiently large so that

$$\frac{\left[\dfrac{s^2}{\omega_1^2\left(\dfrac{s^2}{\omega_A^2} + \dfrac{2\zeta_A s}{\omega_A} + 1\right)} + \dfrac{2\zeta_1 s}{\omega_1\left(\dfrac{s}{\omega_{RG}} + 1\right)} + 1\right] K_oK_2}{K_1\left(\dfrac{s}{\omega_{HS}} + 1\right)\left(\dfrac{s^2}{\omega_0^2} + \dfrac{2\zeta_0 s}{\omega_0} + 1\right)} \gg 1, \quad (24)$$

then Equation 23 is reduced to

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_1}{\dfrac{s^2}{\omega_1^2\left(\dfrac{s^2}{\omega_A^2} + \dfrac{2\zeta_A s}{\omega_A} + 1\right)} + \dfrac{2\zeta_1 s}{\omega_1\left(\dfrac{s}{\omega_{RG}} + 1\right)} + 1} . \quad (25)$$

Two important results may be seen immediately. The dynamics of the hydraulic servo or of the airframe do not appear in Equation 25. In fact any element in the forward path of the control system (Fig. 7) has no effect on the over-all dynamics of the system at high loop gains (it is assumed that

these elements are linear). Also, no matter now large the gain K_2 is chosen, the system will not approach the ideal characteristics defined by the transfer function

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_1}{\frac{s^2}{\omega_1^2} + \frac{2\zeta_1 s}{\omega_1} + 1} \quad . \tag{26}$$

The reason for this is the dynamic effect of the accelerometer and the rate gyro. These effects may be evaluated in Equation 23 by inserting different values of the terms ω_{HS}, ω_{RG}, ω_A and ζ_A.

It is convenient to first rewrite Equation 23 as follows:

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_0K_2}{\left(\frac{s}{\omega_{HS}}+1\right)\left(\frac{s^2}{\omega_0^2}+\frac{2\zeta_0 s}{\omega_0}+1\right) + \frac{K_0K_2}{K_1}\left[\frac{s^2}{\omega_1^2\left(\frac{s^2}{\omega_A^2}+\frac{2\zeta_A s}{\omega_A}+1\right)} + \frac{2\zeta_1 s}{\omega_1\left(\frac{s}{\omega_{RG}}+1\right)} + 1\right]} \tag{27}$$

The dynamic characteristics may now be obtained by computing the roots of the equation formed by setting the denominator of Equation 27 equal to zero as follows:

$$\left(\frac{s}{\omega_{HS}} + 1\right)\left(\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1\right) + \frac{K_0K_2}{K_1}$$

$$\left[\frac{s^2}{\omega_1^2\left(\frac{s^2}{\omega_A^2} + \frac{2\zeta_A s}{\omega_A} + 1\right)} + \frac{2\zeta_1 s}{\omega_1\left(\frac{s}{\omega_{RG}} + 1\right)} + 1\right] = 0. \tag{28}$$

Figures 9, 10 and 11 show the loci of the short period roots of Equation 28 for different values of ω_{HS}, ω_{RG}, ω_A and ζ_A as the gain parameter $\frac{K_o K_2}{K_1}$ is varied from zero to infinity.

Four combinations of free airframe characteristics were chosen to encompass the widest possible range of flight conditions; these are as follows:

ω_0	ζ_0
1	0
1	1
10	0
10	1

The ideal characteristics were chosen as follows:

$$\omega_1 = 2 \text{ radians per second}$$
$$\zeta_1 = 0.707.$$

The separate effects of the hydraulic servo, the rate gyro, and the accelerometer are discussed below.

<u>Effect of the Hydraulic Servo</u>

The effects of the hydraulic servo alone (Fig. 9) were obtained by neglecting the rate gyro and accelerometer dynamics in Equation 27.

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_o K_2}{\left(\frac{s}{\omega_{HS}}+1\right)\left(\frac{s^2}{\omega_0^2}+\frac{2\zeta_0 s}{\omega_0}+1\right)+\frac{K_o K_2}{K_1}\left(\frac{s^2}{\omega_1^2}+\frac{2\zeta_1 s}{\omega_1^2}+1\right)} \cdot \qquad (29)$$

As $\frac{K_o K_2}{K_1}$ approaches zero, the feedback path becomes open circuited, and Equation 29 approaches

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_o K_2}{\left(\frac{s}{\omega_{HS}}+1\right)\left(\frac{s^2}{\omega_0^2}+\frac{2\zeta_0 s}{\omega_0}+1\right)} \cdot \qquad (30)$$

As $\frac{K_o K_2}{K_1}$ approaches infinity, Equation 29 approaches

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_1}{\frac{s^2}{\omega_1^2} + \frac{2\zeta_1 s}{\omega_1} + 1} . \tag{31}$$

These results indicate that with no feedback the short period characteristics are defined by the roots of the equation

$$\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1 = 0, \tag{32}$$

and with a large feedback loop gain the characteristics are defined by the roots of the equation

$$\frac{s^2}{\omega_1^2} + \frac{2\zeta_1 s}{\omega_1} + 1 = 0 . \tag{33}$$

Therefore, at both extremes of loop gain, the short period response is independent of the hydraulic servodynamics.

In the midrange values of loop gain, the system dynamics are defined by the roots of the equation

$$\left(\frac{s}{\omega_{HS}} + 1\right)\left(\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1\right) + \frac{K_o K_2}{K_1}\left(\frac{s^2}{\omega_1^2} + \frac{2\zeta_1 s}{\omega_1} + 1\right) = 0. \tag{34}$$

In Figure 9 the effect of loop gain is shown for two finite values of hydraulic servo characteristic frequency, namely, 20 and 50 radians per second and an infinite frequency. The curves for which the frequency is infinite represent the system in which an ideal hydraulic servo is used, that is, one

with a unity transfer function. As the characteristic frequency, ω_{HS}, is decreased, it can be seen from Figure 9 that the short period roots deviate further and further from the ideal case.

From previous analysis it has been shown that at both extremes of loop gain the servodynamics do not affect the short period response. The maximum deviation resulting from the servodynamics occurs between these two loop gain extremes. This midrange effect of the hydraulic servo appears to be more pronounced for those conditions in which the free airframe short period natural frequency, ω_0, is the largest. As would be expected, this result indicates that the closer together are the values of ω_0 and ω_{HS}, the greater will be the interactive effect between the servodynamics and the short period dynamics.

<u>Effect of the Rate Gyro</u>

The effects of the rate gyro alone (Fig. 10) are calculated from Equation 27 by neglecting the hydraulic servodynamics and the accelerometer dynamics. This equation may then be written in the following form:

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_oK_2}{\left(\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1\right) + \frac{K_oK_2}{K_1}\left(\frac{s^2}{\omega_1^2} + \frac{2\zeta_1 s}{\omega_1\left(\frac{s}{\omega_{RG}}+1\right)} + 1\right)} \tag{35}$$

As $\frac{K_oK_2}{K_1}$ approaches zero, Equation 35 approaches

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_oK_2}{\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1}, \tag{36}$$

and as $\frac{K_o K_2}{K_1}$ approaches infinity, Equation 35 approaches

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_1}{\frac{s^2}{\omega_1^2} + \frac{2\zeta_1 s}{\omega_1\left(\frac{s}{\omega_{RG}} + 1\right)} + 1} . \tag{37}$$

These results indicate that with no feedback the short period characteristics are those of the free airframe defined by Equation 32. With a very large loop gain, these characteristics are defined by the roots of the equation

$$\frac{s^2}{\omega_1^2}\left(\frac{s}{\omega_{RG}} + 1\right) + \frac{2\zeta_1 s}{\omega_1} + \frac{s}{\omega_{RG}} + 1 = 0 \quad . \tag{38}$$

The roots of Equation 38 are functions of the rate gyro dynamics; therefore, it is seen that even for an infinite loop gain the system does not attain the ideal characteristics. Since the ratio of the gyro characteristic frequency to the ideal short period frequency is of the order of 20 or 30, the short period roots of Equation 38 do not deviate markedly from the ideal roots. This is shown in Figure 10 by the variation in the terminal points of the loci.

In the midrange of values of loop gain, the system dynamics are represented by the roots of the equation

$$\left(\frac{s}{\omega_{RG}} + 1\right)\left(\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1\right) + \frac{K_o K_2}{K_1}\left[\frac{s^2}{\omega_1^2}\left(\frac{s}{\omega_{RG}} + 1\right) + \frac{2\zeta_1 s}{\omega_1} + \frac{s}{\omega_{RG}} + 1\right] = 0. \tag{39}$$

In Figure 10 the effect of loop gain is shown for two finite values of ω_{RG}, namely, 40 and 60 radians per second and an infinite characteristic frequency. The set of curves for which ω_{RG} is infinite represents the system with an ideal

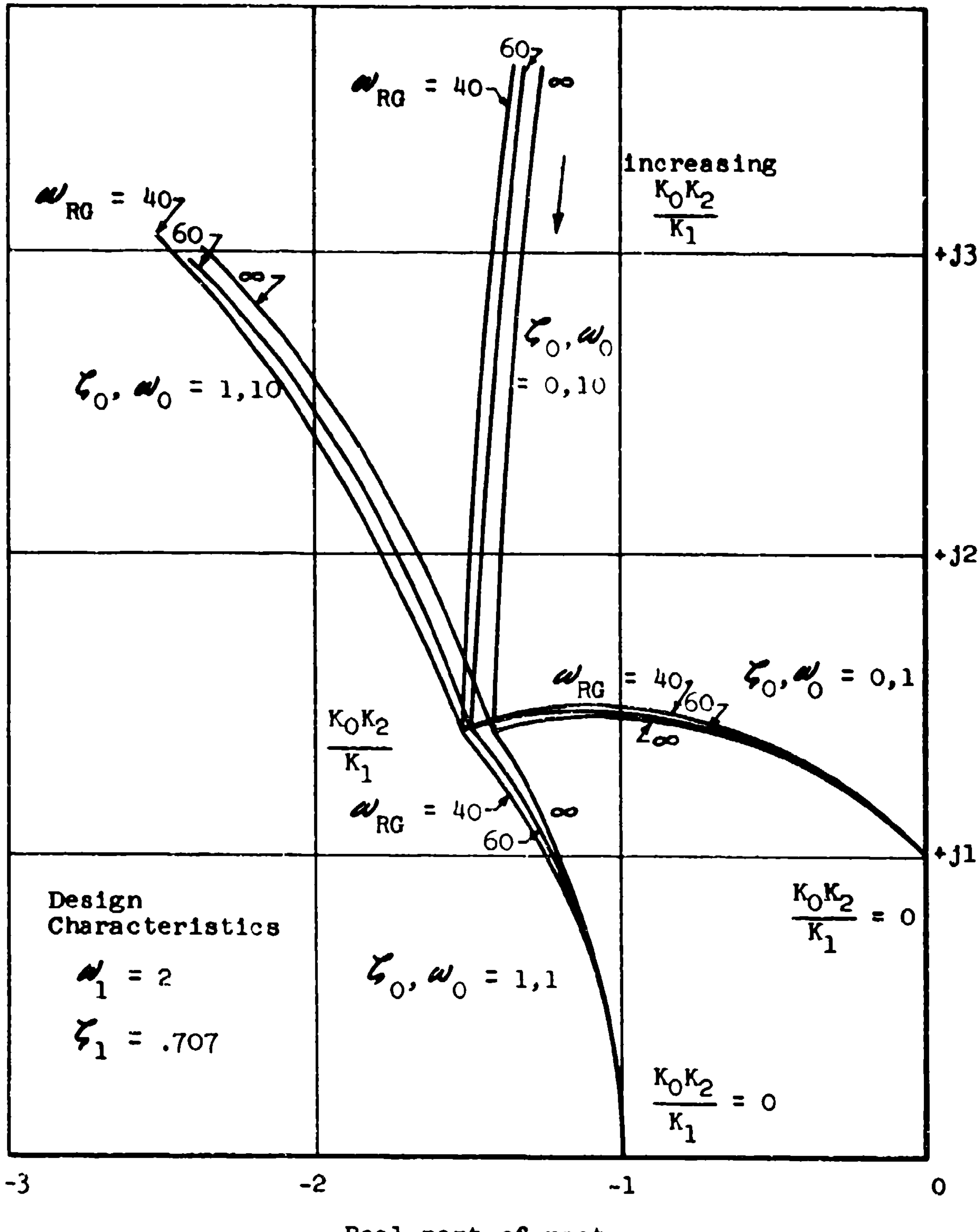


Fig. 10 EFFECT OF RATE GYRO ON CLOSED-LOOP, SHORT PERIOD CHARACTERISTICS FOR DIFFERENT LOOP GAINS AND FREE AIRFRAME CHARACTERISTICS

gyro. As the characteristic frequency of the gyro is decreased, the loci of the short period roots deviate further from the ideal case. In Figure 10 it is shown that these loci shift towards larger negative real parts as the gyro frequency decreases.

Effect of the Angular Accelerometer

The effects of the accelerometer alone (Fig. 11) are obtained from Equation 27 by omitting the hydraulic servo-dynamics and the rate gyro dynamics. This equation may then be written as follows:

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_1}{\left(\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1\right) + \frac{K_o K_2}{K_1}\left[\frac{s^2}{\omega_1^2\left(\frac{s^2}{\omega_A^2} + \frac{2\zeta_A s}{\omega_A} + 1\right)} + \frac{2\zeta_1 s}{\omega_1} + 1\right]}. \quad (40)$$

As $\frac{K_o K_2}{K_1}$ approaches zero, Equation 40 approaches

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_o K_2}{\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1}. \quad (41)$$

As $\frac{K_o K_2}{K_1}$ approaches infinity, Equation 40 approaches

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_1}{\frac{s^2}{\omega_1^2\left(\frac{s^2}{\omega_A^2} + \frac{2\zeta_A s}{\omega_A} + 1\right)} + \frac{2\zeta_1 s}{\omega_1} + 1}. \quad (42)$$

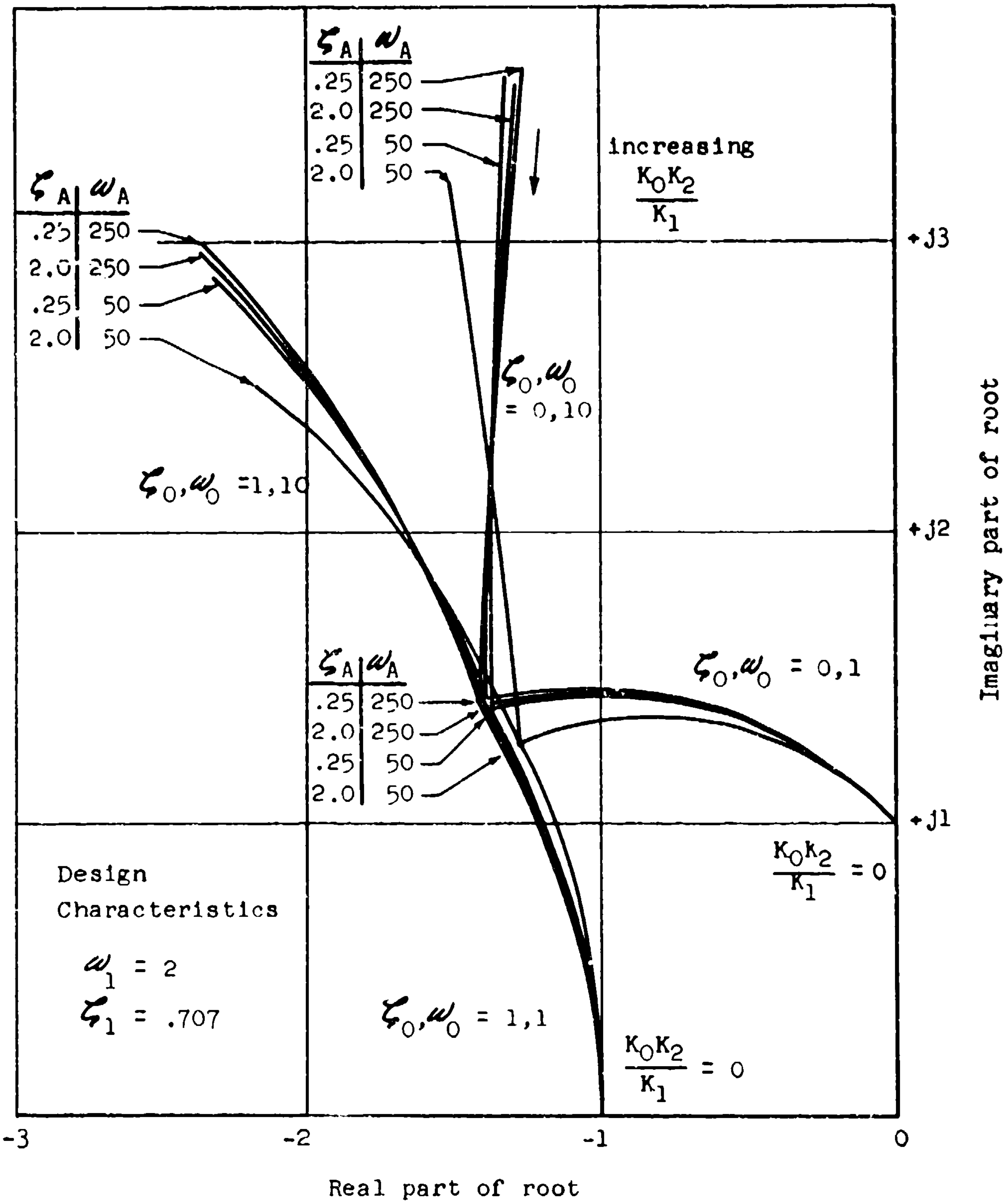


Fig. 11 EFFECT OF ACCELEROMETER ON CLOSED-LOOP, SHORT PERIOD CHARACTERISTICS FOR DIFFERENT LOOP GAINS AND FREE AIRFRAME CHARACTERISTICS

With no feedback, the short period characteristics are equal to those of the free airframe. With a very large feedback gain, these characteristics are defined by the roots of the equation

$$\frac{s^2}{\omega_1^2} + \left(\frac{s^2}{\omega_A^2} + \frac{2\zeta_A s}{\omega_A} + 1\right)\left(\frac{2\zeta_1 s}{\omega_1} + 1\right) = 0. \qquad (43)$$

The short period roots of Equation 43 are functions of the natural frequency and damping ratio of the accelerometer; therefore, the system does not attain the ideal characteristics even for an infinite loop gain. In the range of ratios of accelerometer natural frequency to short period frequency, of the order of 25 to 125, the effect of accelerometer dynamics at very large feedback gains is very small. The greatest deviation of the terminal point from the ideal point occurs for accelerometer characteristics of $\omega_A = 50$ radians per second and $\zeta_A = 2.0$. These results are illustrated graphically by the terminal points of the loci in Figure 11. In the midrange of values of loop gain, the system dynamics are represented by the roots of the equation

$$\left(\frac{s^2}{\omega_0^2} + \frac{2\zeta_0 s}{\omega_0} + 1\right)\left(\frac{s^2}{\omega_A^2} + \frac{2\zeta_A s}{\omega_A} + 1\right) + \frac{K_0 K_2}{K_1}\left[\frac{s^2}{\omega_1^2} + \left(\frac{2\zeta_1 s}{\omega_1} + 1\right)\left(\frac{s^2}{\omega_A^2} + \frac{2\zeta_A s}{\omega_A} + 1\right)\right] = 0. \qquad (44)$$

In Figure 11 the effect of loop gain, $\frac{K_0 K_2}{K_1}$ on the short period roots of Equation 44 is shown for four combinations of accelerometer natural frequency and damping ratio as follows

ω_A	ζ_A
50	0.25
50	2.0
250	0.25
250	2.0

The distortion of the loci from that of the ideal case is small except for the case $\omega_A = 50$ and $\zeta_A = 2.0$. In this instance the loci terminal point appears to be shifted towards the origin in relation to the ideal terminal point. This indicates a reduction in the short period frequency which is attained with little change in damping ratio.

The shifts of the root loci and the deviation of the terminal points of these loci from the design value resulting from the accelerometer and the rate gyro dynamics may be utilized to the designer's advantage. The use of these phenomena in the synthesis of the system will be discussed in the following section.

Effects of Added Components on System Stability

The closed-loop short period natural frequency and damping ratio that are produced by the self-adaptive control system previously discussed have been shown to vary significantly with changes in loop gain but to vary only slightly in terms of variations in sensor and servodynamics. The realistic system shown in Figure 7 contains components which generate additional modes of motion that are superimposed upon the short period motion. The interaction between these additional modes and the short period mode is evident in Figures 9, 10, and 11 and is shown by the displacement of the short period roots. The actual roots which caused these displacements are not shown in Figures 9, 10, and 11. They become important only when their real parts approach positive values indicating system instability.

Stability problems resulting from these additional components may be studied by conventional methods of analysis in which Root Locus, Bode, or Nyquist diagrams are used. The application of any one of these three methods to the multiple loop control system previously discussed would be tedious and mathematically complicated because the effects of changes in component dynamics and feedback gains enter into all three of the feedback loops. The most desirable analytical technique is that which requires the least recalculation when any of the above parameters are varied. For this reason a relatively new technique, the dual Nyquist method, was selected at most applicable on this analysis.

Derivation of the Dual Nyquist Diagram

The dual Nyquist diagram is a graphical procedure for determining the stability of feedback systems. A mathematical derivation of this technique is described in Reference 2; a short introduction to the method is included in Appendix A. The particular application of this method to the present problem may be approached by representing the closed-loop transfer of the system in several ways. The approach which was chosen separates the forward path of the loop from the feedback path which in turn separates the effects carried by sensor dynamics from those caused by airframe and hydraulic servodynamics.

The complete closed-loop transfer function, Equation 23, may be written as follows:

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_1}{\frac{K_1}{K_o K_2}\left(\frac{s}{\omega_{HS}}+1\right)\left(\frac{s^2}{\omega_0^2}+\frac{2\zeta_0 s}{\omega_0}+1\right)+\left[\frac{s^2}{\omega_1^2\left(\frac{s^2}{\omega_A^2}+\frac{2\zeta_A s}{\omega_A}+1\right)}+\frac{2\zeta_1 s}{\omega_1\left(\frac{s}{\omega_{RG}}+1\right)}+1\right]} \tag{45}$$

or

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_1}{A(s) + B(s)}$$

where

$$A(s) = \frac{K_1}{K_o K_2}\left(\frac{s}{\omega_{HS}}+1\right)\left(\frac{s^2}{\omega_0^2}+\frac{2\zeta_0 s}{\omega_0}+1\right) \tag{46}$$

and

$$B(s) = \frac{s^2}{\omega_1^2\left(\frac{s^2}{\omega_A^2}+\frac{2\zeta_A s}{\omega_A}+1\right)}+\frac{2\zeta_1 s}{\omega_1\left(\frac{s}{\omega_{RG}}+1\right)}+1. \tag{47}$$

It should be noted that the forward path transfer function is equal to $\frac{K_1}{A(s)}$ and that the feedback path transfer function is equal to $\frac{B(s)}{K_1}$. A change in flight condition or servodynamics affects only A(s), and a change in sensor dynamics affects only B(s).

In order to apply the dual Nyquist theory, the maps of the complex functions A(s) and B(s) must be plotted as the complex variable s traces out a contour enclosing the right half of the complex plane. The general shapes of these loci may be approximated by considering the five significant values of the variable s, as follows:

$$s = 0 + j0$$

$$s = 0 + j\omega_1, \quad 0 < \omega_1 < \infty$$

$$s = 0 + j\infty$$

$$s = \infty + j\infty$$

$$s = \infty + j0.$$

A sketch of the location of these five points on the complex plane is shown in Figure 12. The numbers on this curve locate the particular points selected above. In Figure 12 is shown only that part of the contour in the first quadrant of the complex plane.

The shape of the plots of A(s) and B(s), as s traces this contour, are shown in Figures 13 and 14, respectively. The numbers on these curves correspond to those in Figure 12. These figures are maps of the functions A(s) and B(s) as s traces that part of the contour in the first quadrant of the complex plane. To complete the dual Nyquist plot, the fourth quadrant must be plotted. By reflecting each of the curves into the axis, the complete figures are obtained.

By applying the dual Nyquist procedures as explained in Appendix A, the functions A(s) and -B(s) must be plotted simultaneously, and their points of intersection examined. A general sketch of the dual Nyquist diagram is shown in Figure 15. In order for the over-all system to be stable, the frequency on the locus of A(s) at the point of intersection must be less than that on the locus of -B(s) at the point of intersection.

Since neutral stability occurs when the two curves intersect at the same frequency, it is evident that the relative values of frequencies at the intersection provides at least a qualitative measure of system stability.

Stability Considerations in the Synthesis of the Final Control System

The vehicle to which the dual Nyquist method of stability analysis was applied consisted of an air-to-surface powered supersonic missile of a type originally considered for use in conjunction with a supersonic bomber. For the purpose of making this stability analysis, it is not necessary to know all of the usual aerodynamic data. It is

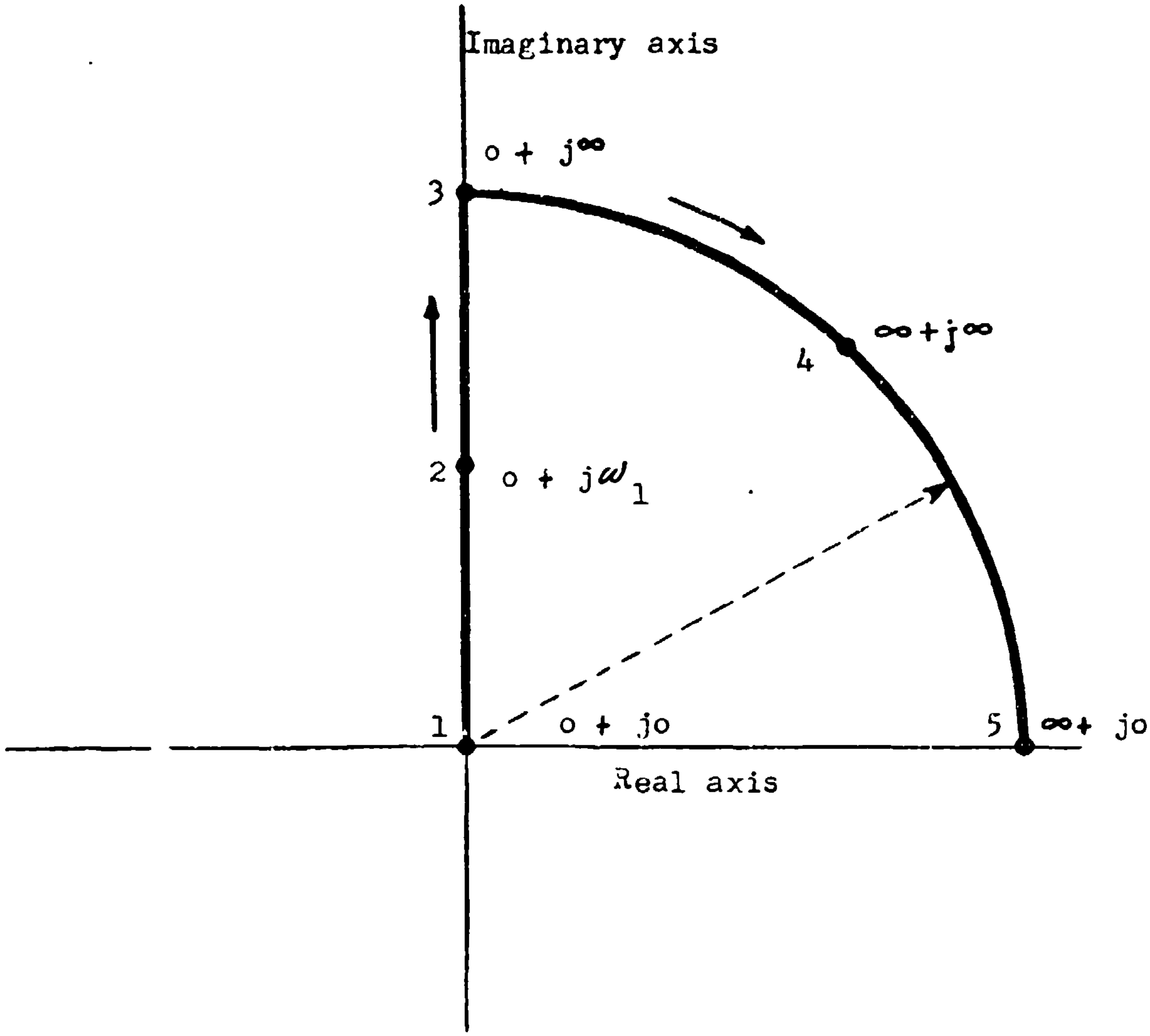


Fig. 12 CONTOUR ON COMPLEX PLANE ENCLOSING THE RIGHT HALF PLANE

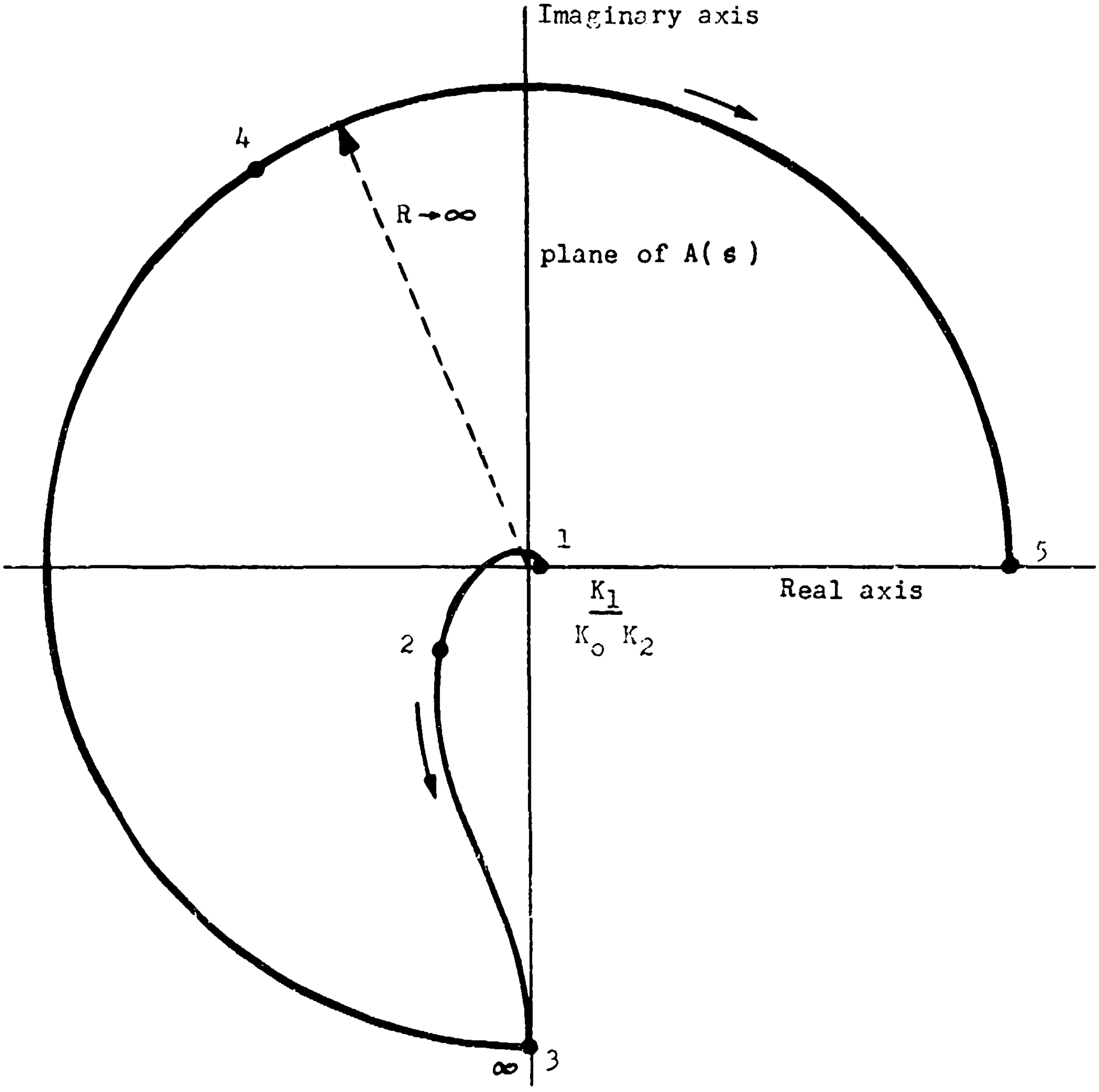


Fig. 13 MAP OF FUNCTION A(s) ON COMPLEX PLANE

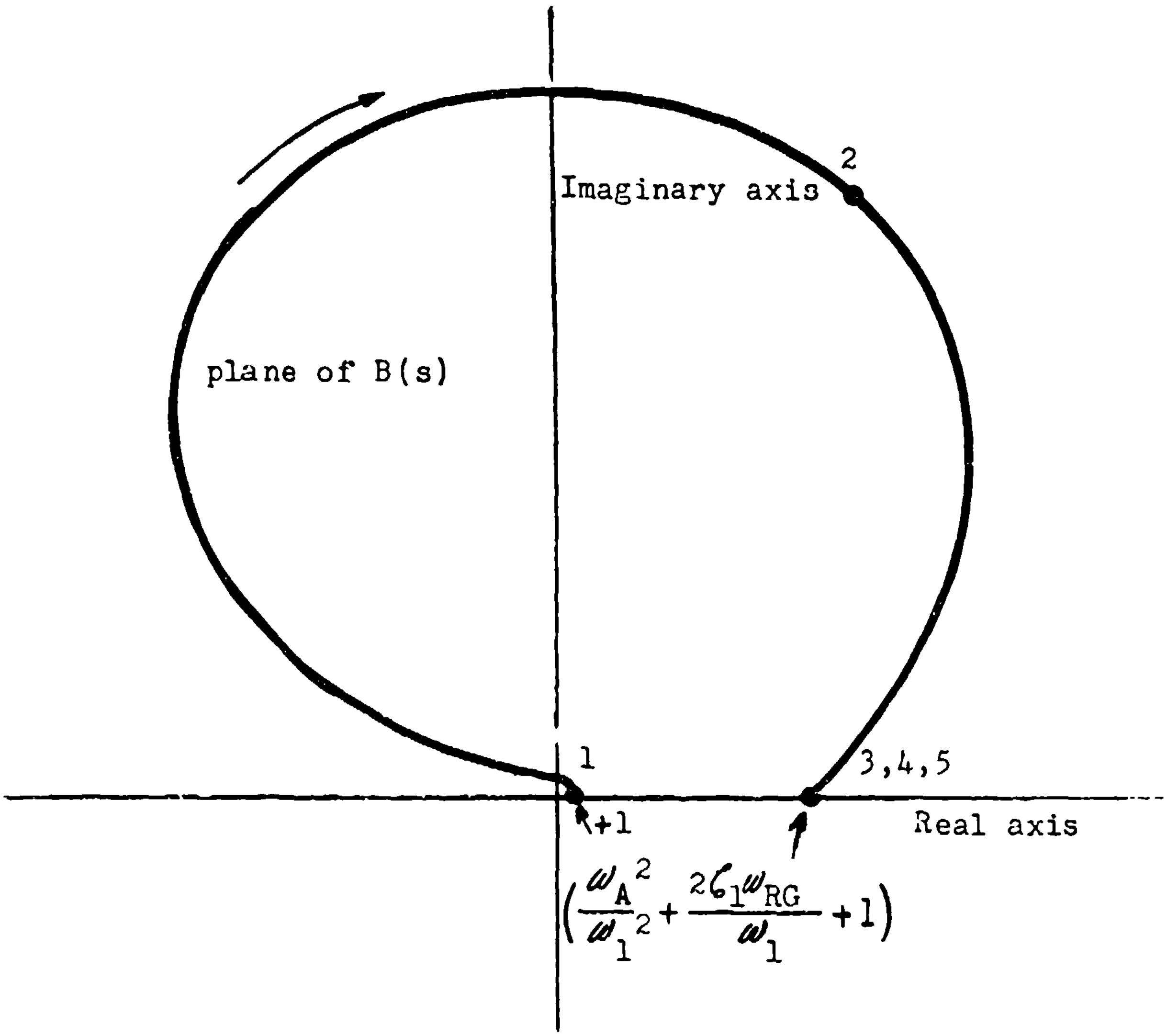


Fig. 14 MAP OF FUNCTION B(s) ON COMPLEX PLANE

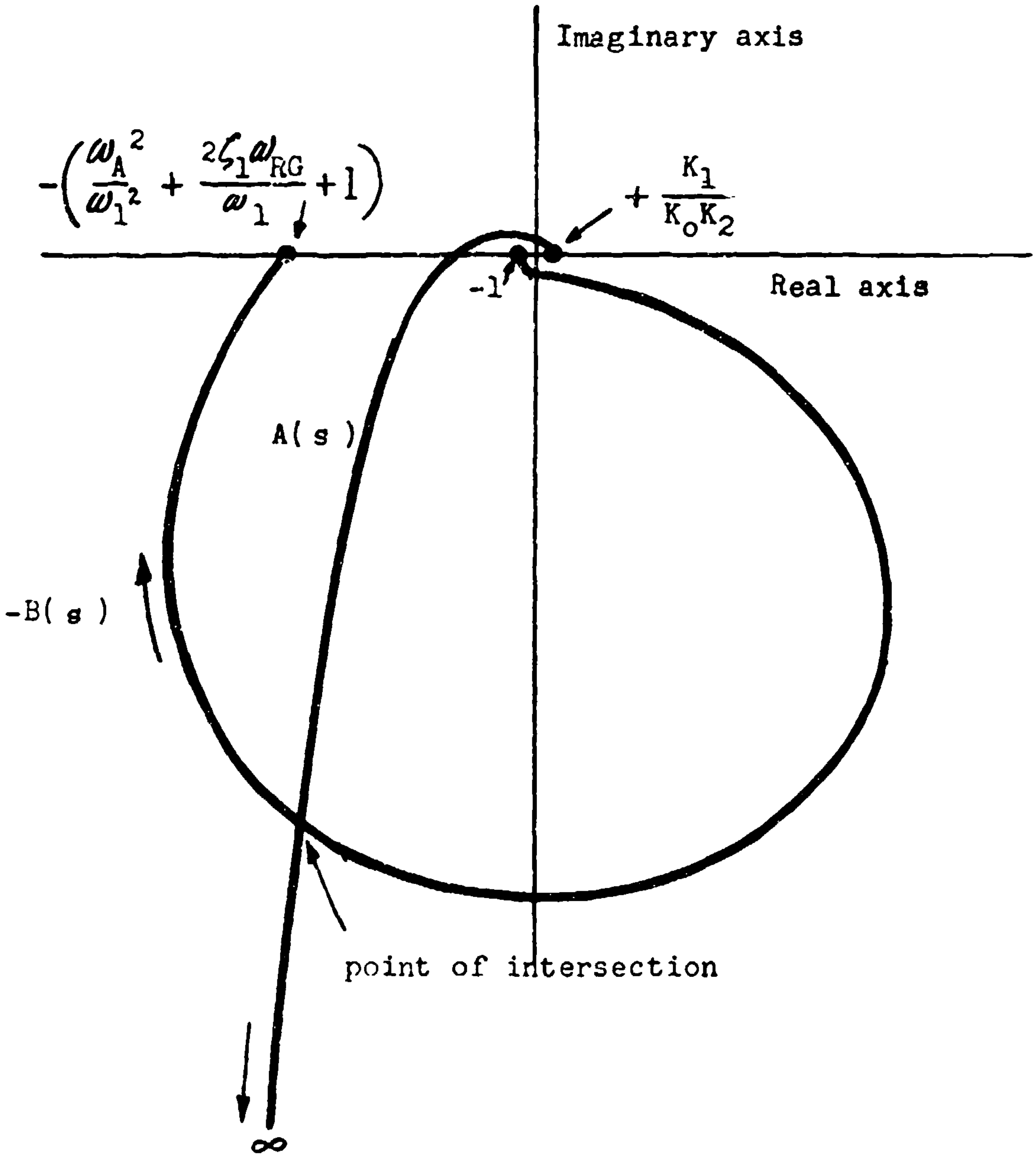


Fig. 15 DUAL NYQUIST DIAGRAM

necessary to determine only the extremes in undamped natural frequency, steady state gain, and damping ratio of the free airframe. The first two are important in determining both the over-all system performance and the gain limitations for stability. The last, damping ratio, is important only as a limiting factor in obtaining optimum performance, and, as will be illustrated, large changes in damping ratio do not complicate the stability problem to any considerable degree.

Table I contains the values of the significant parameters at each of three flight conditions which were chosen for illustration.

TABLE I

FREE AIRFRAME SHORT PERIOD CHARACTERISTICS AT ILLUSTRATIVE FLIGHT CONDITIONS

Condition	Steady Stage Gain, K_o	Undamped Natural Frequency, ω_0	Damping Ratio, ω_0
1	5.7	1.1	0.05
2	2.69	2.8	0.20
3	1.016	4.84	0.30

The range of frequency and damping which are represented is typical of a large number of modern aircraft so that the values chosen constitute a good illustration for a self-adaptive control synthesis.

Effect of Flight Condition Upon Stability - Figure 16 is a diagram containing graphs of the loci of the functions A(s) and -B(s) defined previously in Equations 46 and 47. The locus of A(s) depends upon flight condition, control loop gain, and hydraulic servodynamics. Three curves are shown corresponding to the flight conditions listed in Table I. The gain factor, K_2/K_1, was assumed equal to unity to facilitate computation and the hydraulic servo characteristic frequency, ω_{HS}, was assumed to be 20 radians per second. The locus of -B(s) depends upon rate gyro and accelerometer dynamics and upon the desired closed-loop system short period characteristics. The assumed rate

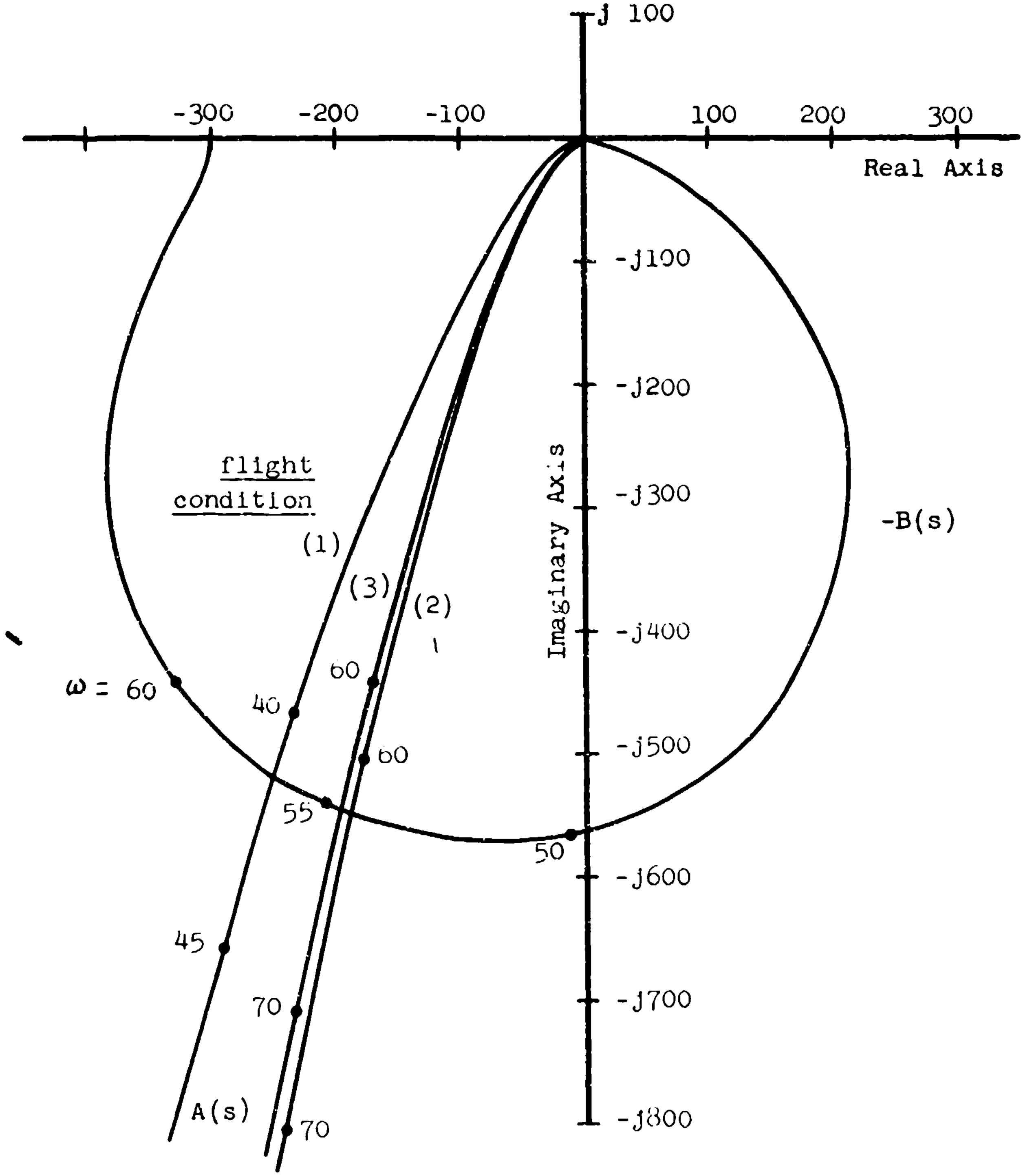


Fig. 16 DUAL NYQUIST DIAGRAM SHOWING EFFECT OF FLIGHT CONDITION UPON STABILITY

gyro characteristic frequency was $\omega_{RG} = 40$ radians per second. The accelerometer characteristics were $\omega_A = 50$ radians per second and $\zeta_A = 0.25$. The component characteristics chosen for this illustration correspond to the lowest values of frequency and damping ratio in the range of component parameters previously used in this report. The desired short period characteristics were $\omega_1 = 3.0$ radians per second and $\zeta_1 = 0.707$. These were chosen within the "good" range of pilot's preference ratings shown in Reference 3. This range lies between natural frequencies of 2 to 4 radians per second and damping ratios of 0.45 to 1.0. These values represent the desired limits for manned aircraft. For pilotless aircraft other design points may be more desirable from the point of view of structural limitations, range, altitude, speed, performance, accuracy, and other operational considerations.

It is seen by inspection of Figure 16 that for the gain, K_1/K_2, equal to unity, the frequency on the curve representing A(s) at the point of intersection is lower than that corresponding to B(s) for condition (1) and higher for conditions (2) and (3). Therefore, the system is stable at condition (1) and unstable at conditions (2) and (3). The condition for which instability is the greatest is condition (3). This is to be expected since at condition (3) occurs the highest natural frequency, closest to the component frequencies.

Effect of Loop Gain Upon System Stability - Because of the manner in which the system transfer function was written in order to develop the dual Nyquist diagram, the reciprocal of the gain factor, K_1/K_2, appears as a term in the over-all system loop gain. The system loop gain is $\frac{K_2 K_Q}{K_1}$ (see Fig. 7). Thus, if the over-all loop gain is reduced sufficiently, by increasing the factor K_1/K_2, to stabilize the system response at the least stable condition (condition (3)), this value of gain will also produce a stable response at the other conditions shown in Table I.

A simple graphical method of computing the loop gain required to produce neutral stability is shown in Figure 17. The solid curves are identical to the curves representing flight condition (3) and -B(s) in Figure 16. As can be seen, a vector has been drawn from the origin through the point $\omega = 55.5$ radians per second on the curve A(s). This vector also intersects the curve -B(s) at the point $\omega = 55.5$ radians per second. This is the only frequency at which such an intersection is possible. It is the neutral stability frequency.

The vector which determines the neutral stability frequency may be found graphically by trial and error. In Figure 17 the vector distance from the origin to the curve A(s) was increased by a factor of 1.587. Therefore the loop gain at neutral stability for condition (3) must be

$$\frac{K_2K_o}{K_1} = \frac{1}{1.587} \cdot 1.016 = 0.64 . \qquad (48)$$

The dashed line in Figure 17 represents a locus of the curve of A(s) at condition (3) for this new loop gain. It is shown merely for illustration and is not necessary in the determination of the proper gains.

The loop gains obtained at the other two flight conditions by increasing the factor K_1/K_2 to 1.587 are calculated as follows. For condition (2)

$$\frac{K_oK_2}{K_1} = \frac{2.69}{1.587} = 1.695 \qquad (49)$$

and for condition (1),

$$\frac{K_oK_2}{K_1} = \frac{5.7}{1.587} = 3.59. \qquad (50)$$

The gain margins for conditions (1) and (2) for these loop gains become 3.42 and 1.123, respectively.

A method was described above for determining the loop gain for neutral stability at one flight condition. It appears that a gain margin of 1.414 (3 db.) is sufficient for the system described in this report. The choice of gain margin depends, of course, upon the accuracy of measurement of system parameters and variations of component dynamics resulting from environmental changes. In the self-adaptive system which is discussed it is best not to overdesign because a reduction in loop gain causes the system characteristics to depart from the ideal characteristics.

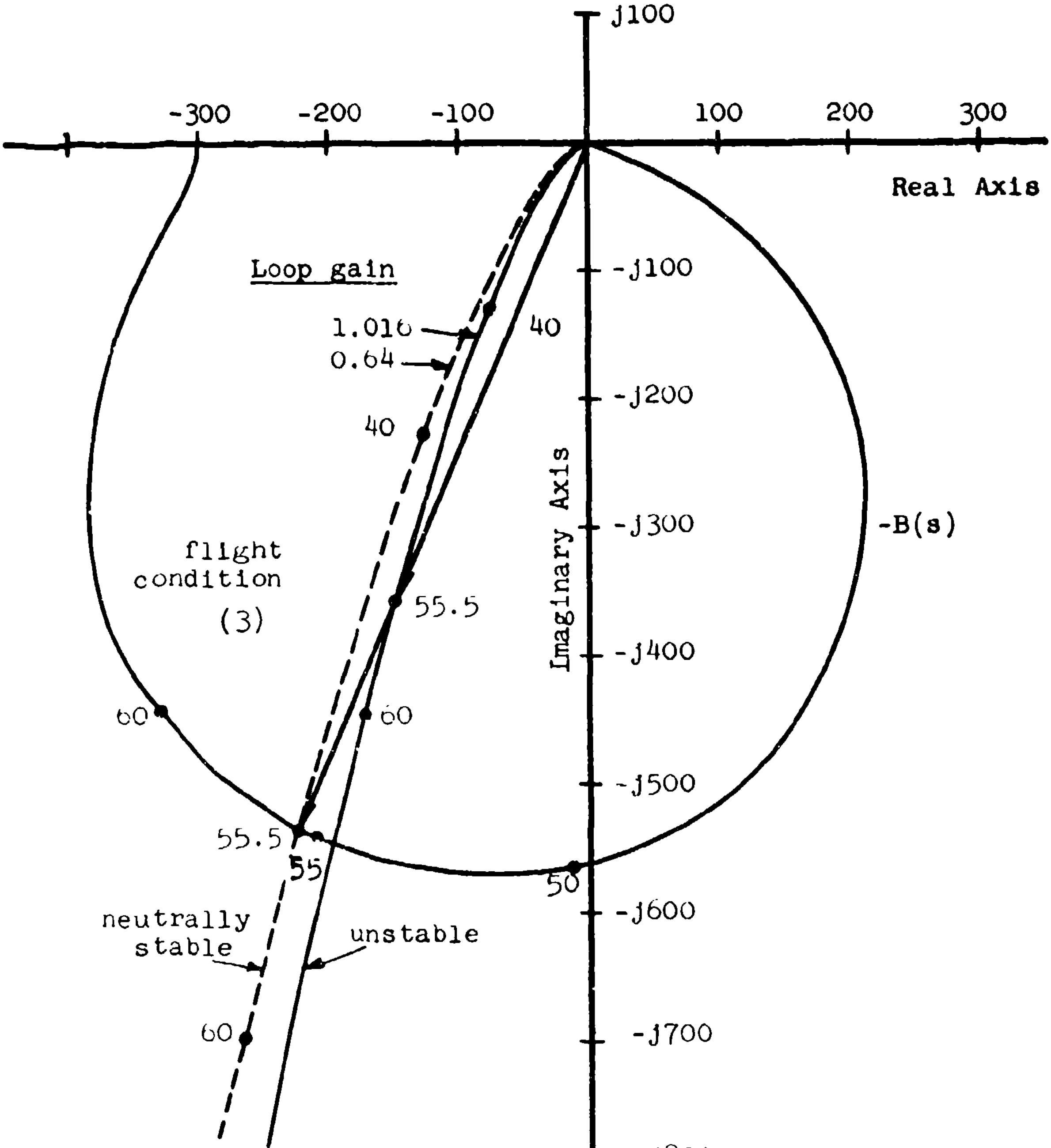


Fig. 17 DUAL NYQUIST DIAGRAM SHOWING EFFECT OF LOOP GAIN UPON STABILITY

Effect of Hydraulic Servo Characteristic Frequency Upon System Stability - Figure 18 represents a dual Nyquist diagram which shows the effect of a change in servo characteristics. Two values of servo characteristic frequency are shown at each of the three flight conditions listed in Table I. They are $\omega_{HS} = 20$ radians per second and $\omega_{HS} = 50$ radians per second. The curves which represent a servo frequency of 20 radians per second are identically the same as those shown in Figure 16. The additional curves representing a frequency of 50 radians per second show an unexpected phenomenon: an increase in servo frequency causes the system to become less stable. For example, in order to bring the response at condition (3), with $\omega_{HS} = 50$ (the least stable) to neutral stability, the loop gain must be reduced by a factor of 2.07 instead of only 1.587 with $\omega_{HS} = 20$.

The term $\left(\frac{s}{\omega_{HS}} + 1\right)$ in Equation 46 at any particular value of $s = j\omega$, has less phase lead and has less magnitude when $\omega_{HS} = 50$ than when $\omega_{HS} = 20$. The first effect, that of producing less phase lead when $\omega_{HS} = 50$, results in a shift of the curve of A(s) in the clockwise direction around the origin. This causes the locus of A(s) to cross the locus of B(s) at a higher frequency point on the locus of B(s) which, taken alone, implies an improvement in stability. The second effect, that of decreasing magnitude, results in an increase in the frequency on the locus of A(s) at the point of intersection which, taken alone, implies a decrease in stability.

In the range of servo frequencies which are of interest in this problem, the decreased gain more than compensates for the decreased phase lead, and, as a result, the system becomes less stable. At frequencies above the range of interest, the reverse compensation takes place, and the system becomes more stable again.

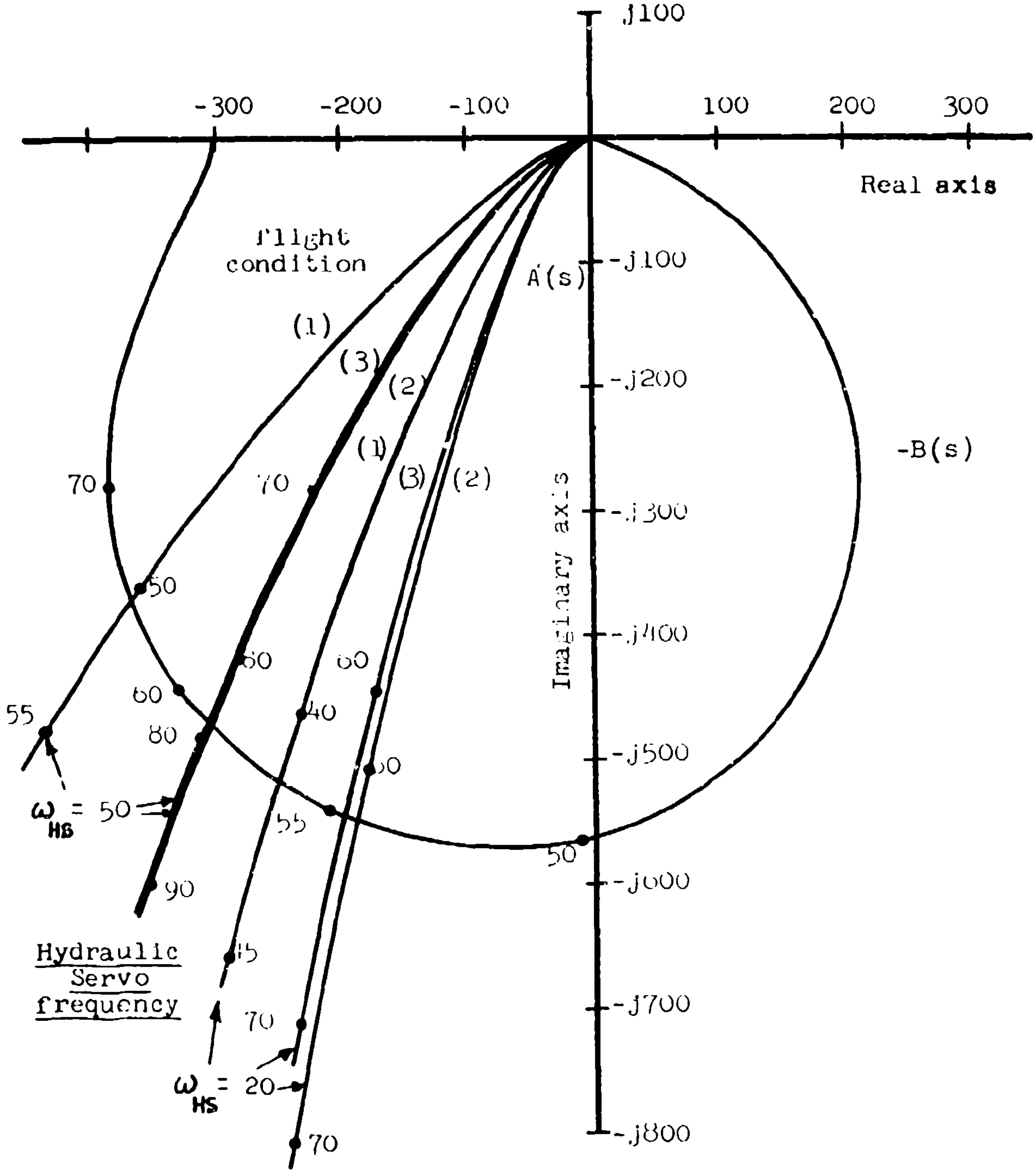


Fig. 18 DUAL NYQUIST DIAGRAM SHOWING EFFECT OF HYDRAULIC SERVO UPON STABILITY

Effect of Rate Gyro Characteristic Frequency Upon System Stability - Figure 19 represents a dual Nyquist diagram which shows the effect of a change in rate gyro characteristics. Two values of rate gyro characteristic frequency are represented: $\omega_{RG} = 40$ radians per second and $\omega_{RG} = 60$ radians per second. These effects appear as changes in the locus of $-B(s)$ and are very small. The three loci of $A(s)$ represent the three flight conditions mentioned previously and are identical with the corresponding curves in Figure 16.

An increase in rate gyro frequency causes the locus of $-B(s)$ to expand radially away from the origin. As a result the frequency at the point of intersection on the locus of $A(s)$ increases. This effect implies a decrease in stability. The above effect is easily proved to be small by expressing Equation 47 in the following form

$$B(s) = \frac{\left(\dfrac{s}{\omega_p} + 1\right)\left(\dfrac{s^2}{\omega_q^{\,2}} + \dfrac{2\zeta_q s}{\omega_q} + 1\right)}{\left(\dfrac{s}{\omega_{RG}} + 1\right)\left(\dfrac{s^2}{\omega_A^{\,2}} + \dfrac{2\zeta_A s}{\omega_A} + 1\right)} \qquad (51)$$

By using this form, it can be shown that

$$\zeta_q \approx \zeta_1 = .707,$$

$$\omega_q \approx \omega_1 = 3.0,$$

$$\omega_p \approx \omega_{RG},$$

and

$$\omega_{RG} - \omega_p \approx 2\zeta_q\omega_q \approx 4.242.$$

Thus in the range of usable rate gyro frequencies ($\omega_{RG} = 40$ to 60), ω_p and ω_{RG} differ by at most about 10 percent of their value. As a result the term $\left(\dfrac{s}{\omega_p} + 1\right)$ in the numerator

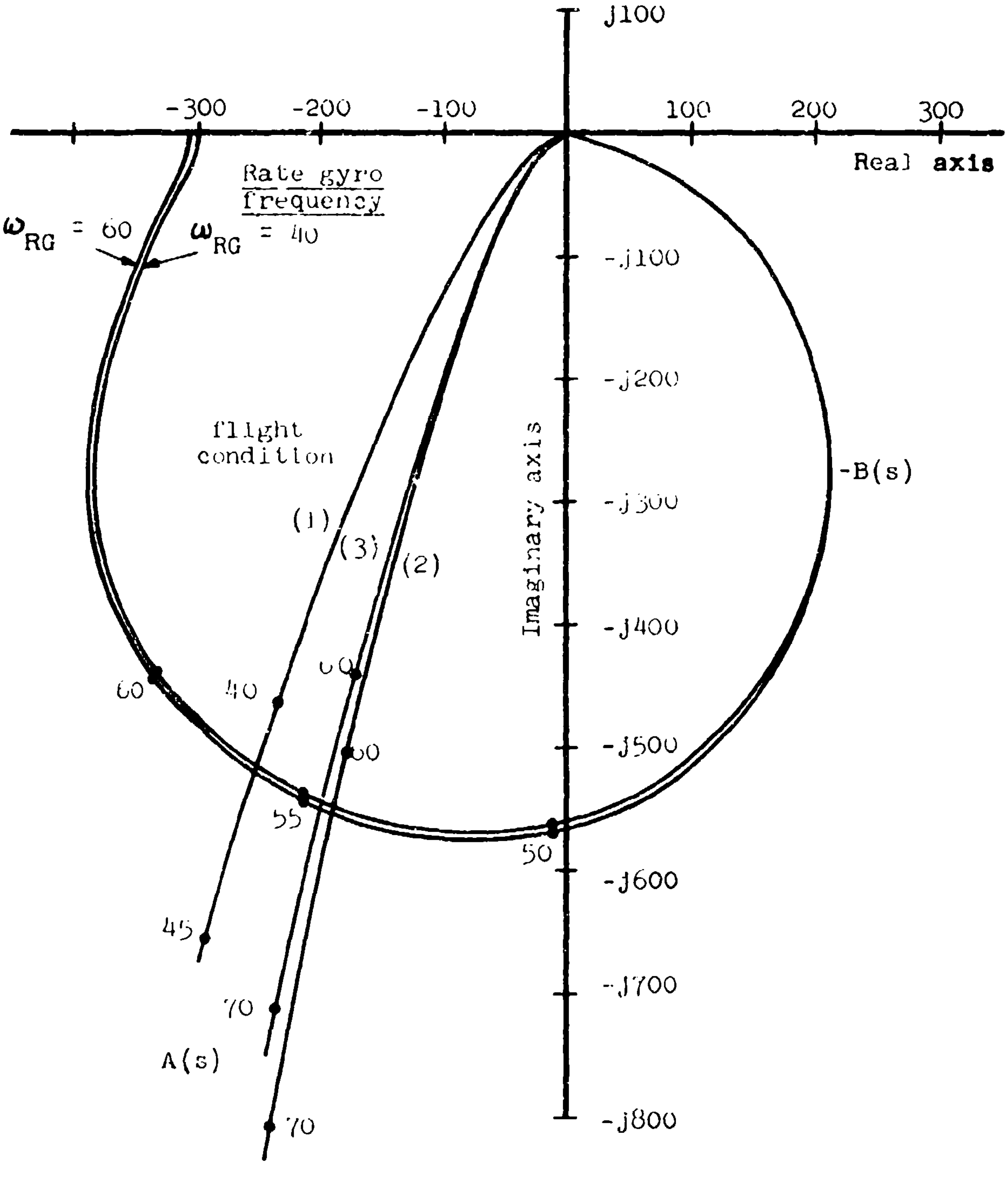


Fig. 19 DUAL NYQUIST DIAGRAM SHOWING EFFECT OF RATE GYRO UPON STABILITY

of Equation 51 almost exactly cancels the term $\left(\frac{s}{\omega_{RG}} + 1\right)$ in the denominator. Consequently, a significant change in stability resulting from a variation in ω_{RG} would hardly be expected. As a first approximation then, the locus of B(s) may be computed from

$$B(s) \approx \frac{\frac{s^2}{\omega_1^2} + \frac{2\zeta_1 s}{\omega_1} + 1}{\frac{s^2}{\omega_A^2} + \frac{2\zeta_A s}{\omega_A} + 1}. \tag{52}$$

It has been mentioned previously that there is no significant change in the effects of rate gyro dynamics on system stability whether these dynamics are represented by a first order transfer function, as has been assumed in this analysis, or by a second order system with typical characteristics $\zeta_{RG} = 0.7$ and $\omega_{RG} = 100$. This unappreciable effect on system stability is easily shown by examining the true expression for B(s), Equation 47. Only the second term contains the rate gyro characteristics and it varies with s to the first power. The first term varies with s to the second power; therefore, it becomes far more significant than the second term at large values of s. For this reason alone variations in rate gyro dynamics will not cause significant changes in B(s) at large values of s. The difference in effect on B(s), between use of the second order rate gyro representation mentioned above and use of the first order representation considered throughout the analysis, was computed. Results of this comparison are listed below:

B(s)	First order rate gyro $\omega_{RG} = 40$	Second order rate gyro $\zeta_{RG} = .7$ $\omega_{RG} = 100$
at s = j50	12.4 + j564	16.6 + j572
s = j60	331 + j442	336 + j453
s = j100	350 + j118	367 + j111

Reference to Figure 19 shows that the above variations between first and second order representations are insignificant and are approximately equal to the variations between a first order system with $\omega_{RG} = 40$ and $\omega_{RG} = 60$.

Effect of Accelerometer Undamped Natural Frequency Upon System Stability - Figure 20 represents a dual Nyquist diagram which shows the effect of a change in accelerometer undamped natural frequency. Two values of accelerometer frequency are represented: $\omega_A = 50$ radians per second and $\omega_A = 250$ radians per second. The curves representing the loci of A(s) and the locus of -B(s) for $\omega_A = 50$ radians per second are identical to the curves plotted in Figure 16.

An increase in accelerometer frequency, with no change in accelerometer damping ratio, ζ_A, causes the locus of -B(s) to expand about the origin. This expansion does not decrease stability as occurred in the case of an increase in rate gyro frequency because the frequency points are shifted along the locus of -B(s) in a counterclockwise direction as ω_A is increased. The general result is an increase in stability with increasing accelerometer frequency.

As a first approximation, it was shown that the effects of rate gyro characteristic frequency may be neglected. As a result, the function B(s) was approximated by a ratio of two quadratic polynomials in s as shown by Equation 52.

Since ω_A is much greater than ω_1, the phase angles of the numerator of Equation 52 are almost + 180 degrees for frequencies that are greater than or equal to ω_A. Hence the phase angle of -B(s) is approximately equal to - [180 degrees - phase angle of denominator, Equation 52] for all values of frequency that are greater than or equal to ω_A.

Figure 20 reveals that the phase angle of the locus of -B(s) for $\omega_A = 50$ radians per second at a frequency equal to an arbitrary fixed multiple (greater than or equal to unity) of 50 radians per second is almost identically the same as the phase angle of the locus of -B(s) for $\omega_A = 250$ radians per second at a frequency equal to the same multiple of 250 radians per second. This observation may be written concisely as follows

$$\text{phase of } \left[-B(s)_{\omega_A=50}\right]_{\omega=50K} \approx \text{phase of } \left[-B(s)_{\omega_A=250}\right]_{\omega=250K} \tag{53}$$

where K is an arbitrary multiplier.

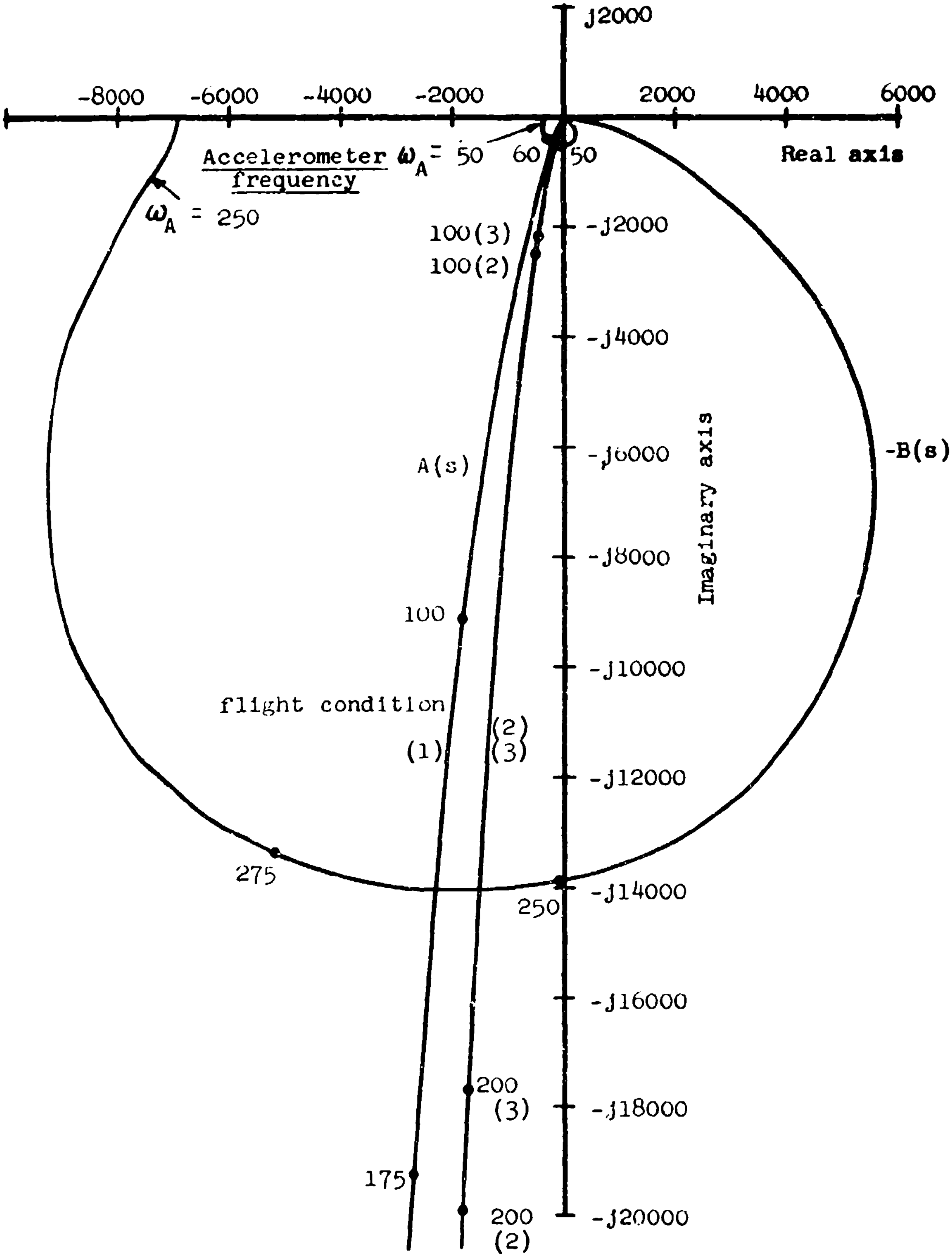


Fig. 20 DUAL NYQUIST DIAGRAM SHOWING EFFECT OF ACCELEROMETER UNDAMPED NATURAL FREQUENCY UPON STABILITY

Inspection of Equation 52 reveals that the magnitudes of the loci of -B(s) (i.e., the vector distances from the origin to the loci of -B(s) shown in Fig. 20) are related as follows for frequencies that are greater than or equal to ω_A.

$$\frac{\left| -B(s)_{\omega_A=50} \right|_{\omega=50K}}{\left| B(s)_{\omega_A=50} \right|_{\omega=250K}} = \frac{(50K)^2}{(250K)^2} = \left(\frac{50}{250}\right)^2 . \quad (54)$$

Thus, from Equation 53, it is seen that, when a straight line is drawn from the origin intersecting the two loci of -B(s), the frequencies at the points of intersection on the loci of -B(s) are near identical multiples of the respective values of ω_A. Also, it is seen from Equation 54 that the ratio of the magnitudes of the above distance from the origin to each point of intersection is equal to the square of the ratio of the respective values of ω_A.

From an inspection of Figure 20 it is evident that the phase angle of the locus of A(s) changes very slightly between the point where it intersects the locus of -B(s) for $\omega_A = 50$ radians per second and the locus of -B(s) for $\omega_A=250$ radians per second. Thus the approximation may be made that the locus of A(s) is almost a straight line from the origin. By referring to Equation 46, it can be seen that the magnitude of A(s) increases as the cube of the frequency for sufficiently large frequencies.

On the basis of the observations stated in the previous two paragraphs, it is therefore deduced that, as ω_A is increased, the intersection frequency on the locus of -B(s) becomes larger at a faster rate than the intersection frequency on the locus of A(s). Hence the system becomes more stable as ω_A is increased.

Effect of Accelerometer Damping-Ratio Upon System Stability - Figure 21 represents a dual Nyquist diagram which shows the effect of a change in accelerometer damping ratio. Two values of damping ratio are represented: $\zeta_A = .25$ and $\zeta_A = 2.0$. The curves representing the loci of A(s) and the locus of -B(s) for $\zeta_A = .25$ are identical to the curves in Figure 16.

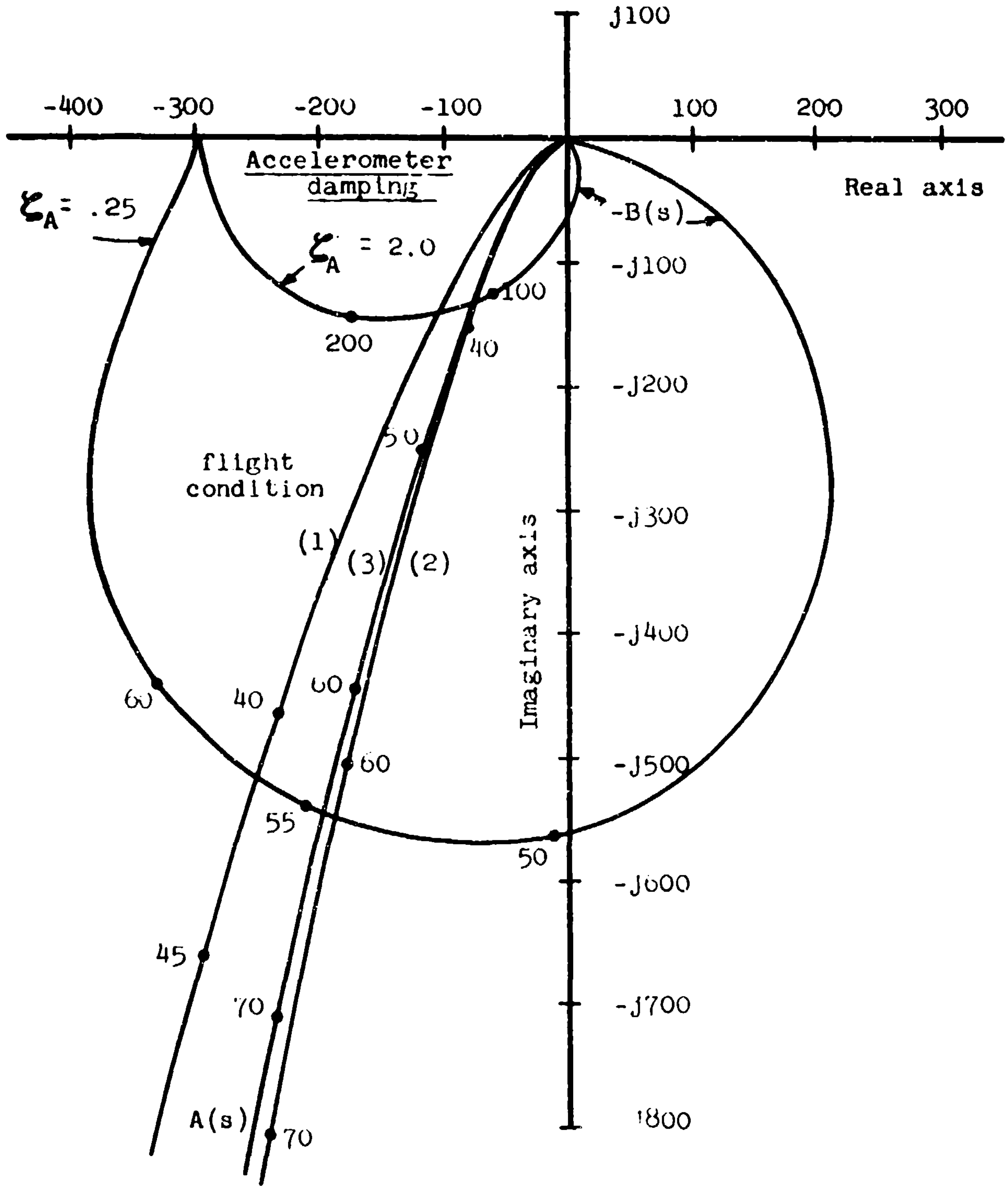


Fig. 21 DUAL NYQUIST DIAGRAM SHOWING EFFECT OF ACCELEROMETER DAMPING-RATIO UPON STABILITY

An increase in accelerometer damping ratio decreases the magnitude of the resonant peak of -B(s); consequently, the locus of -B(s) shrinks toward the origin as ζ_A is increased. This shrinking in itself implies an increase in stability since the locus of -B(s) crosses the locus of A(s) at a lower frequency on the locus of A(s). An increase in ζ_A also causes a decrease in phase lag of -B(s) at frequencies greater than ω_A. As a result, the frequency points are shifted along the loci of -B(s) in a counterclockwise direction which further increases stability. The total result is a considerable increase in stability with increasing accelerometer damping ratio.

<u>Effect of Design Short Period Damping Ratio Upon System Stability</u> - Although the design damping ratio, ζ_1, and natural frequency, ω_1, are not characteristics of components in the system, in Reference 3 it is indicated that there is some tolerable variation of these parameters. Therefore, although there may be a unique optimum design point, there appears to be enough acceptable variation in this point so that consideration ought to be given to the effect of variation in the design point upon the stability of the over-all system. The values of design points which have been considered herein all lie within the range of pilot's ratings considered good in Reference 3.

Figure 22 represents a dual Nyquist diagram which shows the effect of a change in desired short period damping-ratio. Three values of damping ratio are represented: $\zeta_1 = .45$, $\zeta_1 = .707$ and $\zeta_1 = 1.00$. The curves representing the loci of A(s) and the locus of -B(s) for $\zeta_1 = .707$ are identical to the curves in Figure 16. The curves in Figure 22 show that a change in desired damping ratio does not complicate the stability problem. The effect of a change in ζ_1 is so small as to be considered negligible.

An examination of Equation 47 reveals that the second term,

$$-\frac{2\zeta_1 s}{\omega_1\left(\dfrac{s}{\omega_{RG}}+1\right)}$$

is the only term in the expression for -B(s) which contains ζ_1. It can be shown by expressing the three terms of -B(s) as vectors and by computing their magnitudes and phase

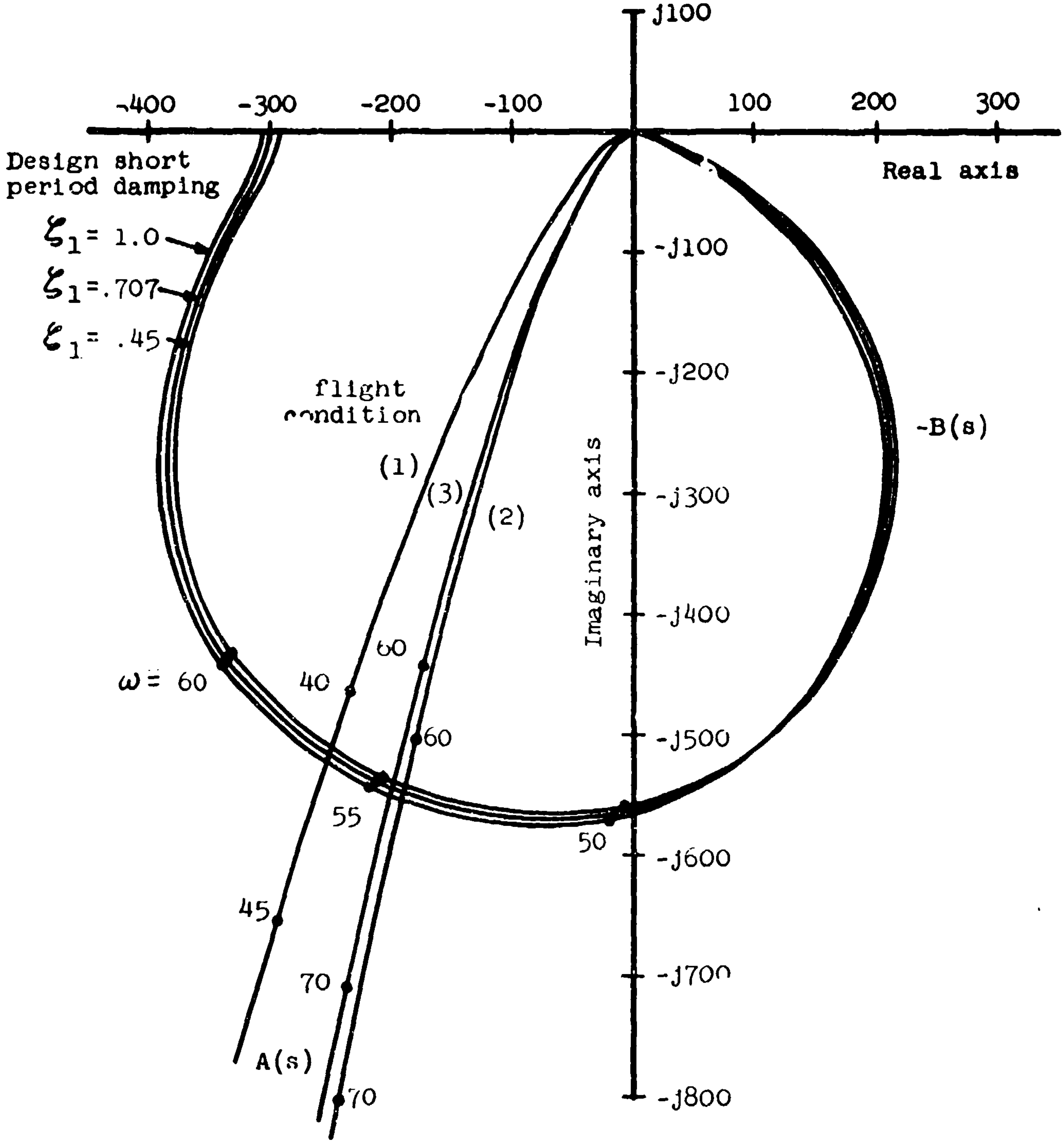


Fig. 22 DUAL NYQUIST DIAGRAM SHOWING EFFECT OF DESIGN SHORT PERIOD DAMPING-RATIO UPON STABILITY

angles that this second term contributes a nearly negligible magnitude to the total vector at high frequencies (frequencies equal to and greater than ω_{RG}). Furthermore a change in ζ_1 has no effect on the phase angle of the second term of -B(s). An increase in design damping ratio merely causes the points on the locus of -B(s) to shift slightly, ostensibly towards a more negative real part of -B(s). For these reasons, variations in ζ_1, within the range of values considered herein, have a negligible effect on the system stability for a given flight condition, loop gain, and particular servo and sensing instrument dynamics.

<u>Effect of Design Short Period Natural Frequency Upon System Stability</u> - Figure 23 represents a dual Nyquist diagram showing the effects of a change in desired short period natural frequency, ω_1. Three values are represented: ω_1 = 2 radians per second, ω_1 = 3 radians per second, and ω_1 = 4 radians per second. The curves in Figure 23 indicate that an increase in ω_1 causes the locus of -B(s) to shrink radially towards the origin. This shrinking implies an increase in stability since the locus of B(s) will cross the locus of A(s) at a lower frequency on A(s). Therefore, the higher the desired short period frequency is made, the more stable the system becomes.

It has been shown previously that B(s) may be approximated by Equation 52. This approximation may be further simplified when it is applied to represent the loci of B(s) at the points of intersection with the loci of -A(s) in Figure 23. The loci of A(s) cross the loci of -B(s) at frequencies that are greater than ω_A. Since ω_A is much greater than ω_1, it follows that, at frequencies greater than or equal to ω_A, Equation 52 may be reduced to

$$B(s) \approx \frac{\dfrac{s^2}{\omega_1^2}}{\dfrac{s^2}{\omega_A^2} + \dfrac{2\zeta_A s}{\omega_A} + 1} . \qquad (55)$$

It is evident from Equation 55 that the magnitude of B(s) at any frequency greater than ω_A decreases as ω_1 is increased while the phase angle of B(s) remains unchanged. It may be concluded that, at high frequencies (in the range of the point of intersection on the dual Nyquist diagrams), increasing the value of ω_1 affects the stability of the system in much the same manner as decreasing the loop gain by the second power.

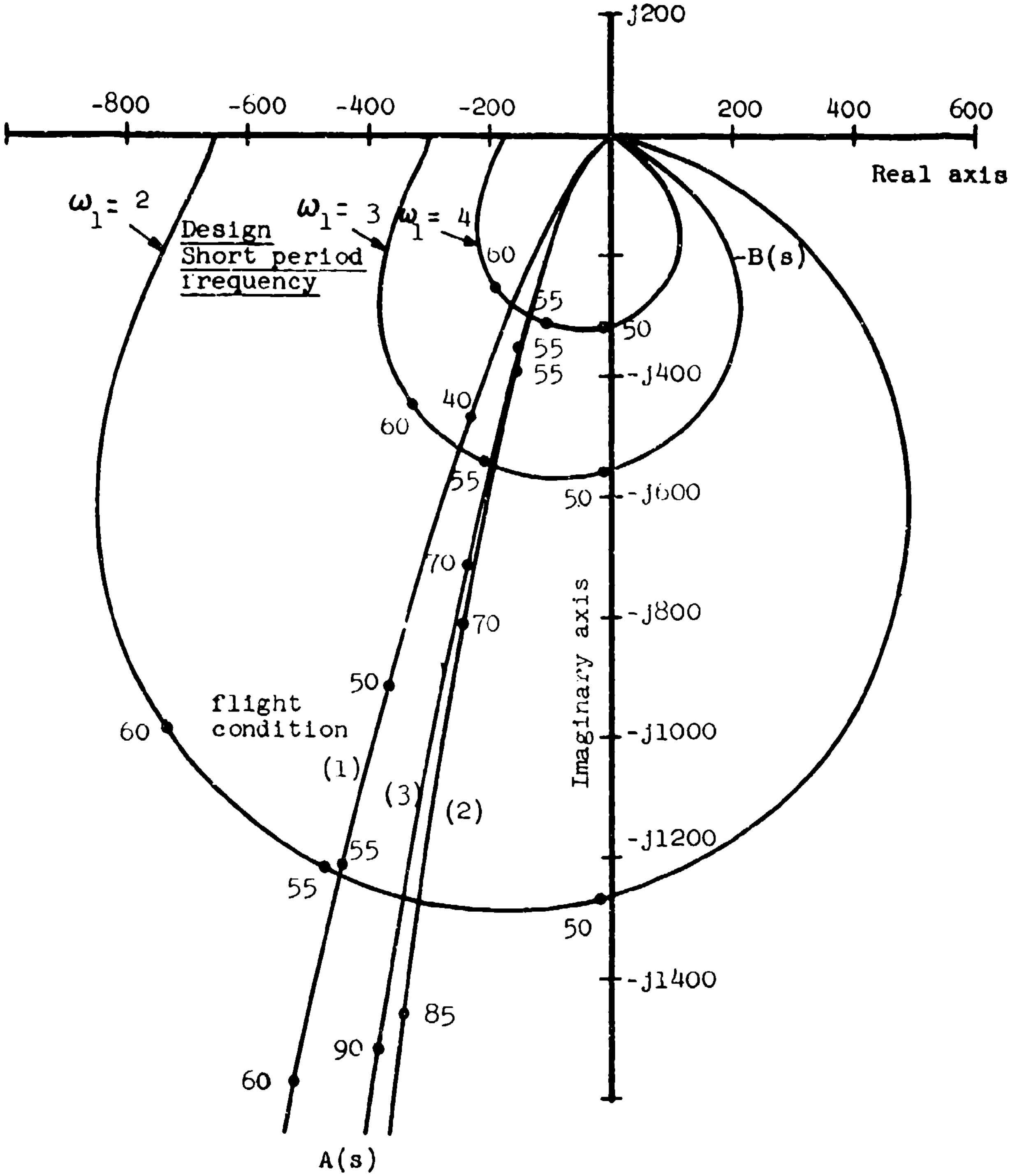


Fig. 23 DUAL NYQUIST DIAGRAM SHOWING EFFECT OF DESIGN SHORT PERIOD NATURAL FREQUENCY UPON STABILITY

Design Considerations in the Synthesis of the Final Control System

Selection of Servo Actuator and Sensing Element Characteristics - From the preceding discussion, several important conclusions may be drawn regarding the selection of servo actuator and sensing elements for the vehicle studied.

A hydraulic servo with the lowest characteristic frequency of those considered, 20 radians per second, appeared to result in the highest stability.

The rate gyro characteristic frequency could be selected arbitrarily within the range of frequencies which were examined (40 radians per second or greater). However, based on cost, the lowest characteristic frequency would appear to be the best choice.

An angular accelerometer with as high a natural frequency as is possible to achieve appears to be the most desirable. Although commercial angular accelerometers are available with natural frequencies as high as 600 radians per second, consideration was given herein to natural frequencies only as high as 250 radians per second. It will be shown in the discussion following that this value is quite adequate.

The larger the damping ratio of the accelerometer, the greater the stability gain margin. However, as will be seen, the dynamic effect of a large damping ratio must be given some consideration; this effect somewhat limits the selection of accelerometer damping ratio. The dynamic effect of damping ratio will be considered in the following discussion.

Considerations of Accelerometer Damping Ratio - In the final selection of the accelerometer damping ratio, consideration should be given to the effects of this parameter on system dynamic characteristics other than those of the short period. The closed loop transfer function, Equation 23,

may be written:

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{K_oK_2\left(\frac{s}{\omega_{RG}}+1\right)\left(\frac{s^2}{\omega_A^2}+\frac{2\zeta_A s}{\omega_A}+1\right)}{\left(\frac{s}{\omega_{HS}}+1\right)\left(\frac{s}{\omega_{RG}}+1\right)\left(\frac{s^2}{\omega_A^2}+\frac{2\zeta_A s}{\omega_A}+1\right)\left(\frac{s^2}{\omega_0^2}+\frac{2\zeta_0 s}{\omega_o}+1\right)+\cdots}$$

$$\cdots\frac{K_oK_2}{K_1}\left[\frac{s^2}{\omega_1^2}\left(\frac{s}{\omega_{RG}}+1\right)+\frac{2\zeta_1 s}{\omega_1}\left(\frac{s^2}{\omega_A^2}+\frac{2\zeta_A s}{\omega_A}+1\right)+\left(\frac{s}{\omega_{RG}}+1\right)\left(\frac{s^2}{\omega_A^2}+\frac{2\zeta_A s}{\omega_A}+1\right)\right] \tag{56}$$

It can be shown that the denominator of Equation 56 has six roots which, for the parameters selected, consist of the short period complex conjugate pair, two negative real roots, and a pair of complex conjugates whose frequency is in the order of 200 radians per second and whose damping ratio varies from 0.1 to 0.4, depending upon the loop gain of the system. In all cases one of the negative real roots is almost exactly cancelled by the root $S = \omega_{RG}$ of the numerator of Equation 56. The other real root is of considerably larger magnitude and may therefore be considered negligible. If an accelerometer damping ratio of 0.25 is selected, the high frequency pair of complex conjugate roots of the denominator is approximately cancelled by the remaining roots of the numerator. Obviously, this cancellation effect diminishes as ζ_A is increased.

For the above reasons, it follows that, with a highly damped accelerometer ($\zeta_A = 2.0$), there will appear a high frequency, lightly damped oscillation in the system when the loop gain is set for a 3db gain margin. A lightly damped accelerometer ($\zeta_A = 0.25$) will essentially cancel out this high frequency oscillation; but, as was shown previously by dual Nyquist diagrams, the system gain margin (for a given loop gain) is decreased as ζ_A is decreased.

The above discussion serves to show that selection of accelerometer characteristics based entirely on obtaining the highest gain margin, for a given loop gain, does not necessarily produce the most desirable closed loop dynamic characteristics. The magnitude of the high frequency oscillation mentioned above increases with ζ_A. However, since ω_A is much greater than ω_1, the magnitude of this oscillation is necessarily small in comparison to that of the short period oscillation. Before selecting an accelerometer damping ratio, the designer must determine the amount of this high frequency oscillation, superimposed upon the short period motion, which can be tolerated.

<u>Selection of Design Short Period Undamped Natural Frequency</u> - There exist many factors such as resonances resulting from structural flexibility, maneuverability requirements, and handling qualities (if the vehicle is manned) which influence the choice of design short period undamped natural frequency. The above factors should be given primary consideration in any new application. In addition to these factors there is considerable merit in choosing the design frequency near the middle of the free airframe frequency range. When this is possible, it is evident that approximately the same degree of compensation is feasible at all flight conditions. On the other hand, the design frequency should be selected to be as large as possible within the limitations of the above considerations because stability margin increases with design frequency. The higher ω_1 is set, the more loop gain can be tolerated and the closer the design frequency can actually be approached. A reasonable compromise among all of these factors can probably be attained for most applications.

Pilots have expressed the opinion that a design short period natural frequency of 3 radians per second seems desirable (Reference 3). In addition, the natural frequencies of the free airframe discussed herein appear evenly distributed about this value. For these reasons a frequency of $\omega_1 = 3$ radians per second was selected.

<u>Selection of Design Short Period Damping Ratio</u> - It has been established that the design short period damping ratio is not influenced by stability considerations. A damping ratio of .707 is considered ideal in almost every application. The free airframe considered in this example is very deficient in damping with respect to the ideal. Therefore, at all flight conditions, the airframe with a self-adaptive control system will approach the ideal damping ratio from lower values of damping ratio. Since the difference between actual damping ratio and the design damping

ratio depends on the loop gain, the selection of ζ_1 depends upon the amount of gain that the system can tolerate. For this reason, it may be desirable to select a design damping ratio, ζ_1, that is larger than 0.707 so that the closed loop damping ratios will be distributed about 0.707. It will be assumed herein, that a closed loop damping ratio less than 0.707 and greater than some other value (.45 is a realistic minimum according to Reference 3) gives the system acceptable performance characteristics. For this reason, a design damping ratio of exactly 0.707 was selected.

<u>Estimate of Short Period Characteristics of Final System</u> - After component hardware dynamics and design short period characteristics have been selected on the basis of stability considerations and other related factors, the next step is to examine the deviations occurring in the closed-loop, short period root loci resulting from the inclusion of component dynamics. For the component dynamics which are being considered in the final system selection, reference to Figures 9, 10, and 11 indicates that the deviation of the root loci from the ideal is small enough for computing a first approximation of the closed loop damping ratio, natural frequency, and steady state gain by using Equations 11, 12, and 13.

The results of using the above component frequencies and of letting $\omega_1 = 3$ radians per second and $\zeta_1 = .707$ are shown in Table II. There are three sets of results listed representing three different choices of accelerometer damping ratio ($\zeta_A = 0.25$, 0.6 and 0.707). Shown are the closed-loop characteristics calculated from Equations 11, 12, and 13 for the three flight conditions previously defined.

Figure 24 is a graph of the results presented in Table II. The points represent the characteristics that are obtained for loop gains which produce a gain margin of 3 db at flight condition (3). It follows, then, that flight conditions (1) and (2) will have gain margins greater than 3 db. It is evident from this figure that all three values of accelerometer damping ratio give excellent results.

<u>Considerations of Effects of Component Dynamics on Short Period Characteristics of Final System</u> - The final system characteristics obtained in the preceding section are not difficult to calculate from Equation 9. This equation represents the transfer function of the closed-loop ideal system. By referring to Equation 23 or 56, it is seen that the exact characteristics are considerably more difficult to calculate since a sixth order polynomial must be factored. An estimate

symbol	gain factor $\frac{K_2}{K_1}$	accelerometer damping ζ_A	
⊙	0.00	——	(free airframe)
⊡	1.75	0.25	
△	4.55	0.60	
○	14.00	2.00	

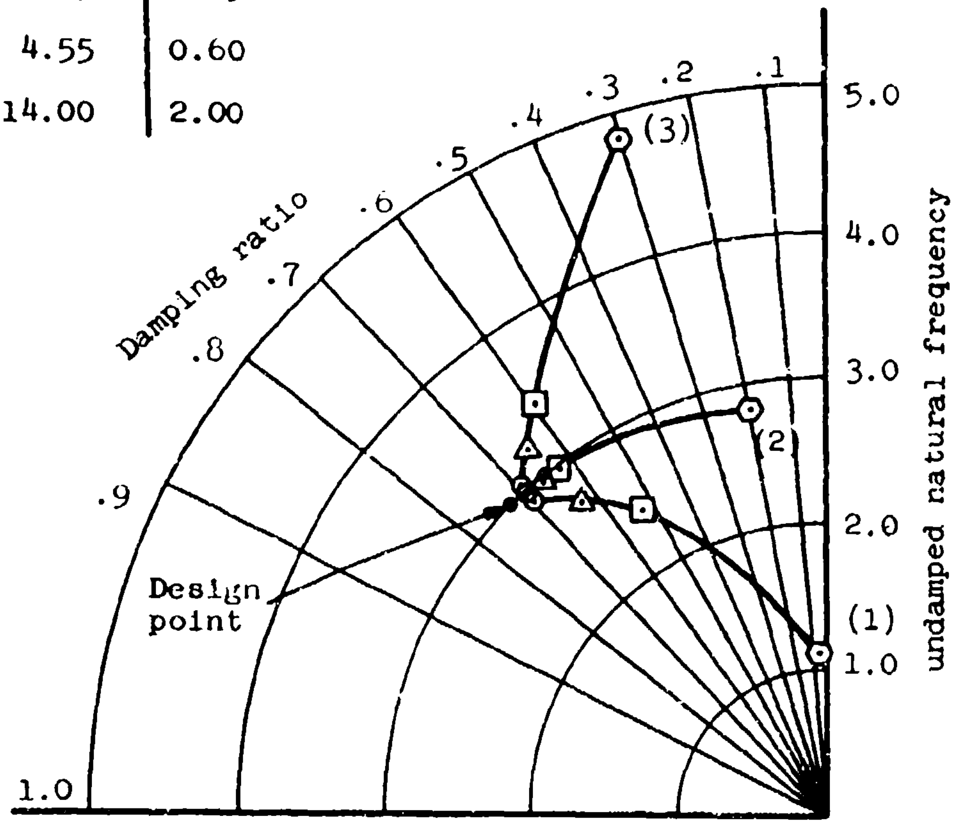


Fig. 24 APPROXIMATE CLOSED-LOOP, SHORT PERIOD CHARACTERISTICS AT A CONSTANT GAIN MARGIN FOR DIFFERENT ACCELEROMETER DAMPING-RATIOS

TABLE II

FIRST APPROXIMATION OF THE CLOSED-LOOP CHARACTERISTICS OF TYPICAL AIRFRAME WITH SELF-ADAPTIVE CONTROL

Gain Factor for 3db gain margin at condition (3) K_2/K_1	Accel-erometer damping ζ_A	Closed-Loop Charac-teristics	Flight Conditions		
			(1) $\omega_0 = 1.1$ $\zeta_0 = .05$ $K_o = 5:7$	(2) $\omega_0 = 2.8$ $\zeta_0 = .2$ $K_o = 2.69$	(3) $\omega_0 = 4.84$ $\zeta_0 = .3$ $K_o = 1.016$
1.75	0.25	K_3	.910 K_1	.825 K_1	.64 K_1
		ζ_3	.511	.614	.591
		ω_3	2.38	2.96	3.40
4.55	0.6	K_3	.963 K_1	.925 K_1	.822 K_1
		ζ_3	.615	.665	.653
		ω_3	2.69	2.99	3.18
14.0	2.0	K_3	.987 K_1	.975 K_1	.935 K_1
		ζ_3	.676	.694	.688
		ω_3	[illegible]	3.00	3.06

of the exact characteristics may, however, be obtained from a consideration of the effects of the component hardware dynamics upon the location of the short period roots. From previous discussion and the graphical results shown in Figures 9, 10, and 11, the following conclusions may be summarized in regard to a vehicle of the type studied.

1. As the hydraulic servo frequency decreases, the closed-loop roots shift from the ideal roots in a counterclockwise direction about the design point.

2. As the rate gyro frequency decreases, the real part of the roots shift farther into the left half plane.

3. The angular accelerometer natural frequency has little effect upon the roots if it is at least as great as 250 radians per second.

4. As the damping ratio of the accelerometer increases, the natural frequency associated with the roots decreases, but the damping ratio appears to remain essentially unchanged.

5. The above deviations of the roots from the ideal root locus are most pronounced in the midrange of values of loop gain.

By applying the above results to the root loci illustrated in Figure 24, the following deductions may be made regarding the exact closed-loop short period characteristic roots. For $\zeta_A = 0.25$, the roots will have a very slightly greater frequency and a more noticeably higher damping ratio than the ideal root because of effects (1) and (2) above. For $\zeta_A = 0.6$, the roots will again be higher in frequency and damping ratio than the ideal roots for the same reason, but the increase in frequency over the ideal will be less than for $\zeta_A = 0.25$ since effect (4) becomes more apparent, and the total shift will be less pronounced because of effect (5). For $\zeta_A = 2.0$, the total change from ideal to actual amounts essentially to an increase in damping ratio because the increase in frequency resulting from effects (1) and (2) is essentially cancelled by effect (4); again the deviation is considerably smaller than for the other two values of ζ_A resulting from effect (5).

In each of the preceding cases, the shift of the roots from their location with an ideal system is greatest for

flight condition (3) because in this condition the loop gain is the smallest of the three. By similar reasoning the smallest root shift occurs for condition (1).

Exact Short Period Characteristics of Final System - Factoring the denominator of Equation 56 was programmed for digital computer solution for each flight condition and three values of accelerometer damping. The final closed-loop short period roots are listed in Table III and illustrated graphically in Figure 25. A comparison of Figures 24 and 25 reveals that the inclusion of sensor and servo-actuator dynamics in the calculation of short period characteristics has actually produced desirable results in spite of the gain limitations imposed thereby. This improvement has been effected because all of the flight conditions shown represent cases where the free airframe is originally deficient in damping. Since the effect of the components [illegible] to shift the roots generally towards greater damping, s[illegible] improvement should be expected for the conditions selected.

For the system illustrated in th[illegible] report the following component and design parameters appear to be optimum:

ω_1 = 3 radians per second

ζ_1 = 0.707

ω_{HS} = 20 radians per second

ω_{RG} = 40 radians per second

ω_A = 250 radians per second

ζ_A = 0.6

$\dfrac{K_2}{K_1}$ = 4.55

symbol	gain factor $\frac{K_2}{K_1}$	accelerometer damping ζ_A	
⊙	0.00	——	(free airframe)
⊡	1.75	0.25	
△	4.55	0.60	
○	14.00	2.00	

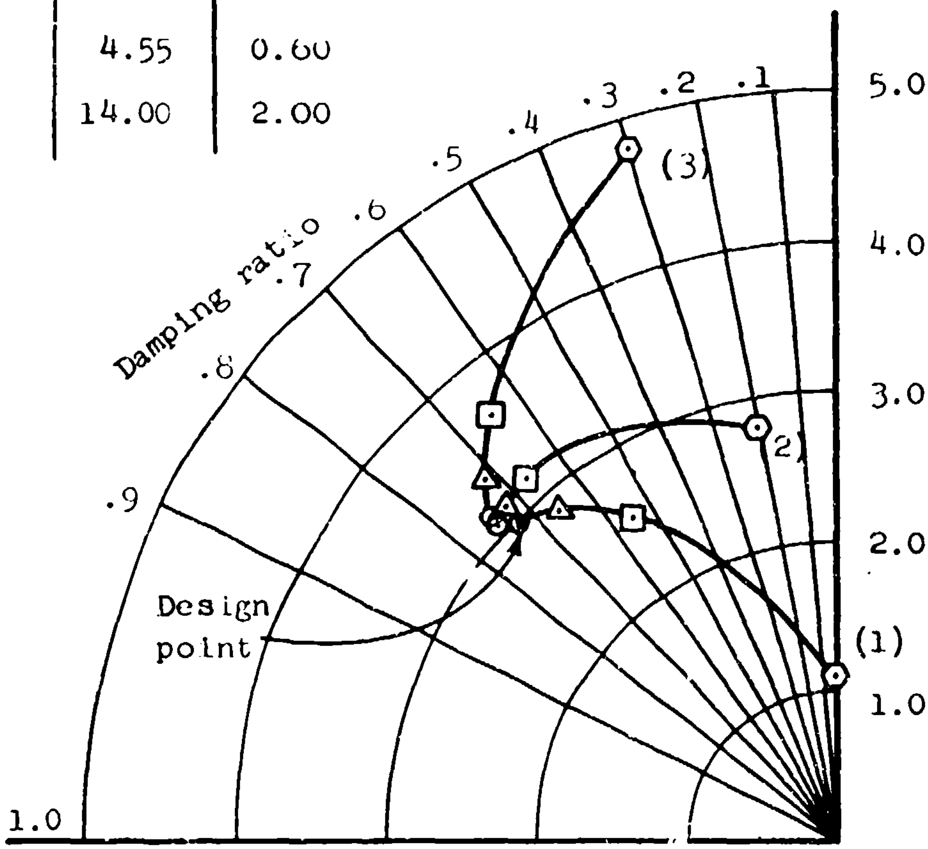


Fig. 25. EXACT CLOSED-LOOP, SHORT PERIOD CHARACTERISTICS AT A CONSTANT GAIN MARGIN FOR DIFFERENT ACCELEROMETER DAMPING-RATIOS

TABLE III

EXACT CLOSED–LOOP CHARACTERISTICS OF TYPICAL AIRFRAME WITH SELF–ADAPTIVE CONTROL

Gain Factor 3db gain margin at condition (3) K_2/K_1	Accel–erometer damping ζ_A	Closed Loop Charac–teristics	Flight Conditions (1) $K_o = 5.7$ $\omega_0 = 1.1$ $\zeta_0 = .05$	Flight Conditions (2) $K_o = 2.69$ $\omega_0 = 2.8$ $\zeta_0 = .2$	Flight Conditions (3) $K_o = 1.016$ $\omega_0 = 4.84$ $\zeta_0 = .3$
1.75	0.25	K_3	.910 K_1	.825 K_1	.64 K_1
		ζ_3	.529	.648	.623
		ω_3	2.54	3.05	3.59
4.55	0.6	K_3	.963 K_1	.925 K_1	.822 K_1
		ζ_3	.642	.706	.697
		ω_3	2.87	3.15	3.34
14.0	2.0	K_3	.987 K_1	.975 K_1	.935 K_1
		ζ_3	.712	.733	.725
		ω_3	2.99	3.07	3.13

CONCLUDING REMARKS

A method of self-adaptive control has been shown to be feasible and applicable in a flight control system for the automatic stability augmentation of the short period response of a typical, powered, air-to-surface missile. The system comprises linear feedback of pitch attitude, pitch rate, and pitch angular acceleration with fixed gains. A second order representation of the airframe short period characteristic was assumed. Typical, present-day, stock control instrumentation (rate gyro, angular accelerometer, and hydraulic servo-actuator) was selected, and the dynamic characteristics of these components were included in the calculations. By using the above representation of the airframe, control instrumentation, and particular feedback arrangement, the following conclusions may be obtained:

1. Extreme feedback loop gains are not essential to reach near ideal short period characteristics at all flight conditions.

2. An adequate margin of stability can be maintained despite dynamic interaction of the control instrumentation with the control system.

3. The selection of sensing and actuating instrumentation is not critical, and it is only necessary to keep the following points in mind. The servo actuator must have a low characteristic frequency, preferably near 20 radians per second. The rate gyro dynamics (whether represented by first order characteristics or by second order characteristics) are not critical. The angular accelerometer should have a damping ratio near 0.6 and must have as high a natural frequency as possible although 250 radians per second appears to be adequate.

REFERENCES

1. Dandois, M., Concepts in Self-Adaptive Controls, Convair-Fort Worth Report FZA-270.

2. Heacock, R., Jones P., "Dual Nyquist Stability Analysis", Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, Memorandum No. 20-98.

3. Campbell, G., Use of an Adaptive Servo to Obtain Optimum Airplane Responses, Cornell Aeronautical Laboratory, Inc., Report No. C.A.L.-84.

APPENDIX A

THE THEORY OF THE DUAL NYQUIST CONCEPT

The dual Nyquist diagram is a graphical procedure for determining the stability of feedback systems. This procedure has advantages over the ordinary Nyquist diagram in certain cases. The dual Nyquist diagram may be used in all instances where the denominator of the transfer function of the system can be written as the sum of two frequency dependent functions.

A detailed discussion of this procedure is outlined in Reference 2. A short introduction to the method is given below.

To illustrate a point pertinent to the dual Nyquist concept, consider the simple feedback control system shown in Figure A-1.

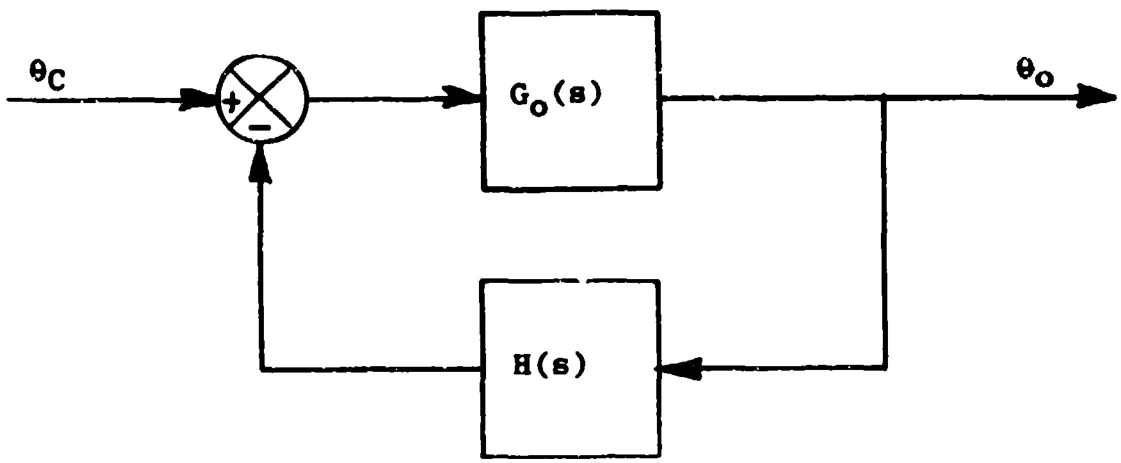


Fig. A-1 ILLUSTRATION OF SIMPLE FEEDBACK CONTROL

The closed-loop transfer function relating the output and input variables of this system is given by

$$\frac{\theta_0(s)}{\theta_C(s)} = \frac{G_0(s)}{1 + G_0(s)H(s)} \quad . \tag{A-1}$$

The factor G(s)H(s) usually consists of a ratio, a sum or a product of frequency dependent functions, or some combination of these forms. By using several of these forms, the following examples can be utilized to illustrate the manner in which the denominator 1 + G(s)H(s) may be written as a sum of two frequency dependent functions.

$$\text{For} \quad G_0(s)H(s) = \frac{P(s)}{Q(s)} \; ; \quad \frac{\theta_0(s)}{\theta_C(s)} = \frac{G(s)Q(s)}{P(s) + Q(s)} \tag{A-2}$$

$$\text{For} \quad G_0(s)H(s) = P(s)+Q(s); \frac{\theta_0(s)}{\theta_C(s)} = \frac{G(s)}{\left[1 + P(s)\right] + Q(s)} \tag{A-3}$$

$$\text{For} \quad G_0(s)H(s) = P(s)Q(s); \frac{\theta_0(s)}{\theta_C(s)} = \frac{\frac{G(s)}{P(s)}}{\frac{1}{P(s)} + Q(s)} \cdot \tag{A-4}$$

In any of the above cases, the denominator of the closed-loop transfer function assumes the form A(s) + B(s).

The ordinary Nyquist criterion in which the above form is used may be stated as follows: "The number of clockwise rotations of the vector $A(j\omega) + B(j\omega)$, on the complex plane, as the complex variable s traces a contour enclosing the positive real half of the complex plane but excluding singularities on the imaginary axis, plus the number of poles of G(s)H(s) with positive real parts must be zero for stability." For most systems, the condition of physical realizability (The function A(s) + B(s) must approach zero as s approaches infinity.) reduces the necessary plot of the s contour to that portion along the imaginary axis. In such systems the portion of the contour which is the infinite semicircle in the right half of the s plane maps into the origin of the A(s) + B(s) plane. In this discussion such a system has been assumed.

It must be understood, however, that certain algebraic manipulations of the transfer function, such as separating the denominator of the transfer function into the functions A(s) and B(s), may result in functions which no longer represent physical elements or loops of the system. In such cases, the functions A(s) or B(s) do not approach zero as s approaches infinity. Such a phenomenon is considered in the stability analysis of the airframe under examination in this report. The discussion which follows in this Appendix should therefore be extended to include the entire contour of s along the infinite semicircle in the right half plane in order to apply the dual Nyquist technique to any system of the type discussed in this report. Since the poles of G(s)H(s) are usually known in advance by inspection of the transfer functions of the forward path G(s) and the feedback path H(s), only the rotation of the vector $A(j\omega) + B(j\omega)$ need be determined. In the dual Nyquist diagram the locus of the vectors representing $A(j\omega)$ and $B(j\omega)$ are plotted separately. The information regarding the rotation of the sum of these two vectors is obtained by inspecting the points of intersection of the two loci.

To illustrate the general method, consider a closed-loop system whose transfer function is of the form

$$\frac{\theta_0}{\theta_C} = \frac{Fo(s)}{A(s) + B(s)} \, .$$

The following procedure is used. The function $F_1(j\omega)$ is plotted for values of ω from 0 to ∞. The function $-B(j\omega)$ is also plotted (Fig. A-2a). It is seen that if a vector is drawn from the $-B(j\omega)$ locus to the $A(j\omega)$ locus, with each end of the vector at the same value of ω, this vector will have the magnitude and direction of $A(j\omega) + B(j\omega)$. In this manner it is possible to visualize the locus of the function $A(j\omega) + B(j\omega)$ without carrying out the summation and plotting of the functions. It is also possible to determine the number of clockwise rotations of this vector by visualizing its successive angular positions as ω is varied from 0 to ∞.

Information regarding the stability of the system can be obtained more rapidly (without drawing the vector) by inspecting the points of intersection of the two loci. Figures A-2b and A-2c were drawn to illustrate the two basic ways in which two loci may intersect. In Figure A-2b the

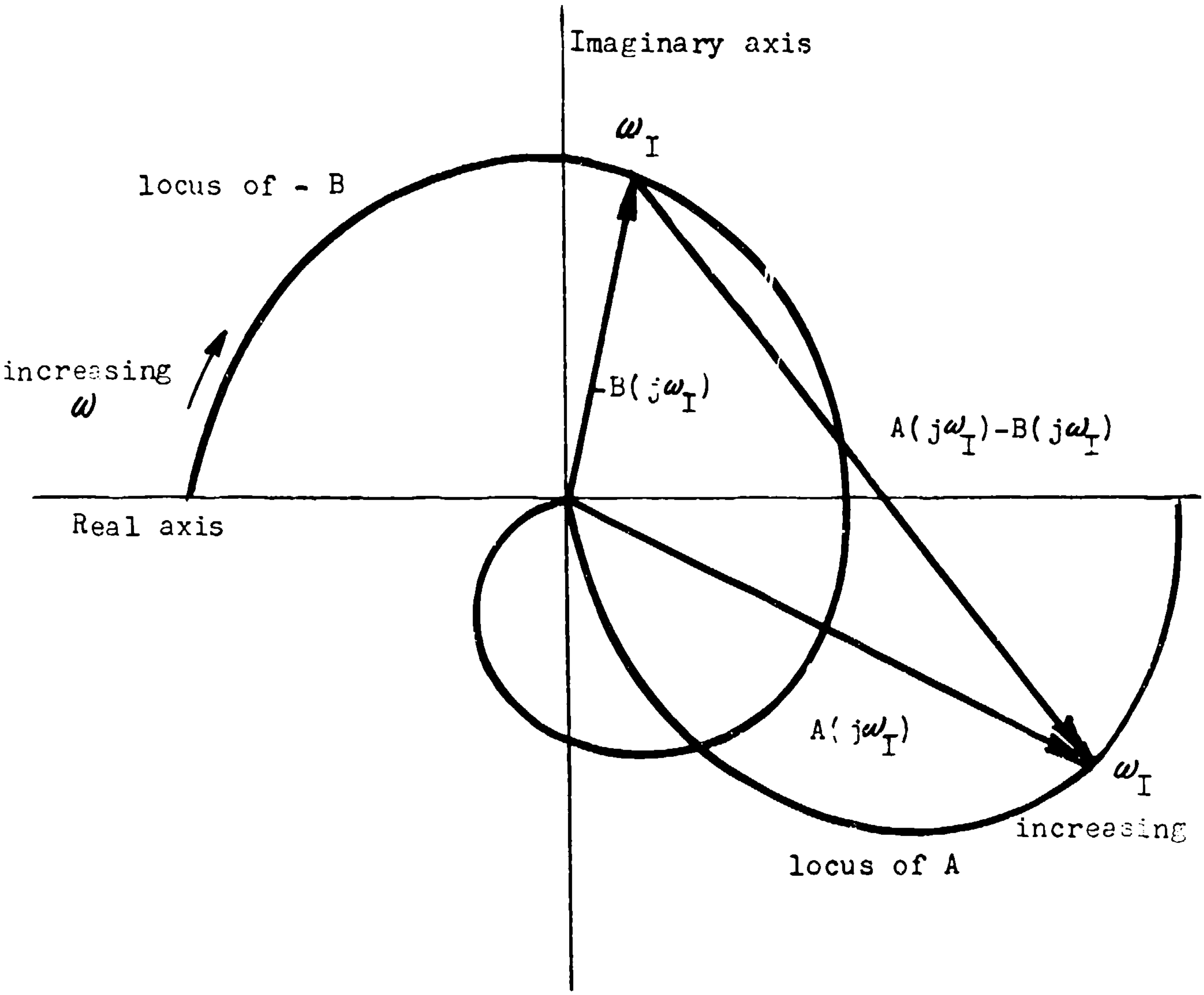


Fig. A-2a ILLUSTRATION OF DUAL NYQUIST DIAGRAM

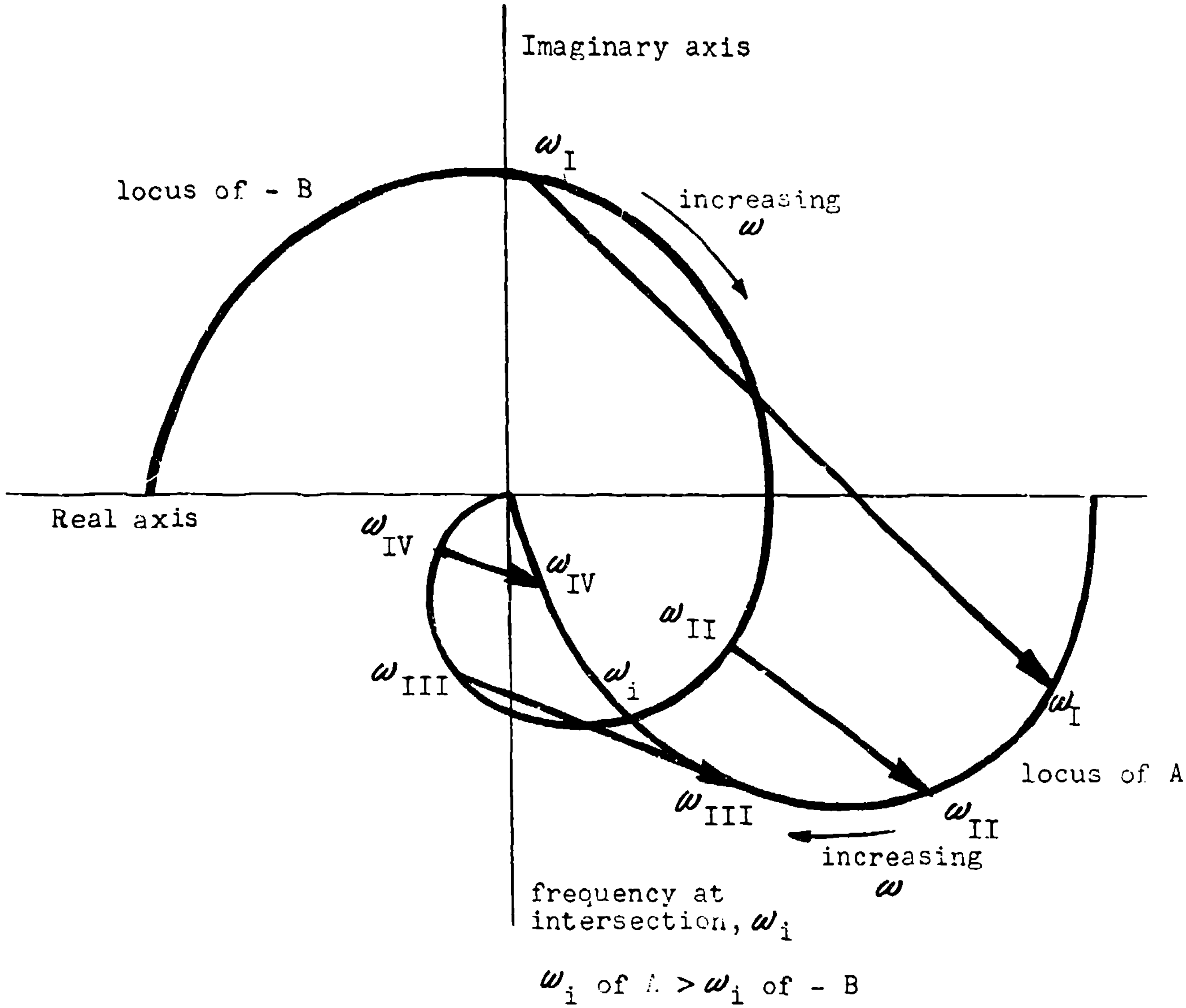


Fig. A-2b ILLUSTRATION OF DUAL NYQUIST DIAGRAM

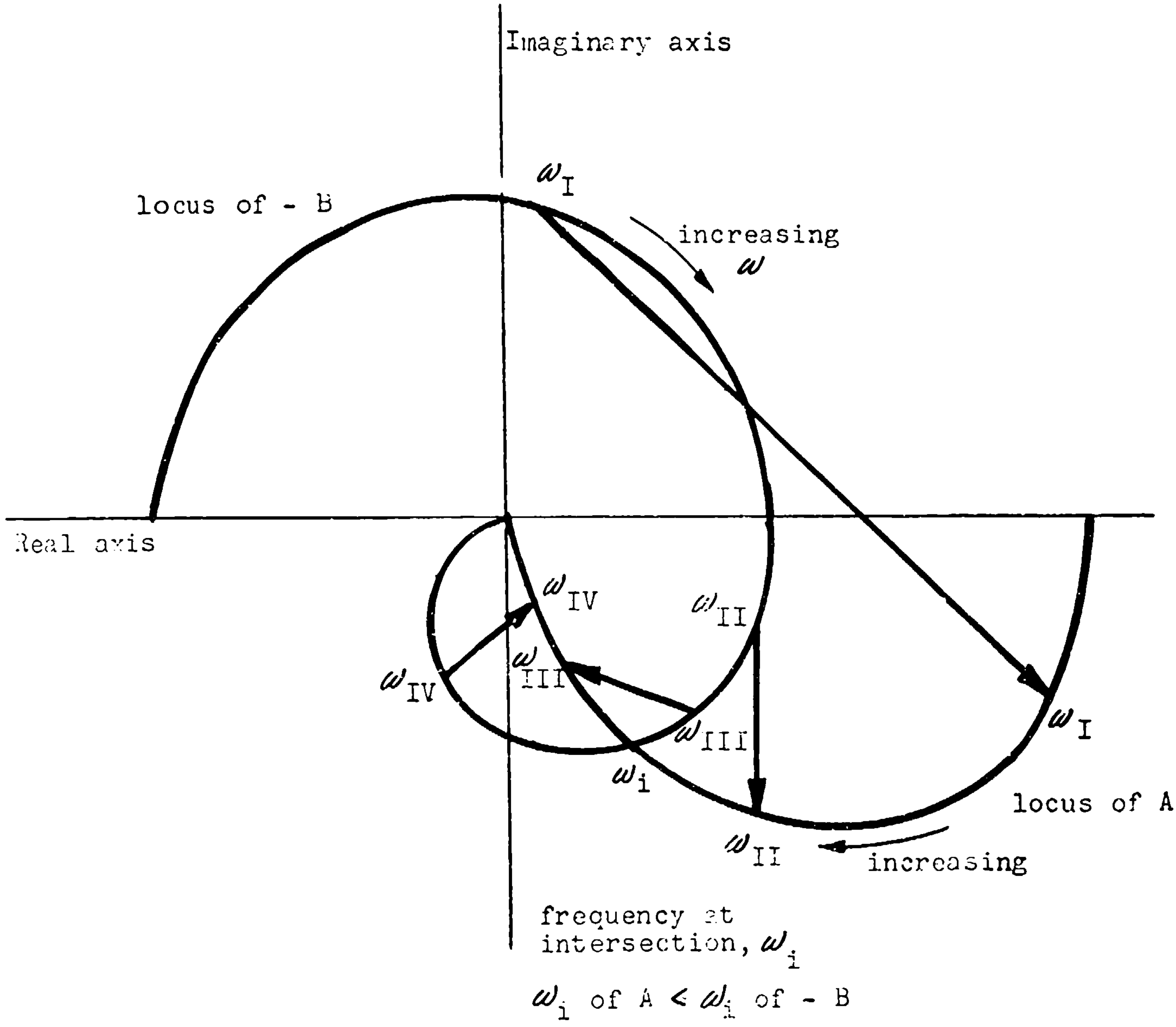


Fig. A-2c ILLUSTRATION OF DUAL NYQUIST DIAGRAM

locus of A enters the region enclosed by the locus of −B at a frequency which is <u>greater</u> than that of −B at the point of intersection. As a result it is seen that the vector $A(j\omega) + B(j\omega)$ makes zero net clockwise revolutions on the complex plane as ω varies from 0 to ∞. In Figure A-2c, on the other hand, the locus of A enters the region enclosed by the locus of −B at a frequency which is <u>less</u> than that of −B at the point of intersoction. As a result, the vector makes one full rotation clockwise as ω ranges from 0 to ∞.

When the dual Nyquist diagram is plotted, it does not matter which of the functions, A or B, is plotted negatively. A general rule may be stated as follows: "Whenever one locus enters a region enclosed by the second locus at a lower frequency (or leaves at a higher frequency) than that existing on the second locus at the point of intersection, the vector $A(j\omega) + B(j\omega)$ makes one full clockwise rotation on the complex plane." The ordinary Nyquist criterion is then applied, as mentioned previously, by using the number of clockwise rotations obtained from the dual locus diagram.

Further remarks on the interpretation of the dual Nyquist diagram in various applications may be found in Reference 2. In applications to multiple loop systems, the dual Nyquist technique is especially useful since it allows the effects of a minor loop parameter to be obtained by plotting the Nyquist diagram of the characteristic equation of the minor loop as one of the two loci. Thus the stability of the inner loop may be obtained at the same time as the over-all stability is being determined. This procedure permits a more rapid determination of the effects of the inner loop parameters on the system stability.

The University of California at Los Angeles (UCLA)

PROGRAM IN BASIC RESEARCH IN ADAPTIVE CONTROL THEORY

G. Estrin, H. C. Hsieh, C. T. Leondes, M. Margolis
N. Richardson, M. Schwartz, E. B. Stear*

UCLA has been the recipient of an Air Force Office of Scientific Research (AFOSR) contract for basic studies in Adaptive Control Theory. Work commenced on this contract on September 22, 1958.

At present there are five main subjects under investigation:

(1) Synthesis of multipole control systems with random signals as inputs.

(2) Time domain synthesis of multipole sampled-data systems.

(3) Synthesis of linear systems with arbitrary deterministic inputs.

(4) An exploratory study of nonlinear methods in mechanics and mathematics for possible application to control systems.

(5) The learning model approach to the design of Adaptive Control Systems.

The areas of investigation as listed above will be discussed in more detail a little later in this paper. But first the approach to the design of Adaptive Control System as taken by UCLA will be discussed.

Philosophy of Adaptive Control

Most of the papers presented at the Symposium were specifically concerned with flight control systems. As such, the work done as outlined in these papers was directed towards the fabrication of systems that would satisfactorily control aircraft. There was neither the desire or the time to generalize the techniques presented.

*All of the College of Engineering, University of California at Los Angeles.

The UCLA program not being constrained to the design of any particular system can afford to take a more general long range view.

Given a complete and accurate description of the vehicle or process dynamics, synthesis techniques exist which permit the design of controls for these vehicles or processes. These design techniques range from intuitive methods to very sophisticated synthesis procedures based on minimizing certain performance indices. But they do depend on a very good description of the dynamics of the system to be controlled. If the system dynamics are not accurately known or if they change during the life of the system poorer performance can be expected.

It is the purpose of the class of Adaptive Control Systems being considered here to somehow compensate for any change or lack of knowledge in the plant (vehicle or process) dynamics in such a way as to optimize system performance at all times. The many different methods proposed at this Symposium are certainly ingenious. But they are tailored to fit a specific situation.

The UCLA approach is more general in that we propose 1) to measure the plant dynamics, 2) use these measured values in the proper control equations to control the plant and 3) update the measurements if the plant dynamics are changing with time.

One of the more difficult tasks in the above procedure is the measurement of the plant dynamics. The measurements are made with the use of a learning model. The same signals which are fed to the plant are also the inputs to the learning model. The output of the learning model is compared with the output of the plant and the difference which we can term the error serves as the forcing function to an adjusting mechanism. It is the duty of the adjusting mechanism to adjust the parameters of the learning model so as to minimize some function

of the error.

If the plant dynamics are changing with time the learning model becomes a parameter tracking model. Certain important questions must be asked of the performance of the learning model measuring technique:

(1) What are the ways in which a learning model can have its parameters adjusted?

(2) Is it stable? Will the learning model have its parameters adjusted so that they are a satisfactory representation of the plant dynamics?

(3) What is the dynamic performance of the learning model? How fast does the learning model respond to change in the plant dynamics? Is the learning model response (i.e., changes in its parameters) suitably damped?

Questions 2 and 3 are difficult to answer because the systems proposed are not only time varying but highly nonlinear. Some progress has been made in the short time devoted to this effort. The really heartening fact in the entire investigation is the new areas in feedback control being opened up by the questions we are asking. In fact, it seems that each question raises two more.

There are additional problems that have not yet been mentioned. One of the most serious of these problems is concerned with the operation of the learning model in a noisy environment. Finally, studies will be made on how to treat the overall design of an Adaptive Control System.

<u>Progress of Work to Date</u>:

(1) The synthesis of multipole control systems with random inputs for a completely free configuration has been solved. The input signals and noise may be cross-correlated or uncorrelated. This work will be presented at a forthcoming IRE national convention in New York City, March, 1959. A technical note is also being prepared for the AFOSR.

The free configuration means that a filter problem has actually been solved. At present, attention is being given to the semi-free configuration, the more nearly realistic control problem. Further investigations will include polynomial inputs in addition to random inputs and the use of finite data instead of assuming an infinite time history of the incoming signals.

(2) Time domain synthesis of multipole sampled data systems. The investigation in this area makes use of the modified Z-transform for the time domain specifications at other than sampling instants.

The multipole control problem is receiving considerable attention because we feel this area has long been neglected from an academic point of view. But more important is the fact that many of our control problems are of the multiple input-output variety and are not being tackled as though they were. In particular aircraft autopilots and jet engine controls are truly multipole systems. All of the problems pertaining to realizability conditions, nonlinearities and the like that have been investigated for single input-output systems deserve attention for multipole system.

(3) Synthesis of linear systems with arbitrary deterministic inputs. Here the input signals are not considered to be simply expressed as a polynomial. Upper bounds on allowable error are one of the specifications. The effort at present is on graphical techniques in the time domain.

(4) An exploratory study of nonlinear methods in mechanics and mathematics for possible application to control systems. The emphasis is equally distributed between searching for methods applicable to synthesis as well as analysis. Specific problems have turned up in our study of adaptive systems. In particular, we are much concerned with a class of nonlinear, time varying

differential equation of the following type:

$$\ddot{x} + a_1 (\dot{x}, x, t) \dot{x} + a_0 (\dot{x}, x, t) = f(t)$$

(5) The learning model approach to the design of adaptive systems has been described. The use of the learning model causes the simplest of these adaptive systems to fall into the general class of multipole nonlinear feedback systems.

CORNELL AERONAUTICAL LABORATORY, INC.
PRESENTATION

P. A. Reynolds

I. INTRODUCTION

Adaptive control systems were first studied at Cornell Aeronautical Laboratory by Graham Campbell. The usefulness of such devices was rapidly becoming apparent in the fields of automatic flight control and also stability augmentation. In June of 1955, the Flight Research Department of C.A.L. submitted a proposal to the Flight Control Laboratory of WADC to investigate servo control loops which could adapt themselves to produce a desired effect, regardless of changing characteristics of the controlled element. This proposal grew out of Campbell's work, which was continued and eventually published as a Master of Science thesis at the University of Buffalo and also as a C.A.L. report. In this report, Campbell investigated the fundamental technique of continuous adaptive control by comparing outputs of the actual system and a dynamic model of desired dynamics.

In March of 1957, a study was initiated under the sponsorship of the Flight Control Laboratory to determine the feasibility of using this servo technique to minimize the variations in the dynamic and static characteristics of high performance aircraft. It is this study which will be of primary concern in this paper.

II. BASIC SERVO LOOP

The basic servo loop, described briefly above, is shown in Figure 1. The input is fed simultaneously to both the controlled element and the model, and

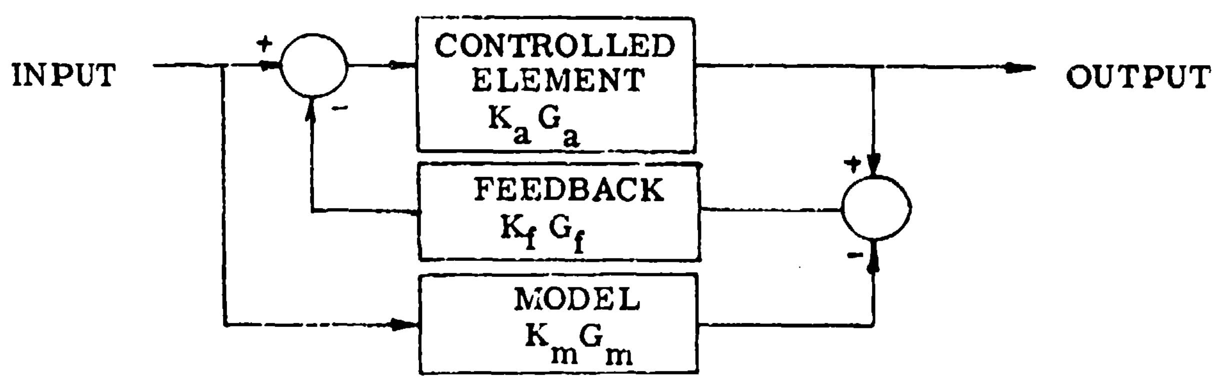


Figure 1

the error fed back directly. (There are other ways in which the error could be used, for example to simply adjust the feedback gain of a single loop system).

The transfer function of this loop is

$$\frac{\text{Output}}{\text{Input}} = \frac{K_aG_a + K_fK_aK_mG_aG_mG_f}{1 + K_fK_aG_aG_f} = K_a'G_a'$$

If K_f is made moderately large, the influence of K_aG_a on the closed loop behavior is reduced. In the limit, the overall transfer function approaches K_mG_m exactly. One can see, then, that although K_aG_a may vary, $K_a'G_a'$ can be restricted to an area around K_mG_m.

The particular advantages of this loop in aircraft control application are the following:

(1) No unproven sensing, computing, or actuating techniques are required. This system is very close to the normal stability augmentation loop found in many aircraft and missiles today.

(2) With the model responding only to the selected input, the device becomes a disturbance alleviator as well as an adaptive servo.

III. CHOICE OF LOOP COMPONENTS

It was desired to apply this technique to the problem of control of high performance aircraft by human pilots. Consideration of the characteristics of the human controller and the necessity for simplicity and reliability of implementation were the prime factors which determined choice of the loop components. Both longitudinal and lateral modes of motion were studied, although this discussion will be restricted to the longitudinal short period mode because of its primary importance in control of the aircraft.

Within the framework of the basic loop, choices had to be made of model static and dynamics, the flight variable to compare with the model output, and the feedback gain and transfer function.

(1) Model Characteristics - The model was chosen giving a steady state output for a steady stick force or position input and having a second order characteristic equation. This made the model quite similar, in first approximation, to the normal aircraft in the short period frequency range. The model steady state gain was fixed at the median value of the aircraft gain so that the steady state error between the two would be minimized. The model natural frequency was set at .5 cps and the damping at .7 of critical. This dynamic behavior has been found to lie close to the optimum for human controllers as determined from pilot opinion data gathered in variable stability airplanes.

Choosing the model in this way would result in the most desirable closed loop behavior, providing the feedback gain can be made high enough to force the aircraft to closely follow the model. However, if the aircraft does not closely follow the model, these model dynamics might not produce optimum closed loop response, and minor changes should be made to optimize the system.

(2) Aircraft Output Variable - Considering aircraft variables which might be compared with the model output, we have normal acceleration (n_z), angle of attack (α), or pitch rate ($\dot{\theta}$), all of which reach a steady value in the short period mode following stick force or motion input. To decide among these, the transfer functions n_z/δ_e, α/δ_e, and $\dot{\theta}/\delta_e$ were examined for the F-101, the F-102 and the F-104 over their entire flight ranges. Pitch rate was chosen for the following reasons. First, it is the most advantageous to measure, requiring nothing external to the aircraft as does measurement of angle of attack and being less susceptible to high frequency noise than an acceleration measurement. Second, its response to elevator motion had the least amount of phase lag at high frequencies. Third, it had less variation (although slightly more than angle of attack). An objection might legitimately be raised here that the pilot flies by stick force and it would be desirable to keep the stick force per "g" constant over the flight envelope. Given a flight control system which produces stick force and elevator deflection proportional to stick position and a servo loop which forces pitch rate per elevator deflection to be constant, the stick force per pitch rate would be constant and stick force per "g" inversely proportional to velocity. One can see, however, that with the slight complication of making the stick force per elevator deflection directly proportional to airspeed, the stick force per "g" is also made constant.

(3) Feedback Gain and Transfer Function - The feedback gain is, of course, limited in any realistic servo loop. The limitation in the adaptive servo loop should be of the same order as that in the conventional stability augmentation loop. This has been shown by manipulation of the block diagram as in Figure 2.

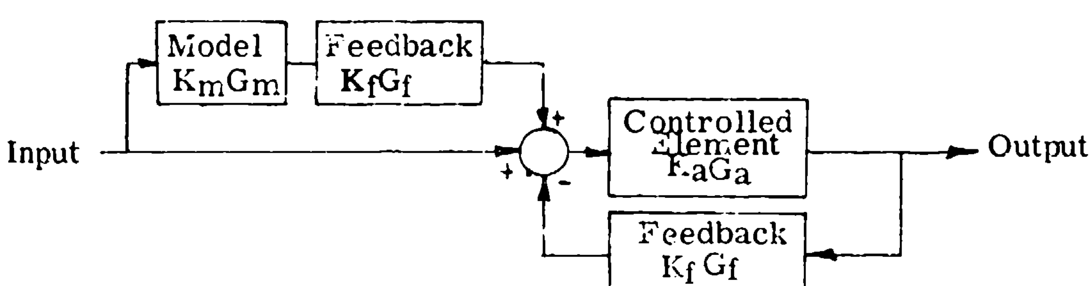


Figure 2

Therefore, it was decided that adaptive servo performance should be determined using loop gains (K_aK_f) no higher than the maximum which has been attained in flight or a reasonable extrapolation of present stability augmentation practice. Based on experience at C.A.L. with variable stability aircraft, a maximum loop gain of five was chosen.

The feedback transfer function used was $(1 + \tau s)$. The lead was necessary to prevent the normal instability at some value of feedback gain when a loop is closed around a third order system (second order short period mode and first order power control lag). The value of τ was .2.

V. PERFORMANCE IN ANALOG F-101, F-102 AND F-104 AIRCRAFT

In Campbell's earlier work, simple second order systems were used for both controlled element and model. Moderate variations in the characteristics of the controlled element were made. Natural frequency varied from .25 to .75 cps, damping ratio from .3 to 1.0, and static gain from .5 to 1.5. With moderate feedback gain, these variations were almost completely nullified. In the recent study, actual aircraft data were used covering the entire flight ranges of the F-101, the F-102, and the F-104. The variations were much wider, especially those of static gain. Natural frequency varied from .2 to 1.7 cps, damping ratio from .06 to .53 and static gain by a factor of 60 to 1. Since the feedback gain had to be set so that the loop gain at maximum forward gain was equal to 5, this meant that the loop gain could get as low as .08.

At this point, it was realized that a desirable complication to the basic loop would be some method of making the feedback automatically variable. But investigation of the performance with constant feedback gain showed that the system, even in this form, has promise. In seven extreme flight conditions, the variations in natural frequency and damping ratio were considerably reduced. The steady state pitch rate per elevator deflection varied considerably, since the low loop gain in some conditions permitted a large steady state error between the model and the aircraft. However, it is felt that this will not cause major difficulty in achieving constant stick force per "g" in view of recent advances in the design of feel systems which produce stick forces by directly sensing the aircraft motion.

VI CONCLUSION

Although this study has not been extensive enough to make firm conclusions, the results are definitely encouraging, especially in view of the possible operational simplicity of an adaptive flight control system of this type.

THE SELF ADAPTIVE FLIGHT CONTROL SYSTEMS
SYMPOSIUM

SESSION VII

Dr. John Aseltine, Chairman
Space Technology Laboratories

Dr. John Aseltine
Space Technology Laboratories

I was thinking in connection with the various definitions that have been made of what an adaptive control is, of a book I read a few years ago called Weismannship by Potter. One of the terms in there was the OK word. You may recall that the OK word is a phrase or word that one uses in the practice of "Weismannship" to gain an advantage. It is a word of which no one is quite sure of the definition, but everyone feels that he should know it; therefore, when confronted with it, everyone feels a sense of embarrassment. I feel a certain ambivalence about destroying the utility of the word adaptive, but from the technical point of view I think that it might be well to work in the direction of defining it well enough so it could be taken off the list of OK words.

Professor Truxal this morning gave us as a definition of an adaptive system, one that was designed from the adaptive point of view. I think this is a pretty good one. I would like to say what I think the adaptive point of view is. I think you need three things in this design of an adaptive system. First, you must have a measure of system performance while the system is operating; second, you must have a means for converting this measure of performance into numbers or some measure of how good the performance is; then, finally, you must have a means of using this number to change the system itself.

I think that most of the systems that we call adaptive have these properties at least inherent in them. I would add this to Professor Truxal's definition. The system designed from this point of view would be an adaptive system. I think maybe we will have a little more to say about that during the panel discussion.

THE AERONUTRONIC SELF-OPTIMIZING AUTOMATIC CONTROL SYSTEM

G. Wm. Anderson, R. N. Buland,
and G. R. Cooper

Aeronutronic Systems, Inc.
Glendale, California

SECTION 1

INTRODUCTION

Aeronutronic System's participation in the adaptive autopilot program began with a contract award in February 1957. This contract called for a feasibility study and analysis of the stability augmentation system to be outlined in this report. Successful demonstration of this approach led to a contract extension for further study and for the fabrication and flight test of a portion of the self-adaptive system. This equipment to be flight tested is designed to continuously measure a system's dynamic performance under normal operating conditions. An experimental model has been fabricated and is presently undergoing shakedown tests. Flight tests in an F-100 aircraft will begin in May of this year.

1.1 BASIC CONCEPTS

The concept of the self-adaptive control system is based on the premise that either implicitly or explicitly such a system must perform the operations shown in Figure 1-1:

1) Continuous measurement of system dynamic performance.

2) Continuous evaluation of performance on the basis of some predetermined criterion.

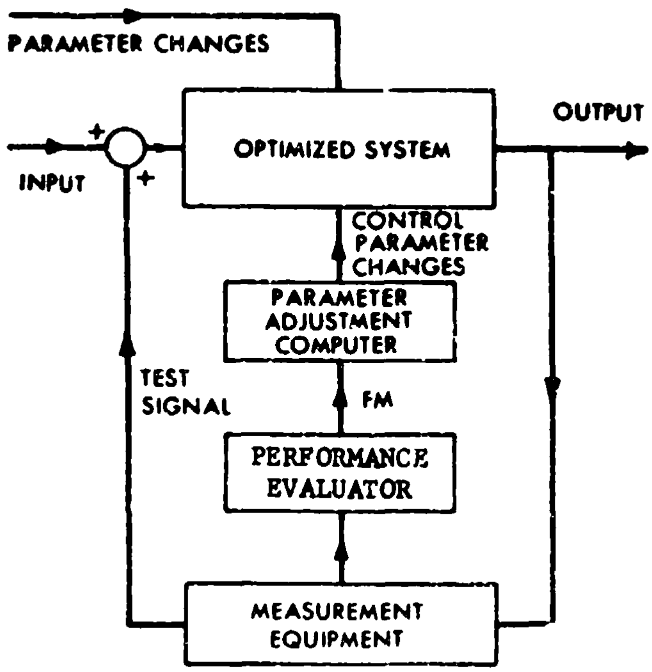


FIGURE 1-1. BASIC SELF-OPTIMIZING SYSTEM

3) Continuous readjustment of system parameters for optimum system performance in accordance with the measurement and the evaluation performed.

Our self-adaptive system consists of an explicit mechanization of these functions.

The measurement of performance should be accomplished with minimum disturbance to the system being optimized. Furthermore, the measurement itself should be relatively immune to the effects of corrupting noise signals. These considerations, in general, lead to the rejection of such standard techniques as are used in the laboratory, including the direct measurement of step or impulse response or the determination of response to steady sinusoidal excitation.

Evaluation of the system's performance can be accomplished by generating a figure of merit from the performance measurement. A figure of merit is defined here as some mathematical function of the measured response, the function being chosen to give emphasis to the specifications of dominant interest.

The most desirable type of FM is one which has zero value when the basic system is "optimally" adjusted and assumes positive or negative values depending on the direction of deviation as the adjustment deviates from "optimum". Such a FM can serve as the adaptive loop's error signal in the true feedback sense and is to be preferred over a FM which has a maximum or minimum at the "optimum" condition. The selection of the proper figure of merit depends mainly on the definition of optimum system performance.

The third operation required of the adaptive loop will vary somewhat with the system being "optimized". Some function (or functions) of the FM must be used to adjust one or more parameters of the "optimized" system.

1.2 PERFORMANCE MEASUREMENT

Several measurement schemes were considered, but the method finally selected depends on a correlation technique usually attributed to Y. W. Lee. If a physical system having an impulse response, $g(t)$, is excited by a noise signal having an autocorrelation function, $\varphi_{ii}(\tau)$, then

$$\varphi_{io}(\tau) = \int_{-\infty}^{\infty} g(t)\ \varphi_{ii}(\tau - t)dt \tag{1.1}$$

where $\varphi_{io}(\tau)$ is the crosscorrelation function of the system input and output. If the excitation noise has a bandwidth considerably larger (three to ten times) than that of the system under test, then $\varphi_{ii}(\tau)$ is effectively an impulse and

$$\varphi_{io}(\tau) = g(\tau) \tag{1.2}$$

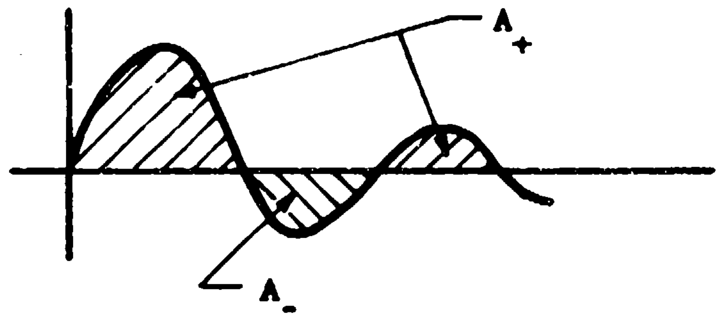
Hence each channel of crosscorrelation provides one point on the impulse response of the system in question.

This approach appears to be ideal for our purposes. The correlation function should be immune to both command signals and to the corruptive signals since the outputs will be uncorrelated with the random noise input. Furthermore, the use of noise excitation allows the excitation energy to be spread over a band of frequencies, and it is expected that the noise amplitude can be kept low enough so that it will only cause a minimum disturbance to the system and pilot.

1.3 PERFORMANCE EVALUATION

The idea of a figure of merit is a fairly common one to the control systems engineer. The most common example is probably the mean square error criterion applied to systems with statistical inputs. A figure of merit which we have considered in some detail employs the damping ratio or relative stability as the criterion for systems performance. As shown in Figure 1-2, it is based on the ratio of the areas of the positive and negative portions of the impulse response and results in a positive or negative quantity as the system damping becomes greater or less than some desired value. By varying the constant K the figure of merit can be made null for different damping ratios. Since the figure of merit reduces to a null for the desired damping ratio and is essentially phase sensitive on either side of the null, this quantity is a suitable error signal for the optimizing system.

AREA RATIO FIGURE OF MERIT



$$F.M. = A_+ - kA_-$$

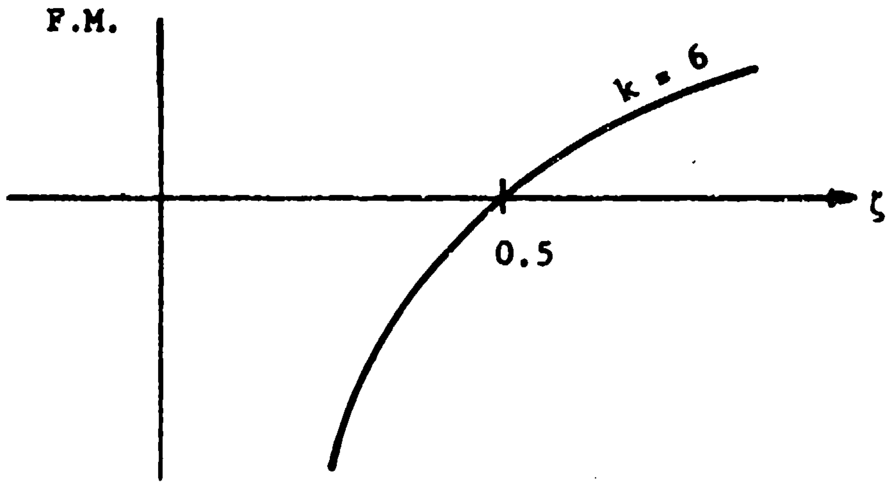


FIGURE 1-2. AREA RATIO FIGURE OF MERIT

1.4 PARAMETER ADJUSTMENT

Controller design may vary somewhat from system to system. In an attempt to develop a universal controller, we investigated the pitch mode of two century series aircraft. A control system which seemed suitable for both aircraft is shown in the block diagram of Figure 1-3. In addition to its being more amenable to a single control parameter, the system was chosen because it behaved more nearly like a second order system. The pole-zero configuration is shown for a typical flight condition. Damping of the dominant pole pair is maintained constant by varying the gain parameter K_a and the compensation time constan T_a.

1.5 SUMMARY

The major emphasis in our program has been placed on the development of the system response measuring equipment. As noted, approaches to the problems of performance evaluation and compensation control have been investigated and found to be feasible, however, considerable work remains to be done in these areas. In the balance of this report, we will present a rather detailed discussion of the theory underlying the approach set forth here, followed by a discussion of the hardware developed and the simulation studies conducted to support this program.

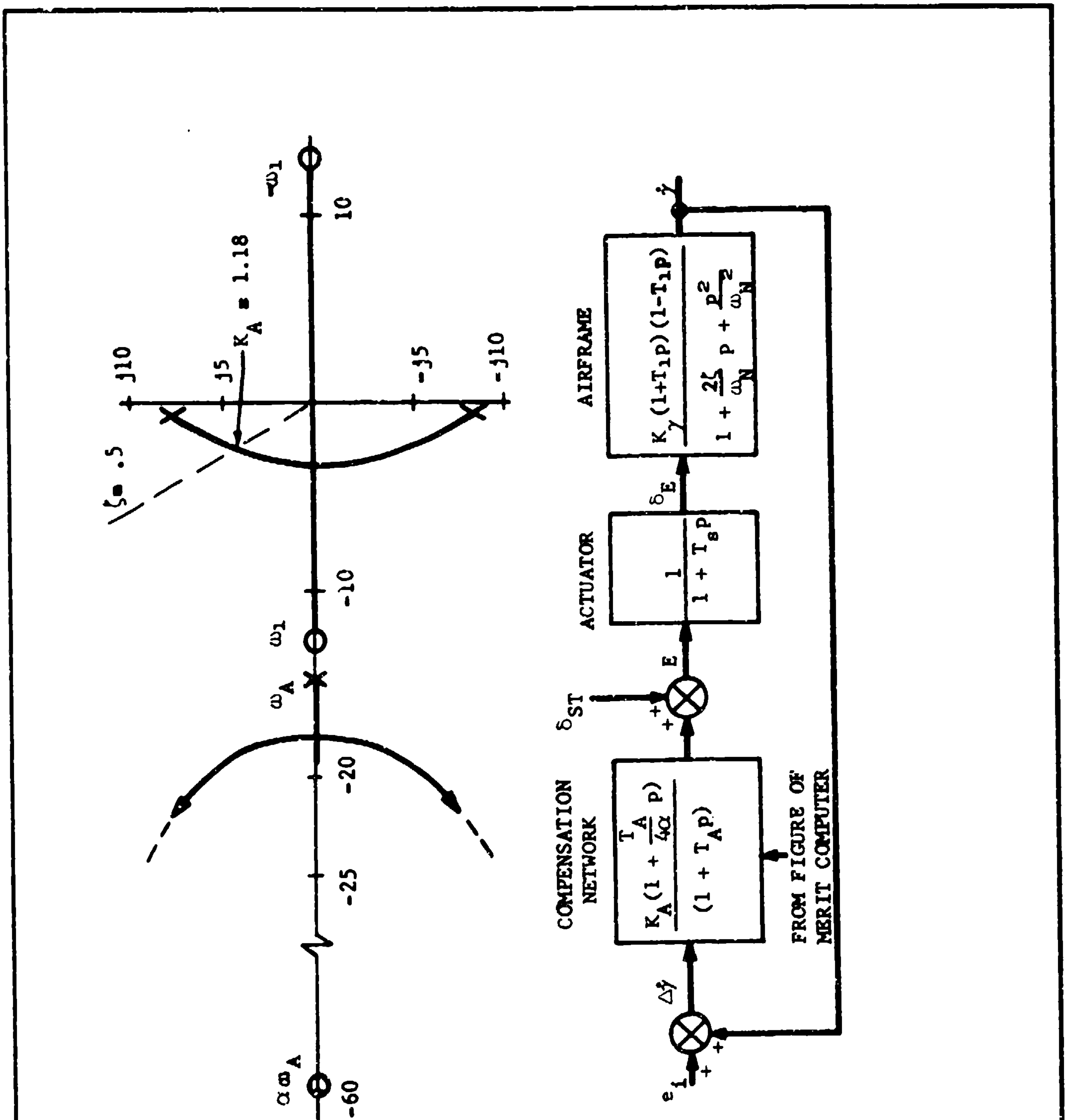


FIGURE 1-3. $\dot{\gamma}$ STABILITY AUGMENTATION

SECTION 2

THEORETICAL DISCUSSION

2.1 CROSSCORRELATOR THEORY

When a linear system is excited with a random input, the system output and the input will be correlated to an extent that depends in part upon the nature of the system. The relationship between this crosscorrelation and the system impulse response forms the theoretical basis for this method of measuring impulse response.

Consider a linear system having an impulse response $g_1(t)$, as shown in Figure 2-1, and an input $x_i(t)$ which is a stationary random variable. The system output $x_1(t)$ is easily obtained from the convolution integral as

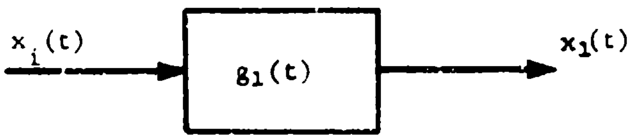
$$x_1(t) = \int_0^\infty x_i(t - \lambda) g_1(\lambda) \, d\lambda \tag{2.1}$$

The crosscorrelation function of $x_1(t)$ and $x_i(t)$ is defined as an ensemble average of their product. Thus

$$\varphi_{i1}(\tau) = E[x_i(t) \; x_1(t + \tau)] \tag{2.2}$$

For stationary inputs, the sequence in which time and ensemble averages are taken can be interchanged and this leads to the result

$$\varphi_{i1}(\tau) = \int_0^\infty \varphi_{ii}(\tau - \lambda) g_1(\lambda) \, d\lambda \tag{2.3}$$



$$x_1(t) = \int_0^{\infty} x_i(t - \lambda)\, g_1(\lambda)\, d\lambda$$

FIGURE 2-1. LINEAR SYSTEM BEING MEASURED

where φ_{ii} is the autocorrelation function of the input.

If the input random variable is sufficiently wide-band so that it may be considered to be white noise, then

$$\varphi_{ii}(\tau) \approx N_x \, \delta(\tau) \qquad (2.4)$$

and the crosscorrelator function becomes

$$\varphi_{i1}(\tau) = N_x \, g_1(\tau) \qquad (2.5)$$

where N_x is the spectral density of $x_i(t)$. Hence, it is clear that determination of the system impulse response can be achieved by measuring the crosscorrelation between the system output and a suitably wide-band random input.

The crosscorrelation function can be measured, in principle, by the method shown in Figure 2-2 which involves the use of ideal delay, ideal multiplication and ideal filtering. The output of the multiplier, $x_o(t)$, has an average value which is equal to the desired crosscorrelation function at $\tau = \tau_m$. That is,

$$\overline{x}_o = E[x_i(t - \tau_m) \, x_1(t)] = \varphi_{i1}(\tau_m) \qquad (2.6)$$

Hence

$$\overline{x}_o = \int_0^\infty \varphi_{ii}(\tau_m - \lambda) g_1(\lambda) d\lambda \qquad (2.7)$$

from which it is clear that the average output is simply the convolution of the impulse response and the autocorrelation function of the input random variable.

A graphical representation of the convolution is shown in Figure 2-3. The average value $\overline{x}_o$ is simply the area under the product of the two curves shown. If the input autocorrelation function is sufficiently narrow compared to the impulse response of the system being measured, then the average value of the multiplier output will be a good indication of the value of the impulse response at $t = \tau_m$.

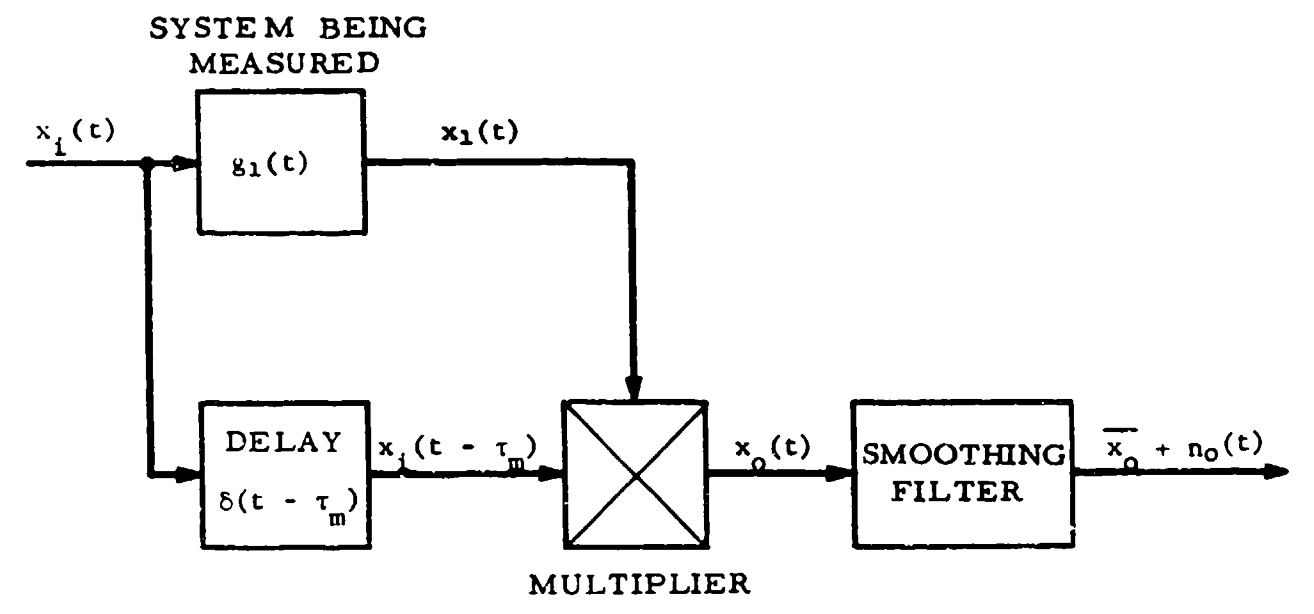


FIGURE 2-2. BASIC CORRELATOR

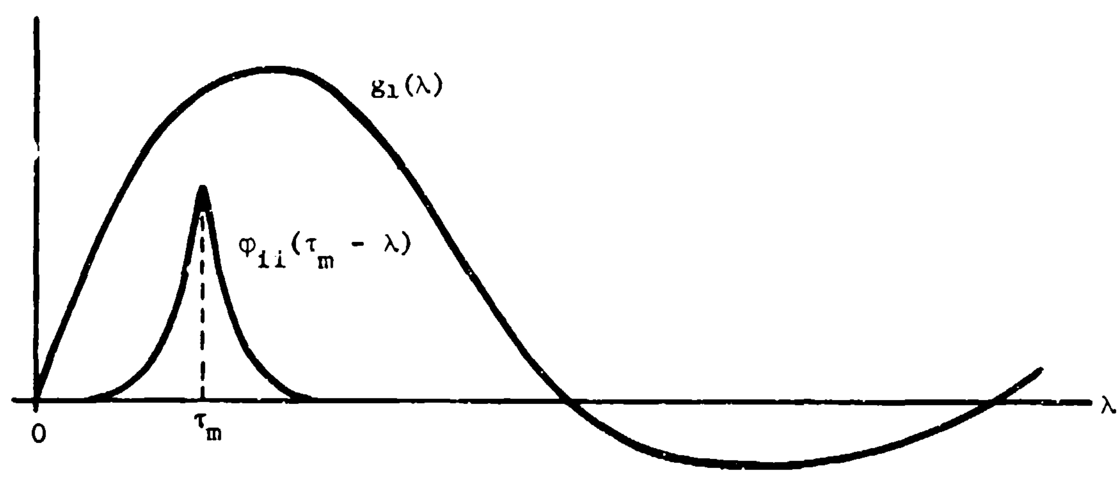


FIGURE 2-3. SHOWING THE CONVOLUTION REQUIRED TO OBTAIN $\bar{x}_o$

Obviously, if the impulse response is to be measured at several different times, it will be necessary to use different values of delay τ_m.

The multiplier output also contains a random component which must be filtered out in order to observe the desired average value. The autocorrelation function of this random component is given by

$$\varphi_{oo}(\tau) = E\left\{[x_o(t) - \bar{x}_o][x_o(t+\tau) - \bar{x}_o]\right\}$$

$$= E[x_o(t)\, x_o(t+\tau)] - (\bar{x}_o)^2 \qquad (2.8)$$

Since $x_o(t)$ can be expressed in terms of the input by

$$x_o(t) = \int_o^\infty x_i(t - \tau_m)\, x_i(t - \lambda) g_1(\lambda) d\lambda \qquad (2.9)$$

the output autocorrelation function can be written as

$$\varphi_{oo}(\tau) = \int_o^\infty d\lambda_1 \int_o^\infty E[x_i(t-\tau_m)x_i(t+\tau-\tau_m)x_i(t-\lambda_1)x_i(t+\tau-\lambda_2)]$$

$$g_1(\lambda_1)g_1(\lambda_2)d\lambda_2 - (\bar{x}_o)^2 \qquad (2.10)$$

The fourth product moment required in (2.10) can be evaluated only after the probability density functions for $x_i(t)$ are specified and this will be done in the following section.

Since the smoothing filter will have a bandwidth small compared to that of the noise out of the multiplier, it is sufficient to find the spectral density of this noise at very low frequencies only. In particular, the spectral density at zero frequency is simply

$$N_o(0) = \int_{-\infty}^{\infty} \varphi_{oo}(\tau) d\tau \qquad (2.11)$$

If the spectral density is assumed to be flat over the bandwidth of

the smoothing filter then the mean square value of the output noise becomes

$$\sigma_o^2 = 2B_2 N_o(0) \tag{2.12}$$

where B_2 is the equivalent noise bandwidth of the smoothing filter in cycles per second. A convenient measure of the goodness of filtering is the signal-to-noise ratio which may be defined as

$$z_o = \frac{(\bar{x}_o)^2}{\sigma_o^2} = \frac{(\bar{x}_o)^2}{2B_2 N_o(0)} \tag{2.13}$$

This may also be expressed in terms of smoothing time, or settling time, as

$$z_o = T \frac{(\bar{x}_o)^2}{N_o(0)} \tag{2.14}$$

in which the smoothing time

$$T = \frac{1}{2B_2} \tag{2.15}$$

has been taken arbitrarily as the time between substantially independent samples of the output noise.

In addition to the response to $x_i(t)$, the output of the system being measured will also contain responses to command inputs and to external disturbances. So far as the measurement of impulse response is concerned, these contributions represent more noise and may be considered as occurring at the output of the system as shown in Figure 2-4.

The component of multiplier output due to external noise only is simply

$$x_n(t) = x_i(t - \tau_m)\, n_1(t) \tag{2.16}$$

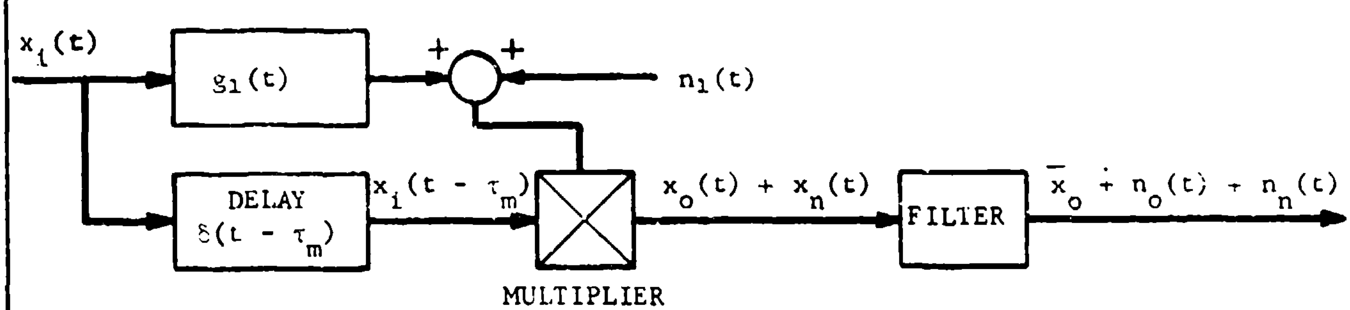


FIGURE 2-4. SYSTEM ASSUMED FOR CONSIDERING EXTERNAL DISTURBANCES

Since x_i and n_1 are statistically independent, the autocorrelation function of x_n is just the product of the autocorrelation functions of the two factors. Thus,

$$\varphi_{nn}(\tau) = \varphi_{ii}(\tau)\ \varphi_{nn_1}(\tau) \tag{2.17}$$

where φ_{nn_1} is the autocorrelation function of n_1.

The spectral density at zero frequency for this component of noise is

$$N_n(0) = \int_{-\infty}^{\infty} \varphi_{ii}(\tau)\ \varphi_{nn_1}(\tau)\,d\tau \tag{2.18}$$

and the mean square value at the output of the smoothing filter is

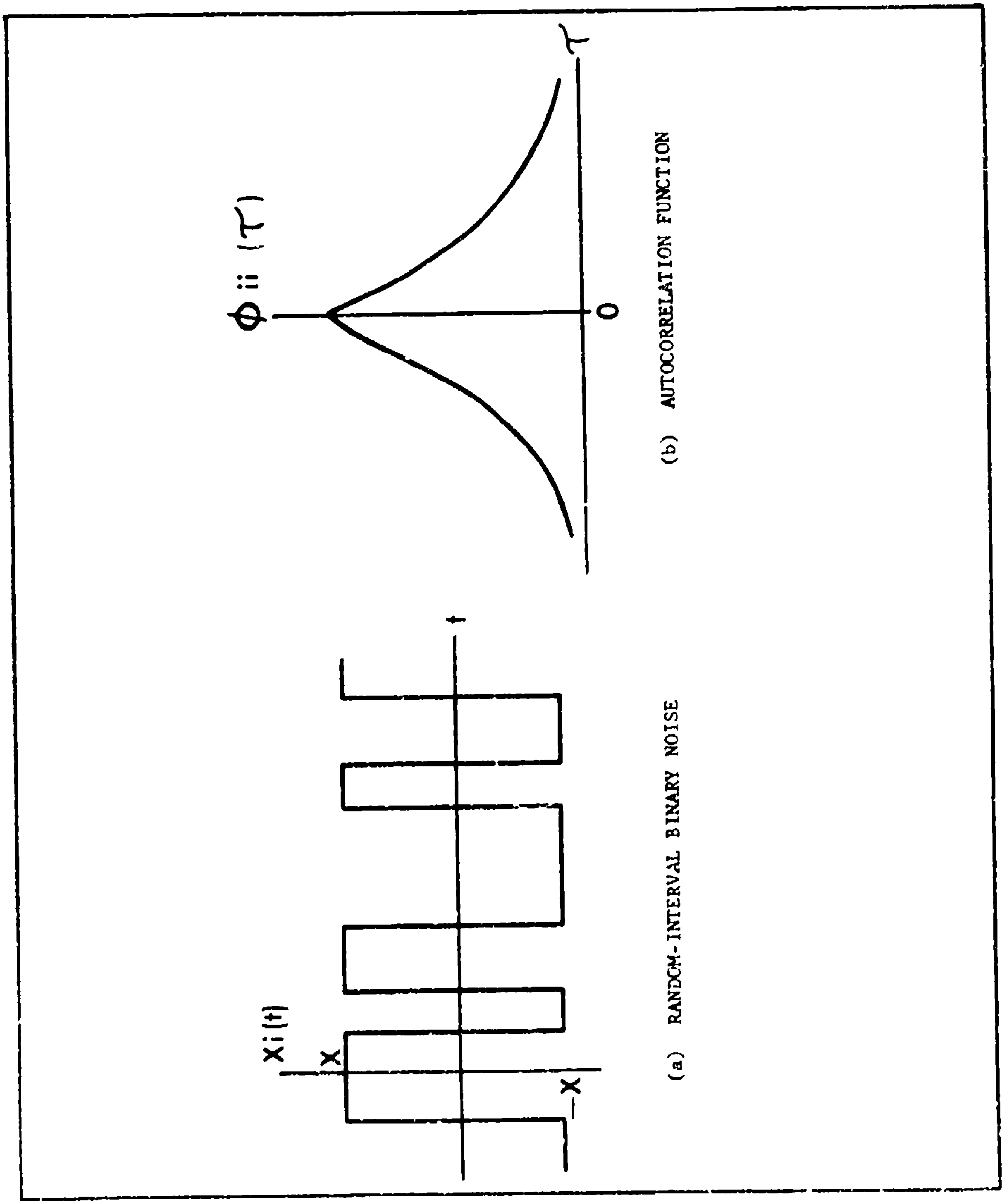
$$\sigma_n^{\,2} = 2B_2\ N_n(0) \tag{2.19}$$

2.2 SYSTEM EXCITATION

The results presented thus far have been completely general in that the nature of the excitation input, $x_i(t)$, has not been specified and can, in fact, be any stationary random process. From the standpoint of ease in mathematical analysis, the assumption that $x_i(t)$ is normally distributed would be highly desirable. However, considerable simplification of hardware, particularly in the delay and multiplication, can be achieved by using an input which has only two states (that is, binary noise). Although there are many different types of binary noise which might be considered, only two will be discussed here.

The first type, which might be called random-interval binary noise, is illustrated in Figure 2-5(a). In this case, the transitions from one state to the other occur independently and the number of such transitions in a long interval is Poisson distributed. These conditions are very nearly fulfilled by a flip-flop circuit operating on pulses from a Geiger counter exposed to a radioactive sample.

If the average number of transitions per second is β, then it can be shown that the autocorrelation function of the random-



(a) RANDOM-INTERVAL BINARY NOISE

(b) AUTOCORRELATION FUNCTION

FIGURE 2-5

interval binary noise is

$$\varphi_{ii}(\tau) = X^2 \epsilon^{-2\rho|\tau|} \tag{2.20}$$

where $\pm X$ are the permitted values of $x_i(t)$. This autocorrelation function is shown in Figure 2-5(b).

The second type, which will be called discrete-interval binary noise, is illustrated in Figure 2-6(a). In this case the times at which transitions can occur are explicitly specified and the state for the succeeding interval is chosen independently of the state in any preceding interval. This type of binary noise might be generated by sampling a very wide band noise source every t_1 seconds and setting $x_1 = X$ if the sample is positive or $x_1 = -X$ if the sample is negative.

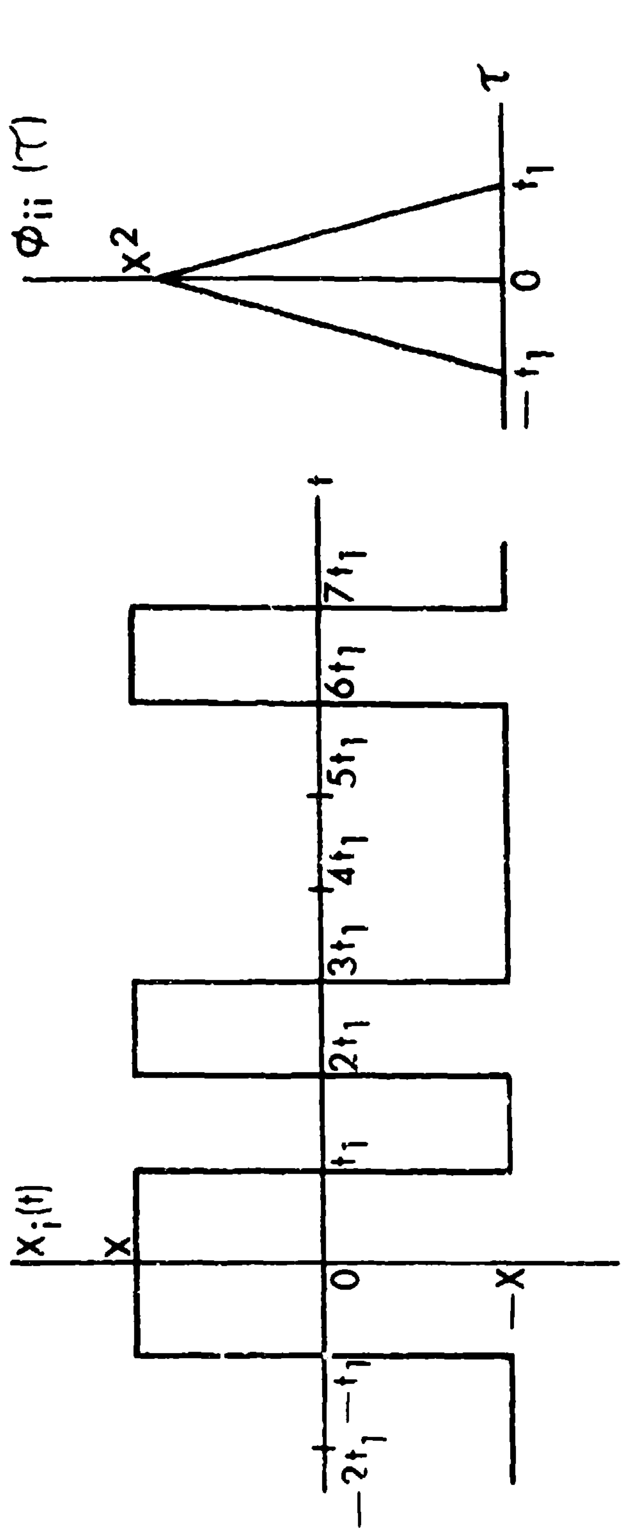
If the minimum interval width is t_1 seconds, the autocorrelation function of discrete-interval binary noise is given by

$$\begin{aligned} \varphi_{ii}(\tau) &= X^2\left[1 - \frac{|\tau|}{t_1}\right] & -t_1 < \tau < t_1 \\ &= 0 & |\tau| > t_1 \end{aligned} \tag{2.21}$$

This autocorrelation function is shown in Figure 2-6(b).

In the discussion of the above two types of binary noise it has been assumed that the input noise sample is of infinite duration. However, there are both practical and theoretical advantages in using an input noise sample of finite length and repeating it periodically. In the first place, the noise sample can be stored in some device which produces the delay and thus eliminates the need for a noise generator. Secondly, the filtering problem becomes easier because the average value can be extracted perfectly (in the absence of external noise) by integrating the multiplier output for exactly one period.

In considering the effect of the periodicity on the measurement of impulse response, it should be recalled that a periodic function has an autocorrelation function which is also periodic with the same period. Thus, if an ideal sample of discrete-interval binary



(a) DISCRETE-INTERVAL BINARY NOISE

(b) AUTOCORRELATION FUNCTION

FIGURE 2-6

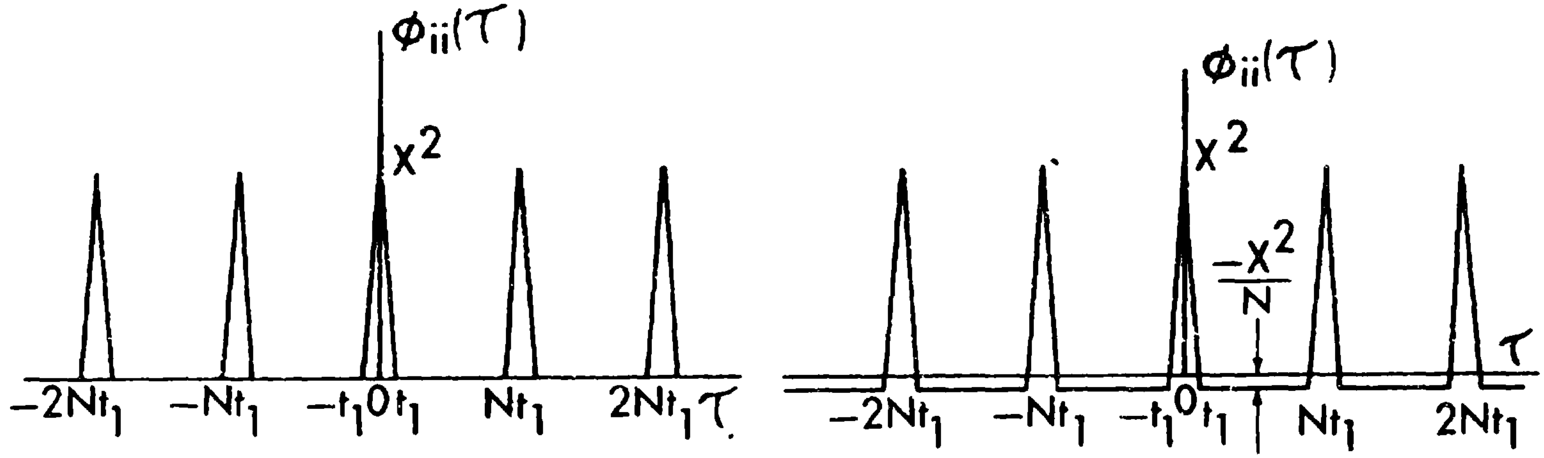
noise, of length Nt_1, is repeated periodically, the resulting autocorrelation function will be as shown in Figure 2-7(a). When this autocorrelation function is convolved with $g_1(t)$ in order to obtain the average multiplier output as was discussed previously, it is clear that the periodicity will have practically no effect if Nt_1 is greater than the length of the significant part of the impulse response.

One problem which does arise, however, is that of selecting an ideal noise sample. If a sample is selected at random, its autocorrelation function may differ greatly from that of the ideal sample unless Nt_1 is made quite large. This variation is a result of the statistics of a given finite length sample differing appreciably from the statistics of the ensemble. It is possible, of course, to try a large number of finite length samples until one is found which is satisfactory, but a more desirable approach would be to synthesize a noise sample having specified characteristics. Such a synthesis procedure is not available in general, but it is found for some special cases of discrete-interval binary noise.

In particular, if N, the number of discrete intervals in one period, is a prime number, it is possible to synthesize a noise sample whose autocorrelation function is of the form shown in Figure 2-7(b). Hence, if N is large (say, greater than 100) this autocorrelation function is nearly as good as that of the ideal sample. Since prime numbers are fairly evenly distributed, there is usually little difficulty in finding one close enough to the desired sample length to be useable. The synthesis procedure consists of applying some of the concepts of number theory to obtain a precise specification of the state of the random variable in every interval throughout the period. The calculation required can be easily programmed for a digital computer so there is no difficulty in obtaining noise samples for even very large values of N.

2.3 THE FILTERING PROBLEM

As has been discussed previously, the purpose of the smoothing filter is to reduce the random component of the multiplier output to a value that is sufficiently small so that the average component can be accurately observed. The computation required to determine if this objective can be achieved started with the evaluation of the autocorrelation function of the multiplier output as given in (2.10) and this in turn requires the evaluation of the fourth product moment of the input excitation.



(a) AUTOCORRELATION FUNCTION OF IDEAL PERIODIC NOISE SAMPLE

(b) AUTOCORRELATION FUNCTION OF SYNTHESIZED PERIODIC NOISE SAMPLE

FIGURE 2-7

For the case of random-interval binary noise it is possible to show that the fourth product moment is given in general by

$$E[x_i(t_1)x_i(t_2)x_i(t_3)x_i(t_4)] = \varphi_{ii}(t_2 - t_1)\varphi_{ii}(t_4 - t_3) \tag{2.22}$$

when the time instants are ordered; that is, when

$$t_4 > t_3 > t_2 > t_1$$

Thus, the evaluation of (2.10) requires breaking the plane of integration into a number of separate regions in which the variables are ordered and then using the appropriate fourth product moment for each. In addition, if the excitation noise is assumed to be very wide-band compared to the system bandwidth so that the approximation

$$\varphi_{ii}(\tau) = X^2 \epsilon^{-2\beta|\tau|} \approx \frac{X^2}{\beta}\,\delta(\tau) \tag{2.23}$$

is valid, then the output autocorrelation function becomes

$$\varphi_{oo}(\tau) = \frac{X^4}{\beta^2}\Big[g_1(\tau_m + \tau)g_1(\tau_m - \tau) + \delta(\tau)\int_0^{\tau_m-\tau} g_1(\lambda)g_1(\lambda+\tau)\,d\lambda$$

$$+ \delta(\tau)\int_{\tau_m}^{\infty} g_1(\lambda)g_1(\lambda+\tau)\,d\lambda\Big] \tag{2.24}$$

From (2.11) the spectral density at zero frequency is simply

$$N_o(0) = \int_{-\infty}^{\infty}\varphi_{oo}(\tau)\,d\tau = \frac{X^4}{\beta^2}\Big[\int_{-\infty}^{\infty} g_1(\tau_m + \tau)g_1(\tau_m - \tau)\,d\tau + \int_0^{\infty} g_1^2(\lambda)\,d\lambda\Big] \tag{2.25}$$

which is seen to be a function of the delay time τ_m. By application of the Schwarz inequality it is easy to show that the first integral of (2.25) can never exceed the second for any value of τ_m so that the spectral density is bounded by

$$N_o(0) \leq \frac{2X^4}{\beta^2}\Big[\int_0^{\infty} g_1^2(\lambda)\,d\lambda\Big] \tag{2.26}$$

The situation is considerably different for the case of periodically repeated samples of discrete-interval binary noise. If the system being measured does not change with time, then the component of multiplier output due to excitation will also be a periodic function of time and its spectral density will contain only discrete components at the fundamental frequency and all higher order harmonics. Hence, the average value can be extracted perfectly by any filter which has zero transmission at these discrete frequencies and the smoothing time need not be greater than one period. If the system does change with time, however, the multiplier output will not be exactly periodic and some residual noise will appear at the output of the smoothing filter. Under most circumstances this noise will be small compared to that which would be obtained with non-periodic excitation.

The use of periodic excitation does nothing to alleviate the difficulty of smoothing the output noise due to external disturbances and this may often be the most important contribution. If the wideband approximation to the input autocorrelation function (2.23) is used, the low frequency spectral density of the multiplier output is obtained from (2.18) as

$$N_n(0) = \frac{X^2}{\beta} \varphi_{nn_1}(0) = \frac{X^2}{\beta} \sigma_1{}^2 \tag{2.27}$$

where $\sigma_1{}^2$ is the mean square value of the external disturbance at the output of the system being measured.

It is now possible to write a lower bound for the signal-to-noise ratio at the output of a smoothing filter having an equivalent smoothing time of T seconds under the assumption of non-periodic, wide-band excitation and the presence of external disturbances. This is

$$z_o \geqslant \frac{T(\bar{x}_o)^2}{N_o(0) + N_n(0)} = \frac{T \dfrac{X^4}{\beta^2} g_1{}^2 (\tau_m)}{\dfrac{2X^4}{\beta^2} \int_0^\infty g_1{}^2(\lambda)\, d\lambda + \dfrac{X^2}{\beta} \sigma_1{}^2} \tag{2.28}$$

A more compact expression can be obtained by writing (2.28) in terms of the ratio of mean square excitation to mean square external disturbance at the output of the system being measured. This ratio, which is

$$z_1 = \frac{\frac{X^2}{\beta} \int_0^\infty g_1{}^2(\lambda)\, d\lambda}{\sigma_1{}^2} \tag{2.29}$$

gives a direct comparison of the relative importance of excitation and external disturbance on the system response. In terms of this ratio (2.28) becomes

$$z_o \geqslant T \frac{g_1{}^2(\tau_m)}{(2 + \frac{1}{z_1}) \int_0^\infty g_1{}^2(\lambda)\, d\lambda} \tag{2.30}$$

If the excitation is periodic, and if T is greater than one period, then the 2 in the denominator of (2.30) can be omitted. This assumes that a filter for the periodic component is used in addition to a more conventional smoothing filter for the external noise.

As an indication of the orders of magnitude involved here, assume that the system being measured is second-order with an impulse response of

$$g_1(t) = \frac{\omega_1}{\sqrt{1 - \zeta_1{}^2}} \epsilon^{-\zeta_1 \omega_1 t} \sin \omega_1 \sqrt{1 - \zeta_1{}^2}\; t \tag{2.31}$$

The value of $g_1(\tau_m)$ will, of course, depend upon the value of τ_m being considered but its maximum is very nearly

$$\mathrm{Max}[g_1(\tau_m)] \approx \frac{\omega_1}{\sqrt{3}} \tag{2.32}$$

when $\zeta_1 = 0.5$. Likewise

$$\int_0^\infty g_1{}^2(\lambda)\, d\lambda \simeq \frac{1}{2}\, \omega_1 \tag{2.33}$$

under the same conditions. The curves of Figure 2-8 indicate the relation among the various parameters for this special case. For periodic excitation the period is taken to be 5 seconds.

The discussion of filters so far has assumed that linear filters will be used. Since the statistics of the noise are not Gaussian it is reasonable to inquire if a nonlinear filter might not produce better results. The situation has been examined as a problem in statistical estimation in order to determine the best that might reasonably be expected without explicitly determining the type of nonlinearity required. The results indicate that for the excitation noise only the smoothing time could be reduced at most by a factor of π in the non-periodic case and not at all in the periodic case (since integration for exactly one period is still required).

When external noise is considered the results depend upon the statistics assumed for the external disturbances. If these are taken to be Gaussian, as is usually done, then the multiplier output will have a first-order probability density function which is Gaussian and a joint probability density function which is nearly Gaussian. Under these circumstances the minimum variance estimator is nearly linear. Hence, it has been concluded that when periodic excitation is used, nonlinear filtering does not offer sufficient improvement to justify its use.

It has also been assumed so far that the parameters of the system being measured do not change with time and on this basis the signal-to-noise ratio out of the correlator can be made arbitrarily large simply by increasing the smoothing time. This assumption is unrealistic, however; in fact, the major purpose of the cross-correlation technique is to make it possible to measure changes in system parameters. It is clear, therefore, that the maximum smoothing times that can be tolerated will depend in some way upon the maximum rate at which the system parameters will change.

If the system impulse response is considered to be time-varying, it may be designated as $g_1(t, \lambda)$ which is defined as the response at time t to an impulse applied at time λ. When non-periodic binary excitation is applied, the signal component out of the multiplier is easily shown to be

$$\bar{x}_o(t) = \frac{X^2}{\beta} g_1(t. t - \tau_m) \tag{2.34}$$

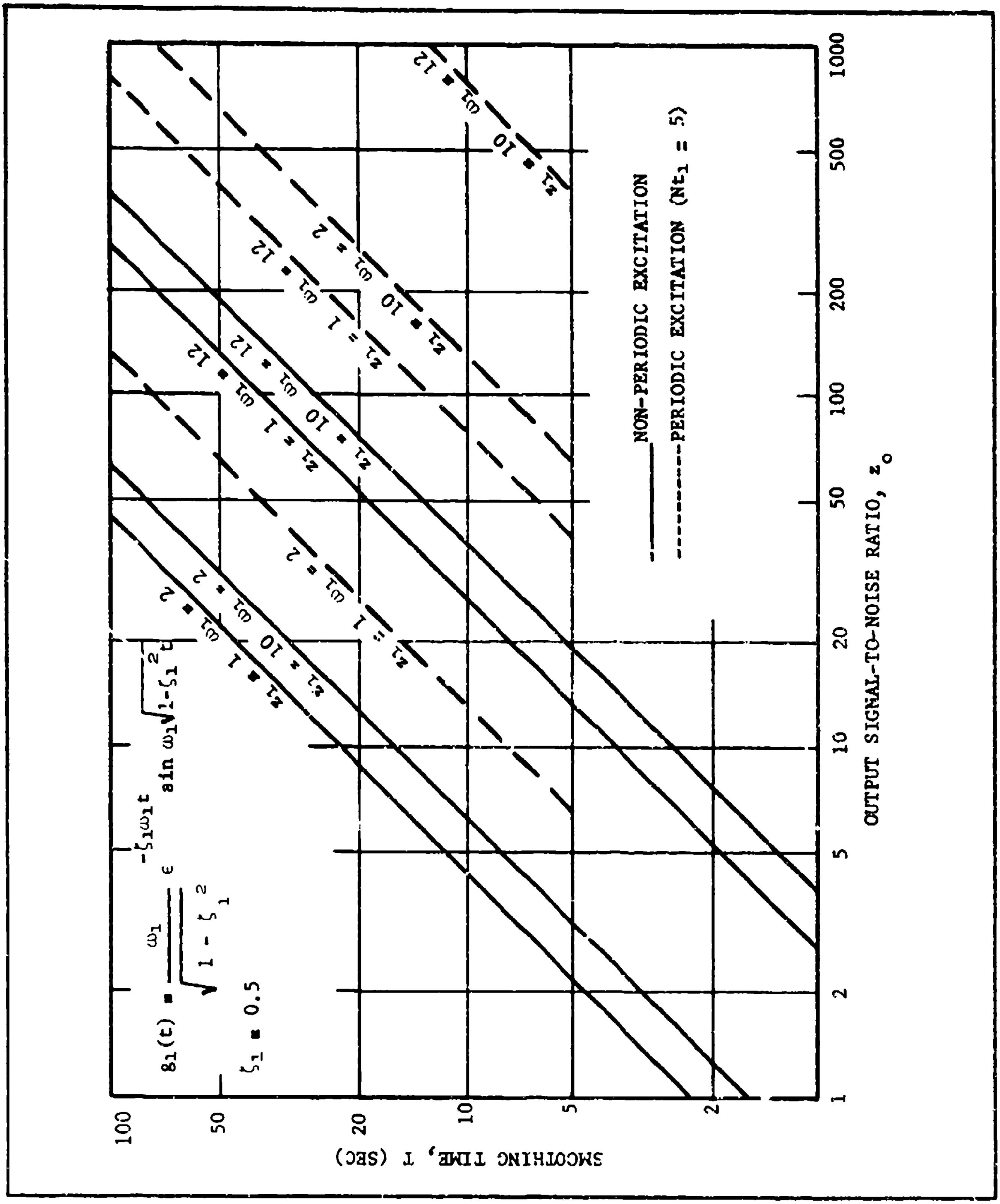


FIGURE 2-8. SMOOTHING TIME VS. SIGNAL-TO-NOISE-RATIO

which is also a function of time. The smoothing time of the filter must therefore be small enough to reproduce $\bar{x}_o(t)$ with any specified degree of accuracy. It is difficult to draw general conclusions here but any specific time variation of the impulse response can be investigated in detail.

It may also be noted from (2.34) that there is a lag of τ_m between the time a change in the system occurs and the time that it may be fully observed in $\bar{x}_o(t)$. As a rough illustration of this, Figure 2-9 shows a hypothetical situation in which the impulse response at τ_m changes from one constant value to another constant value at $t=0$. The average value of correlator output does not completely reach its new value until $t = \tau_m$.

2.4 MEASUREMENT OF SYSTEM DAMPING

It has been demonstrated that the crosscorrelation procedure makes it possible to measure the impulse response of a system at a n[illegible]er of discrete points. A possible application of this procedure is to determine how well the system is damped, but in order to do this it is first necessary to relate some characteristic of the impulse response to the system damping. A characteristic which appears to be suitable for this purpose is the ratio of the positive area of the impulse response to the negative area.

In the case of a second-order system such as defined by (2.31), it can be shown that the area ratio is

$$R = \left|\frac{A_+}{A_-}\right| = \epsilon^{\frac{\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} \tag{2.35}$$

where A_+ is the positive area of $g_1(t)$, A_- is the negative area, and ζ_1 is the damping ratio. It is clear, therefore, that R is a monotonic function for ζ_1 and is independent of the natural frequency ω_1. Thus, a measured value of area ratio uniquely determines the system damping ratio.

For higher-order systems there is no such simple mathematical relation between area ratio and system damping but their physical relationship is essentially the same. This fact may be made more

evident by recalling that the step response of a linear system is simply the area under the impulse response. Hence, the overshoot on the step response is directly related to the amount by which the positive area exceeds the net area which can, in turn, be related to the area ratio.

If the correlator is to be used as a component of an adaptive system which is to control damping, then it is desirable to create an error signal which is zero at the desired value of damping and has opposite polarity on either side of this. Such an error signal, which will be designated as the figure of merit, can be formed by multiplying the negative area of the impulse response by the desired area ratio and adding it to the positive area. Thus,

$$F_m = A_+ + R_o A_- \tag{2.36}$$

where R_o is the area ratio corresponding to the desired system damping.

In the present case the entire impulse response is not available but only a finite set of discrete values. Hence, the appropriate areas can be approximated by multiplying the known ordinates of $g_1(t)$ by the spacing between ordinates and adding. On this basis, the figure of merit becomes

$$F_m = \sum_{m=1}^{M} A_m \, g_1(\tau_m) \tag{2.37}$$

where M is the number of correlators being used and

$$A_m = a_m \qquad g_1(\tau_m) > 0$$

$$= R_o a_m \qquad g_1(\tau_m) < 0$$

$$a_m = \frac{1}{2} (\tau_{m+1} - \tau_{m-1})$$

In actuality, however, the time values of $g_1(\tau_m)$ are not available but only a set of random variables having average values

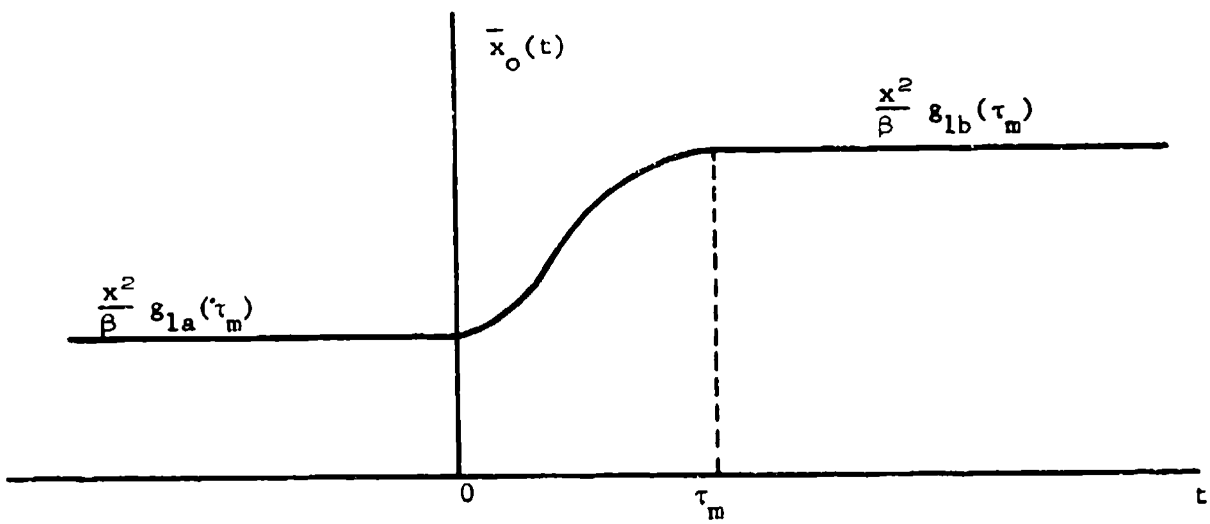


FIGURE 2-9. CHANGE IN OUTPUT CAUSED BY A STEP CHANGE IN IMPULSE RESPONSE

proportional to $g_1(\tau_m)$. Hence, the computed figure of merit will also be a random variable and subject to several kinds of errors. In the first place, the output of any correlator is to be multiplied by R_o or not depending upon whether that output is negative or positive. For those channels in which the average value is small, the output will fluctuate across zero. Because all negative values are multiplied by R_o (which may be about 6), the resulting average will be biased negatively by an amount which depends upon the magnitude of noise. Because of this bias the figure of merit will go through zero at a different value of system damping than it should.

Furthermore, the variations in figure of merit due to the noise may be quite large because the noise out of all the correlators is being added. In addition, for those channels in which $g_1(\tau_m)$ is negative, the noise is being multiplied by R_o.

Two steps can be taken to reduce the above errors in figure of merit. First the weighting factor of R_o can be applied to a given channel on the basis of the sign of $g_1(\tau_m)$ rather than the sign of the correlator output. This can be done by making an independent measurement of natural frequency as discussed in the next section. Secondly, those channels in which the average value is small can be completely eliminated from the computation of figure of merit on the grounds that they contribute little to the result anyway. This procedure eliminates the noise and bias errors that these channels would otherwise produce.

When the parameters of the system being measured change with time an additional error in the figure of merit is produced by the delay in the correlator. As was discussed in the preceding section, there is a lag of τ_m in any channel between the time a parameter change occurs and the time its effect is fully apparent in the correlator output. The resulting lag in the figure of merit will depend upon how the delays for the various channels are distributed. If the delays are spaced approximately exponentially (closer spacing at small delays), a step change in system damping might result in a change in figure of merit as shown in Figure 2-10. It is assumed in this sketch that the system damping changed at $t = 0$ from some value less than the desired value to a greater value.

When the correlator and figure of merit computer are used as part of a closed-loop system to control damping, the resulting errors in damping arise primarily from noise and bias errors in the

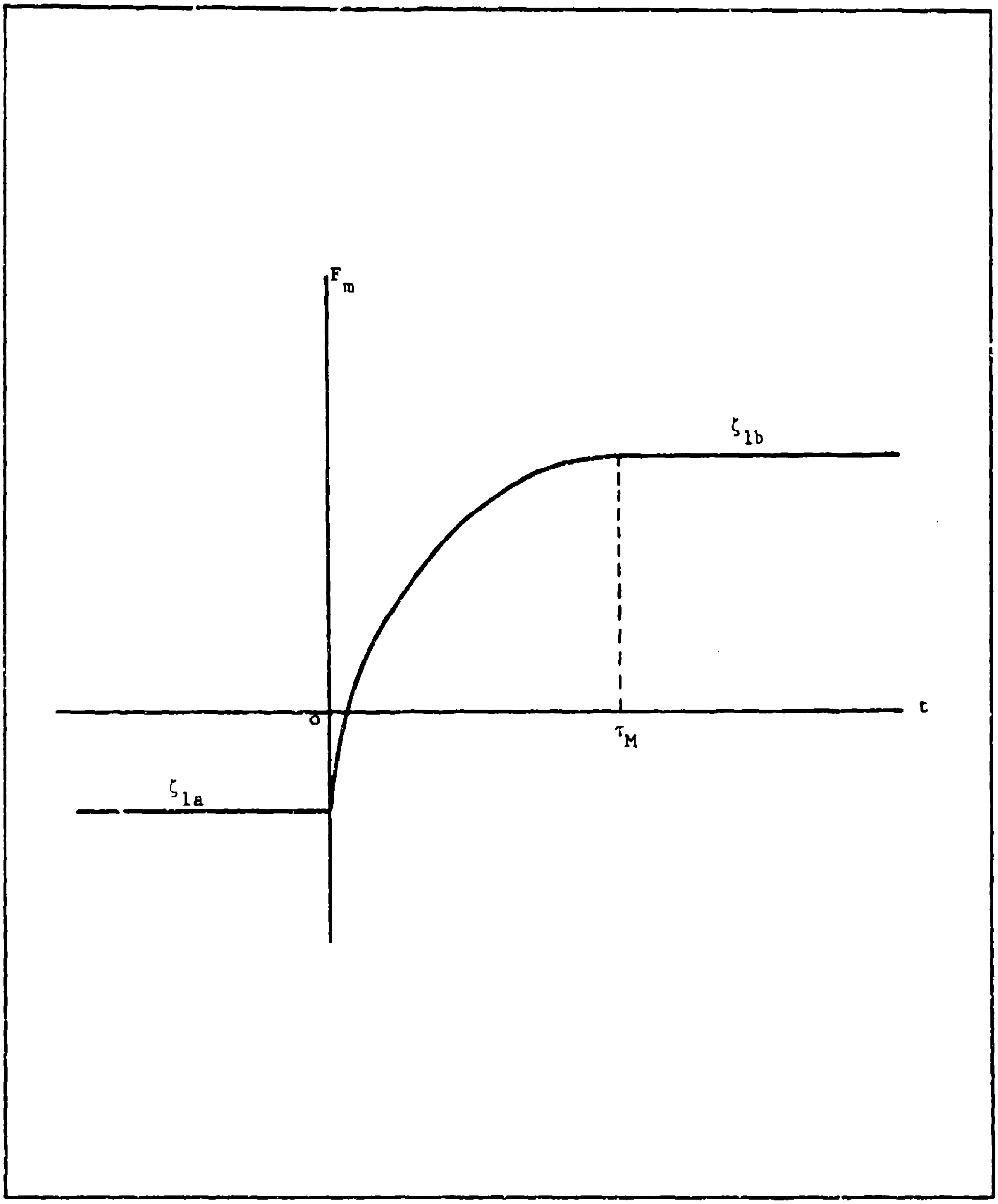


FIGURE 2-10. CHANGE IN FIGURE OF MERIT CAUSED BY A STEP CHANGE IN DAMPING

figure of merit and from the lag introduced by the correlators. The latter can be approximately compensated by a linear network but this increases the error due to noise. The final compensation of the closed-loop system therefore represents a three-way compromise among peak error, velocity error and noise error when the system parameters are assumed to change at some maximum rate.

2.5 MEASUREMENT OF NATURAL FREQUENCY

It may be desirable to measure the natural frequency of a system either for the purpose of controlling it or for the purpose of improving the computation of figure of merit as has just been discussed. It is possible, of course, to determine the natural frequency from a knowledge of the damping and the time that the impulse response goes through its first zero. However, it is of interest to note that an independent measurement is also possible.

Consider a crosscorrelator of the form shown in Figure 2-11 and note that this differs from the original correlator only in that a differentiator has been added. It can be shown that the average value of the multiplier output is

$$\bar{x}_o = \frac{X^2}{\beta} \frac{d\, g_1(\tau)}{d\tau} \tag{2.38}$$

when the excitation is non-periodic binary noise. For a second-order system of the form described by (2.31), and for no delay ($\tau = 0$), this becomes

$$\bar{x}_o = \frac{X^2}{\beta} \omega_1^2 \tag{2.39}$$

from which it is clear that the natural frequency can be obtained independently of the damping.

For higher order systems the initial slope of the impulse response is zero and the measurement $\tau = 0$ gives no information. However, by introducing a small delay, it is again possible to obtain the natural frequency. For example, in a third-order system with a real pole at $-\alpha$ and a small delay of τ_1, the average output becomes

$$\bar{x}_o \approx \frac{X^2}{\beta} \alpha\, \tau_1\, \omega_1^2 \tag{2.40}$$

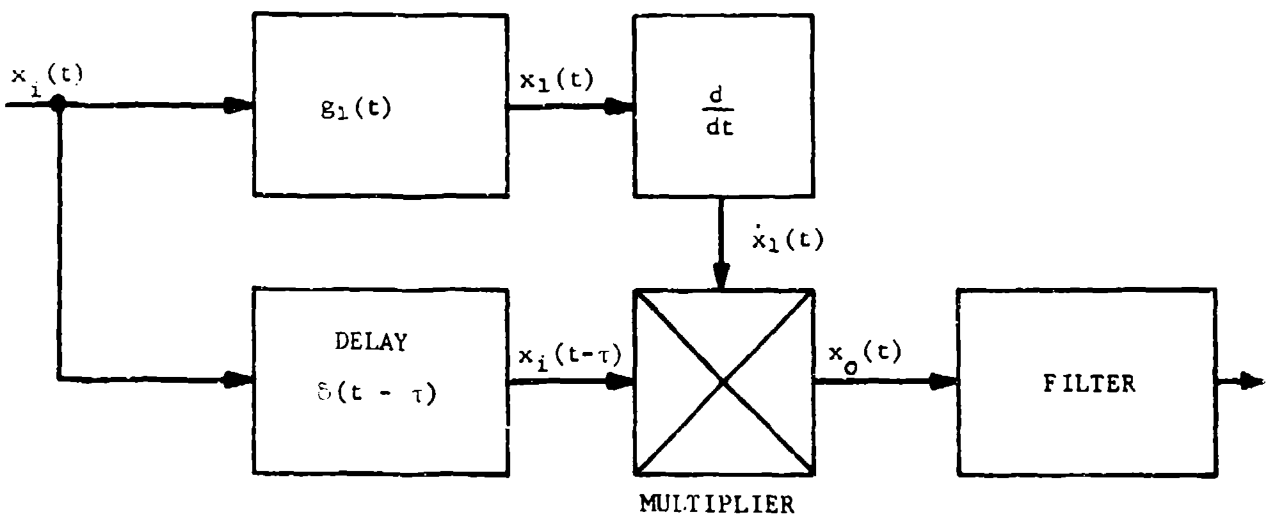


FIGURE 2-11. SYSTEM FOR MEASURING NATURAL FREQUENCY

This result is still independent of the damping ζ_1 but does depend upon α which may or may not be known.

Essentially the same results can be obtained by introducing the differentiation before the system being measured or before or after the delay. In all cases, however, the signal-to-noise ratio out of the correlator is poorer than without differentiation.

SECTION 3

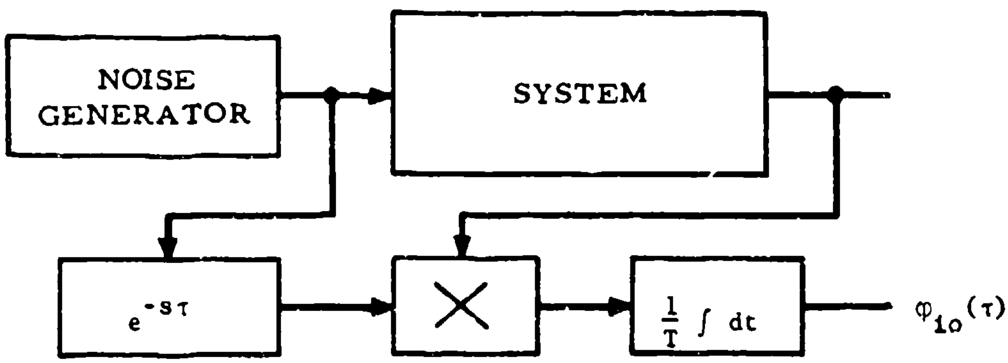
CROSSCORRELATOR MECHANIZATION

Several approaches to the mechanization of the crosscorrelator have been investigated, including the use of analog, digital, and combined analog-digital techniques. Figure 3-1 is a flow diagram of the mathematical operations to be performed. These include noise generation, time delay generation, multiplication, and integration with respect to time.

3.1 NOISE GENERATION

Complexity of the mechanization is determined to a great extent by the waveform generated by the random noise generator. Since there are no restrictions on the statistical properties of the noise as far as the impulse response computation is concerned, there are several techniques for noise generation available to us. Of these, binary noise appears to offer the most advantages as far as simplifying the system mechanization. The resulting system employs a combination of analog and digital techniques.

A typical sample of binary noise is shown in Figure 3-2. The waveform has constant amplitude but random polarity and with a Poisson distribution for the pulse lengths, $\underline{\ell}$. Because the design of the time delay generator is based on periodic sampling of the noise signal, it is necessary to place a lower limit on the pulse length such that no pulse occurs whose length is shorter than $\underline{a}$.



$$\varphi_{io}(\tau) = \lim_{T \to \infty} \int_{-T}^{T} \varphi_{ii}(\tau - t)\, g(t)\,dt$$

FIGURE 3-1. OPERATIONS REQUIRED TO COMPUTE CROSSCORRELATION COEFFICIENTS

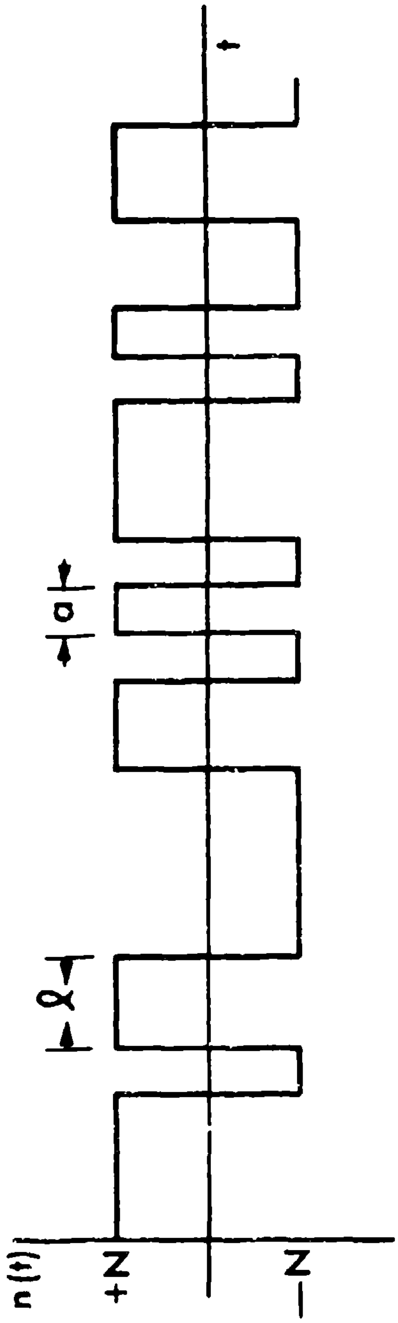


FIGURE 3-2. BINARY NOISE SAMPLE

Figure 3-3 is a block diagram of a suitable random noise generator. Randomly occurring pulses are obtained from a Geiger counter excited by a radioactive sample. These pulses are passed through a one-shot multivibrator serving as the pulse period discriminator. The one-shot insures that successive pulses in its output do not occur closer together than the minimum pulse period a. A flip-flop is triggered by the discriminated pulse train to generate the waveform shown in Figure 3-2. Bandwidth adjustments are made by varying the count rate from the Geiger counter and by varying the minimum pulse period.

An alternative to the use of a random noise generator is the employment of a recorded noise sample which is repeated over and over again. As noted earlier, such an "ideal" noise sample has some very attractive properties which make it a more desirable noise source than the random noise generator.

3.2 TIME DELAY GENERATION

Turning our attention to the time delay generator, we note that the time delay generator accepts the noise signal as an input and generates several outputs, one for each point on the system impulse response. The question arises as to how many points to compute. Certainly a sufficient number of points must be included to obtain all of the significant information available in the system response. In the systems to be evaluated with this correlator, the natural frequencies are expected to vary from 2 rad/sec to 12 rad/sec. As shown in Figure 3-4, it appears that a satisfactory approach is to compute a fixed number of points with exponential spacing between points. In this manner, we can obtain a good spread of the points in the first part of the response at both high and low frequencies. The low frequency limit establishes the requirement for the maximum time delay at approximately four seconds. Experimental evidence indicates that 12 points on the impulse response will give adequte information. Hence the time delay generator must produce 13 outputs, one for the system excitation and 12 delayed signals, with delays from 0.01 seconds to 4.0 seconds spaced exponentially.

A pure time delay of the order of several seconds without significant distortion is usually difficult to obtain. However, the binary characteristic of the noise signal permits us to employ some relatively simple digital techniques to obtain many seconds of delay in increments of hundredths of a second.

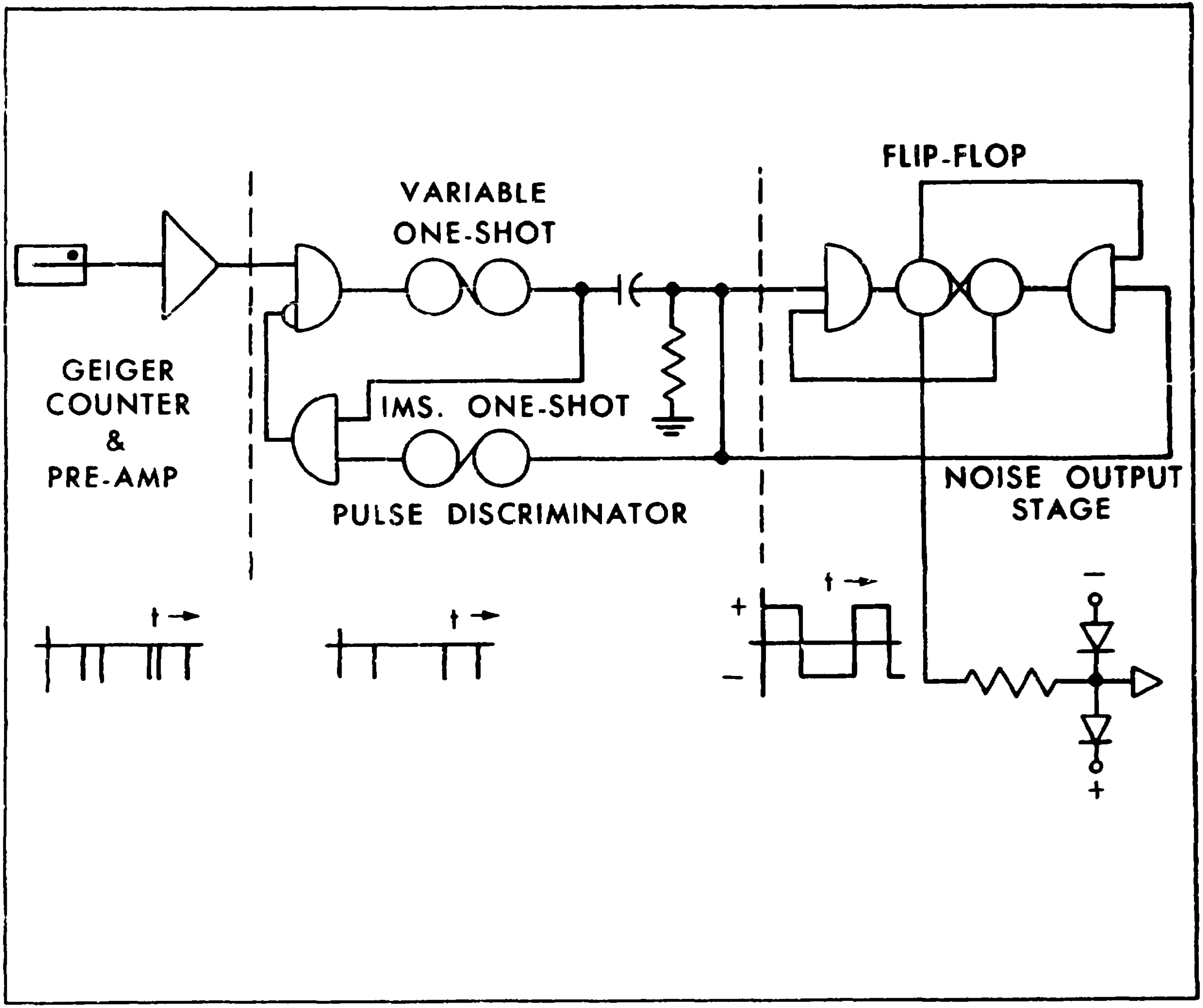
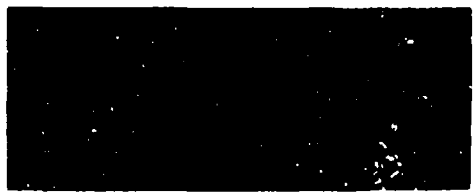


FIGURE 3-3. BLOCK DIAGRAM - RANDOM NOISE GENERATOR

$\omega = 2$ rad/sec. $\zeta = 0.4$



$\omega = 12$ rad/sec $\zeta = 0.4$

FIGURE 3-4. EXPONENTIAL SPACING OF CORRELATOR POINTS

The scheme is based on sampling the random or asynchronous input to generate a periodic or synchronous digital signal which is propagated through a shift register. The shift register is in effect a discrete delay line which can be tapped at several points. Reconstruction of the sampled noise is effected in a one-bit storage unit, a flip-flop, in the output of each channel.

3.3 MULTIPLICATION

Multiplication is also simplified by the binary character of the noise signal. The noise signal passing through the time delay generator is normalized to an amplitude of ± 1, since the digital circuits do not sense amplitude. Multiplication of the system response by ± 1 simply means that we connect either the system response or its inverse to the multiplier output. This is a simple gating operation that may be accomplished either electronically or with relays.

3.4 AVERAGING

Computation of the average value of the multiplier output completes the crosscorrelation computer. The average value is obtained by passing the multiplier output through a low pass filter consisting of one or two long time constant RC sections.

3.5 CROSSCORRELATION HARDWARE

A block diagram of this crosscorrelator is shown in Figure 3-5.

Note that one additional component appears in this diagram, specifically, the readout commutator. This is necessitated by the design of the shift register. To delay several seconds of information arriving at rates of the order of hundreds of bits per second requires a shift register of considerable capacity. For example, in the system under discussion, it is necessary to obtain ten seconds delay at an information rate of 100 bits per second. This requires a shift register with the capacity to store 1000 bits of information. A shift register of this size assembled from flip-flops or magnetic cores proves to be quite expensive, and the degree of flexibility afforded is not required. Consequently, the shift register is mechanized as a precessing circulating register on a magnetic drum. All of the information stored in the circulating register is read out during each revolution of the drum. The

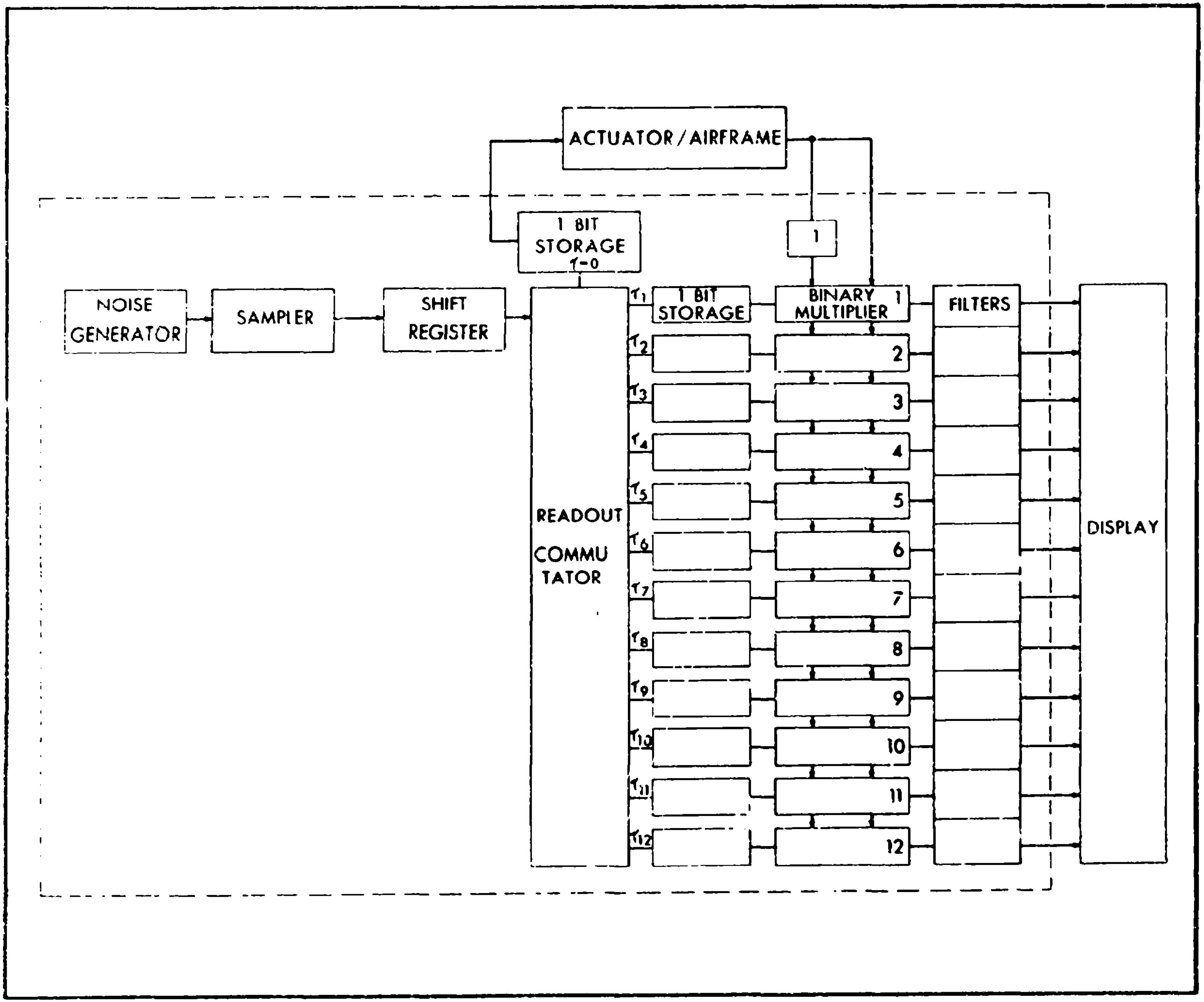


FIGURE 3-5. BLOCK DIAGRAM - CROSSCORRELATOR

readout commutator is required to distribute the proper bits to the output storage flip-flops.

Figure 3-6 is a photograph of a breadboard model of the correlator built for use with the analog computer. Figure 3-7 is a photograph of the experimental model to be flight tested this spring. Etched circuit boards and solid-state components have been employed throughout the unit. However, no attempt has been made to optimize the airborne package since the circuit boards employed were designed for a general purpose computer. Considerable saving in size and weight could be made by employing components tailored to the application.

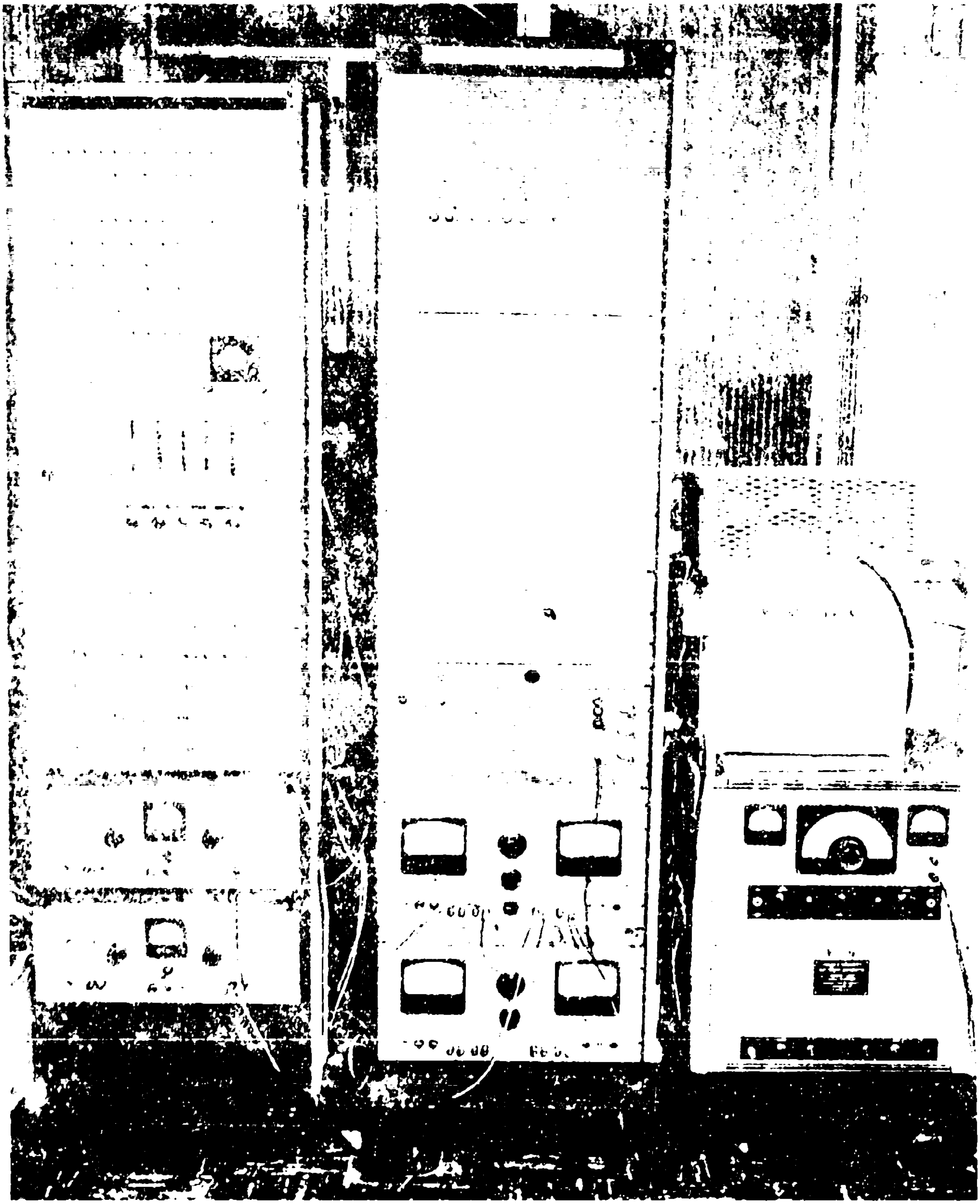


FIGURE 3-7. AIRBORNE MODEL OF CROSSCORRELATOR

SECTION 4

SIMULATION STUDIES

Simulation studies have been conducted throughout this program to support the theoretical analysis and hardware development. Several of these studies will be discussed briefly to illustrate that the Aeronutronic concept of adaptive control is feasible and that the crosscorrelator is capable of measuring the airframe response with good accuracy.

4.1 DEMONSTRATION OF ADAPTIVE PITCH DAMPER

A pitch damper with a variable compensation loop was simulated to demonstrate that adaptive control could be achieved by monitoring the impulsive response of the system. The stability augmenter chosen for this demonstration is shown in Figure 4-1. The rate of change of flight path angle $\dot{\gamma}$ is the control parameter. This type of control was chosen because of simplicity of control and the fact that the closed loop response was predominantly second order. The gain K_a and time constant T_a are controlled by the adaptive loop. The ratio of K_a to T_a is fixed at 17.5. Thus K_a and T_a can both be controlled from one shaft. The actuator is represented by a single lag with a time constant of 0.05 seconds. The airframe is represented by the short period mode with a natural frequency which varied from 2 to 12 rad/sec within the flight regime chosen. Note that the damping of the dominant roots increases with increasing gain. For the conditions simulated, the compensation network gain varied from 0.2 to 3.5 in order to keep the dominant root damping ratio at 0.5.

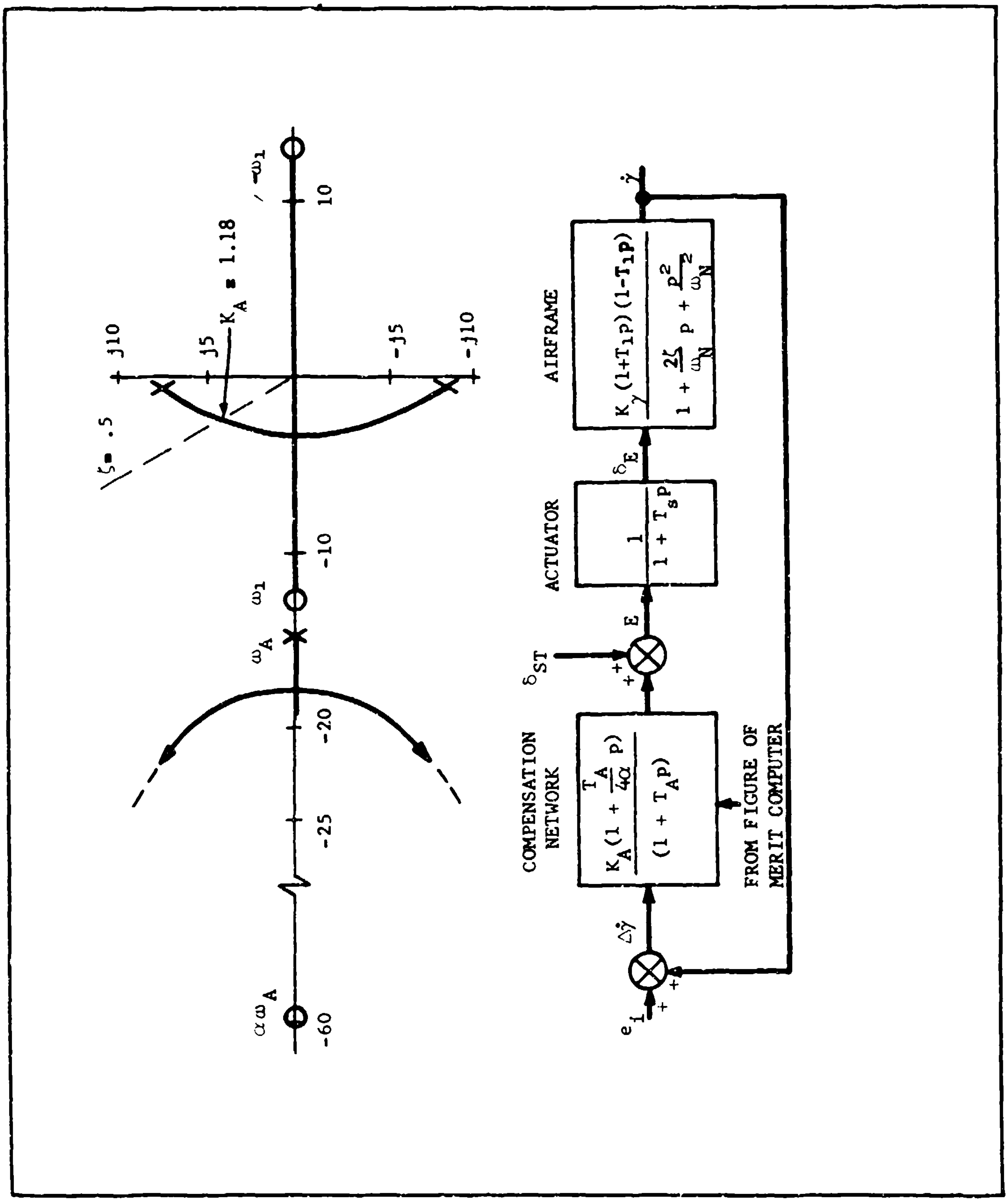


FIGURE 4-1. $\dot{\gamma}$ STABILITY AUGMENTATION

The self-adaptive system block diagram is shown in Figure 4-2. The system impulsive response is obtained by forcing the system with impulses supplied by a pulse train generator. The repetition rate of the pulses was chosen so that the system response decays to zero between successive pulses. The Figure of Merit Computer measures the system damping ratio. When the damping is greater than the desired 0.5, the F_M signal is positive and visa versa. The F_M signal is sampled at the end of each pulse period and held until the end of the following period. Just prior to the end of the following period and after sampling, the Figure of Merit Computer output is reset to zero and a new cycle begins. The Compensation Controller controls the rate of change of K_a and T_a. This rate is directly proportional to the output of the sample and hold circuit, F_{Ms}.

The system operation is actually an iteration process: Assume that initially the system is overdamped, that is, K_a and T_a are too large. When the initial impulse is applied, an overdamped response results and the Figure of Merit is positive. This signal is held during the following cycle and the Compensation Computer decreases K_a and T_a at a constant rate. This continues until the damping ratio is 0.5 and F_M is zero. Thereafter K_a and T_a track the changes in damping ratio as the aerodynamic conditions change.

The results of a climbing flight are shown in Figure 4-3. The velocity is held at Mach 1.2 while the aircraft climbs from sea level to 60,000 feet. When the compensation loop is open, a large variation in system damping is apparent. In contrast, when the compensation control loop is closed, a fairly uniform $\dot{\gamma}$ response is obtained when traversing the identical flight trajectory. Thus an adaptive pitch damper has been achieved.

4.2 CROSSCORRELATOR CAPABILITIES

The crosscorrelator is the heart of the Aeronutronic adaptive autopilot concept. If the ASI system is to function properly, the crosscorrelator must measure the impulsive response of the system continuously and accurately. Simulation techniques have been used to demonstrate that this can be accomplished.

The accuracy of the crosscorrelator was determined by crosscorrelating a second order system and making twenty independent measurements of each correlation coefficient. Typical results are shown in Figures 4-4 and 4-5.

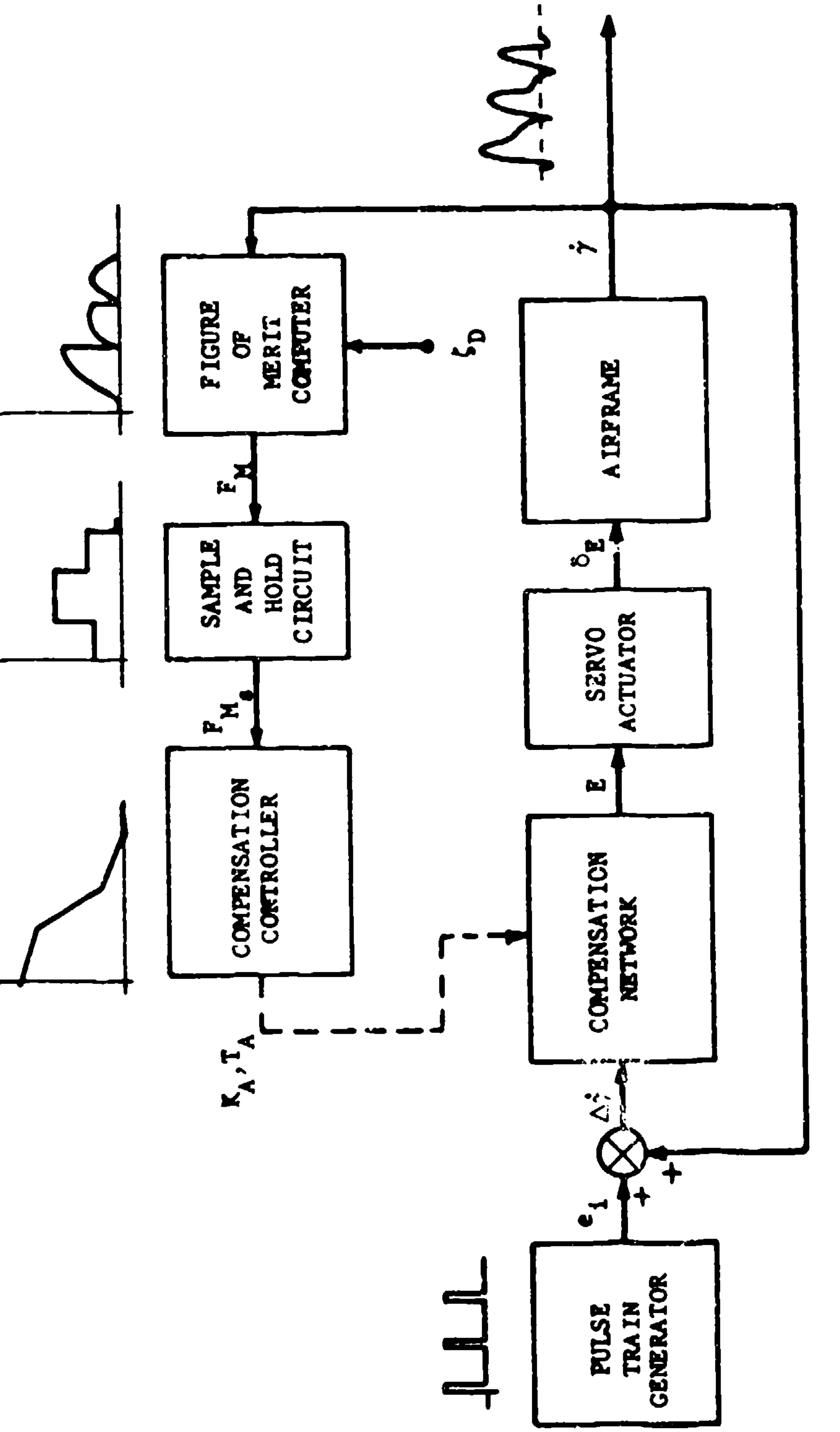


FIGURE 4-2. BLOCK DIAGRAM - IMPULSE EXCITED SOAP SYSTEM

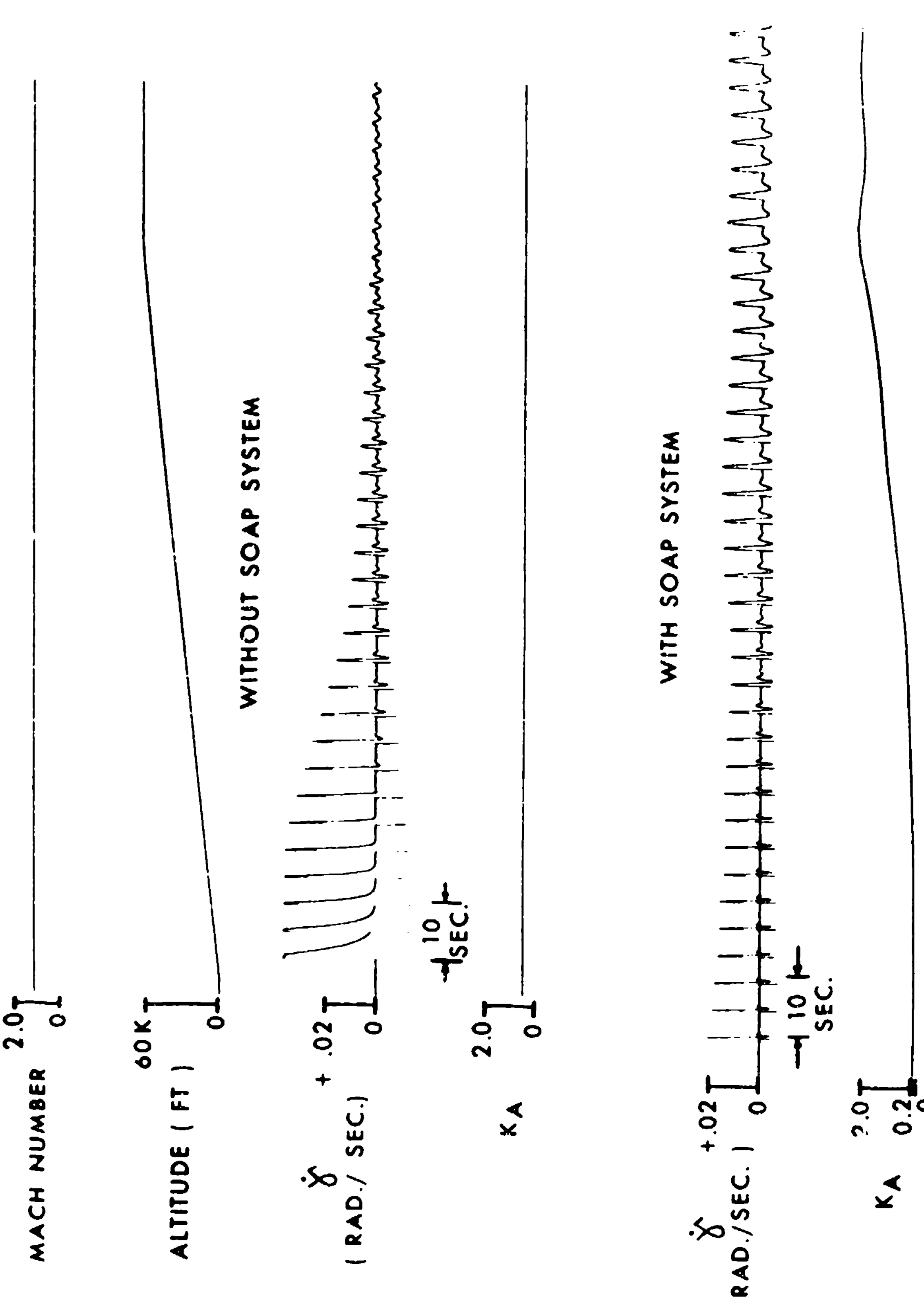


FIGURE 4-3. CLIMBING FLIGHT USING IMPULSE EXCITED SOAP SYSTEM

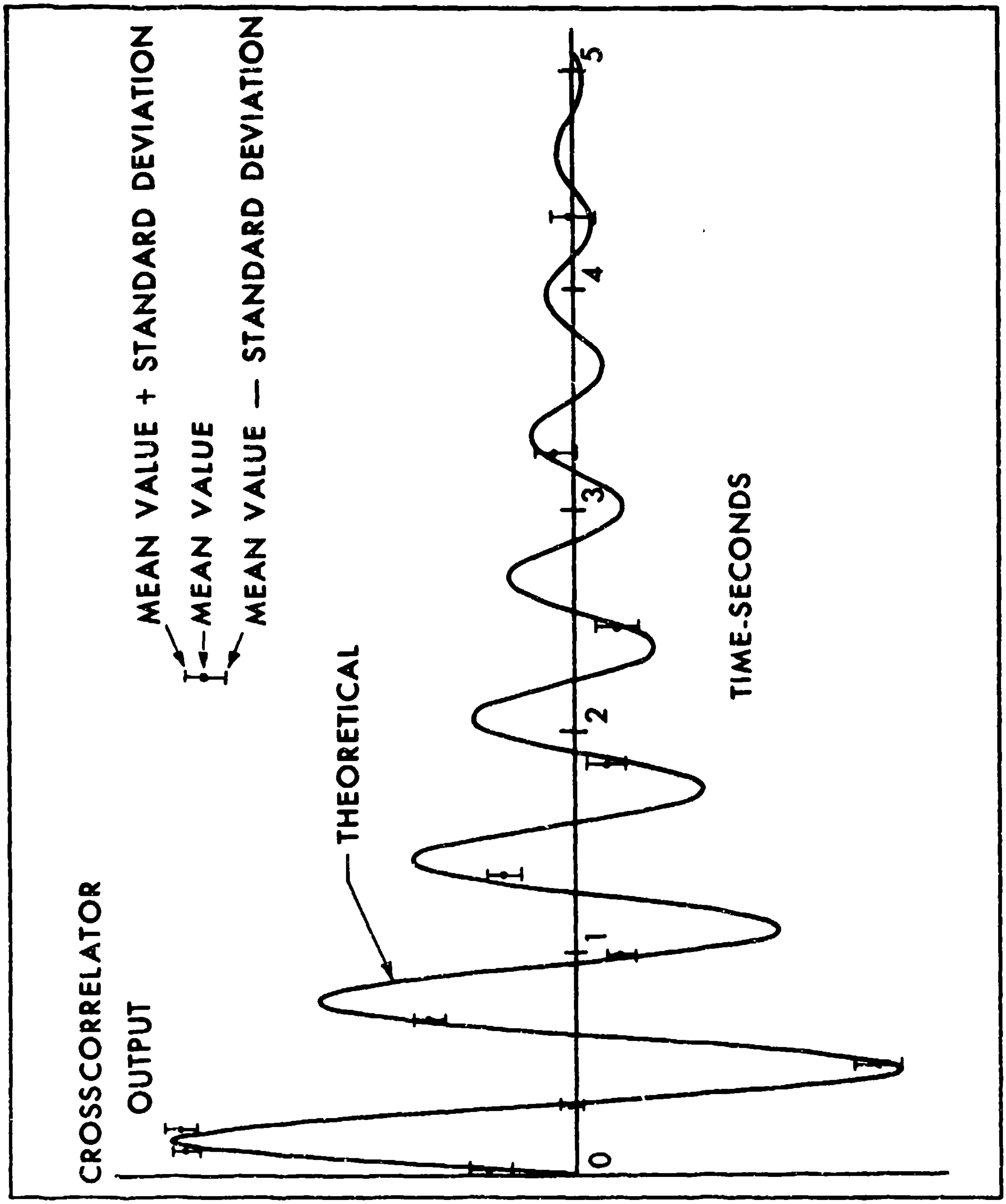


FIGURE 4-4. IMPULSE RESPONSE OF SECOND ORDER SYSTEM WITH ω_n = 10 RPS and ζ = .1

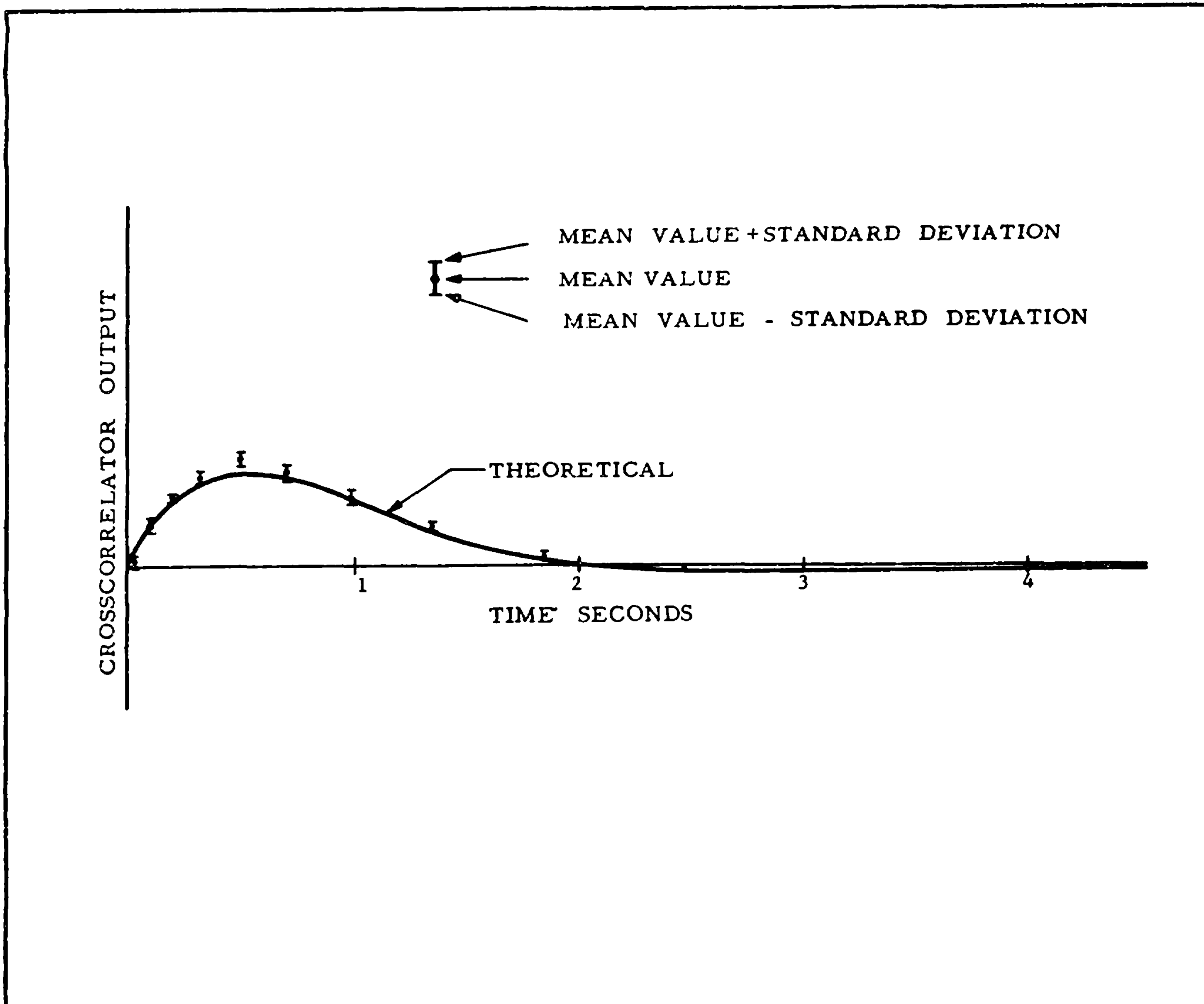


FIGURE 4-5. IMPULSE RESPONSE OF SECOND ORDER SYSTEM WITH ω_n = 2 RPS AND ζ = .7

Figure 4-4 shows the impulsive response of a second order system with a natural frequency of 10 rad/sec and a damping ratio of 0.1. The dots are the mean value of the twenty readings of each coefficient. The two bars above and below each point are plus and minus one standard deviation about each mean. Figure 4-5 is the same type of presentation but for a second order natural frequency of 2 rad/sec and a damping ratio of 0.7. The standard deviation is always less than 10 per cent of the maximum amplitude of the impulsive response and is about 5 per cent in most cases. The measured variance of the correlation coefficients is almost entirely due to the statistical nature of the correlation signal and was predicted from theory. The crosscorrelation equipment does not cause any appreciable error.

ASI is currently engaged in preparing to flight test the crosscorrelator. Studies have shown that it would be highly desirable to obtain data in the air which could be used as a check of the crosscorrelator. However, the nonlinearities of the actuator plus mechanization difficulties make it undesirable to attempt to apply impulses to the control surface and thus to obtain the impulsive response of the system. On the other hand, the airframe response to a small step input is easily mechanized. The system response can be measured by the crosscorrelation method by inserting a pure integration in series with the actuator airframe and making the integrator a part of the system. Practically this is difficult, so a low pass filter with a twenty-second time constant was substituted for the pure integration.

The crosscorrelation and the step response are compared in Figure 4-6 for the simulated pitch mode of a supersonic aircraft. The crosscorrelation coefficients have been corrected for the effect of the low pass filter which makes the coefficients with the longer delays decay toward zero. Thus the crosscorrelation technique may be used to obtain impulse or step response whichever is easiest or most desirable to measure.

The effect of nonlinear components in the autopilot system has been of some concern in studying the crosscorrelator. The effect of such nonlinearities is also important in flight testing the crosscorrelator because we wish to measure the airframe response as accurately as is possible. For the above reasons a separate study was made of the nonlinear actuator which we will have in our F-100C flight test vehicle. The characteristics of the actuator are shown in Figure 4-7.

The actuator is essentially a first order lag in closed loop form. The open loop gain N_1 is nonlinear as is shown. The result is

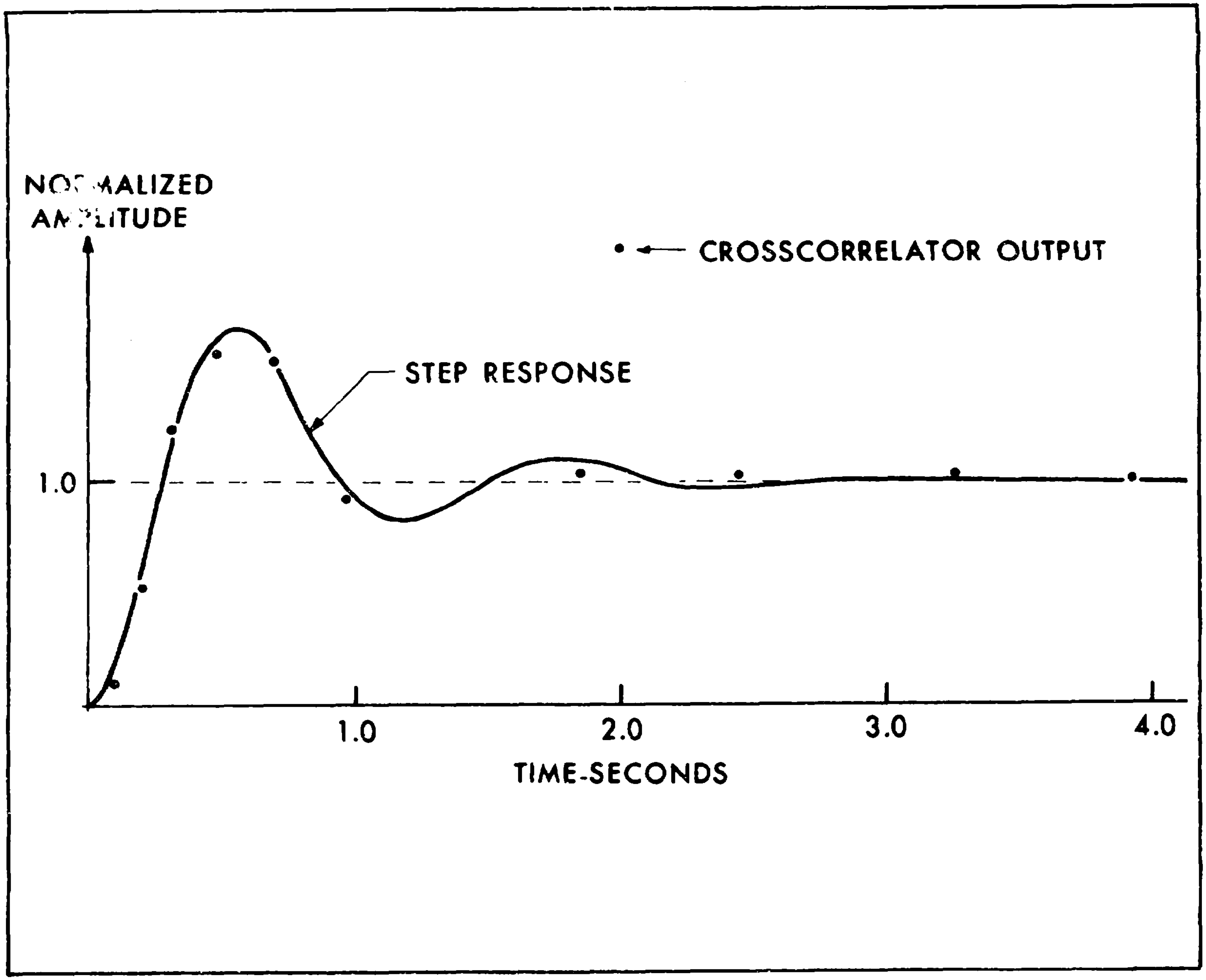


FIGURE 4-6. STEP RESPONSE OF F-100 AIRFRAME FOR LOW ALTITUDE CONDITION

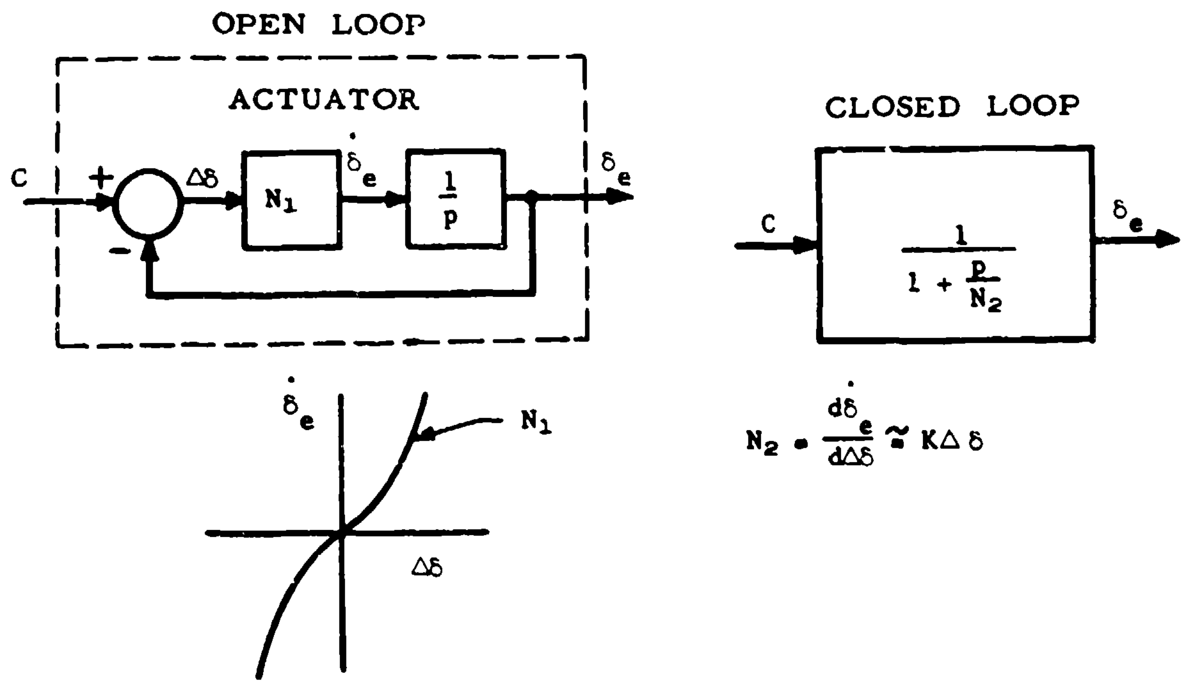


FIGURE 4-7. NONLINEAR ACTUATOR CHARACTERISTICS

that the closed loop time constant $1/N_2$ varies directly as the error $\Delta\delta$. Thus the actuator is sluggish for low level inputs and has a short response time for large inputs.

Figure 4-8 shows the crosscorrelation of the actuator for four different levels of input. Note the first order impulsive response decays faster as the input amplitude is increased. The time constant for each input amplitude was measured from the crosscorrelation coefficients and was computed from the characteristic of N_1. Figure 4-9 shows the variation in N_2 with the input amplitude C for both methods. Thus the crosscorrelator measured the nonlinear characteristic quite accurately.

The F-100C pitch damper system has been studied when backlash and dead space are present. The effect did not appreciably affect the crosscorrelation of the system. It appears that the nonlinearities encountered in the aircraft actuator will not present any problem to the crosscorrelation technique.

The use of the crosscorrelation technique was suggested by Bootan as a means of describing certain types of nonlinearities. Dr. Rideout and his group at the University of Wisconsin have been using the crosscorrelation to obtain generalized error functions for linear and nonlinear servos. It would seem that the crosscorrelation technique has considerable potential in this field.

4.3 SUMMARY

The concept of self-adaptive control by monitoring the system impulsive response has been shown to be feasible. Also the crosscorrelator has been shown to have good accuracy and to have acceptable capability in spite of nonlinearities encountered in aircraft actuators. There remains the demonstration of adaptive control with the correlator in the system. This has been done for a second order system and was reported in the technical literature*.

In the near future, the pitch damper loop will be simulated with the crosscorrelator. However, major emphasis to date has been placed upon the components of the system such as the Figure of Merit computer, the control of natural frequency, and the development of a periodic noise generator.

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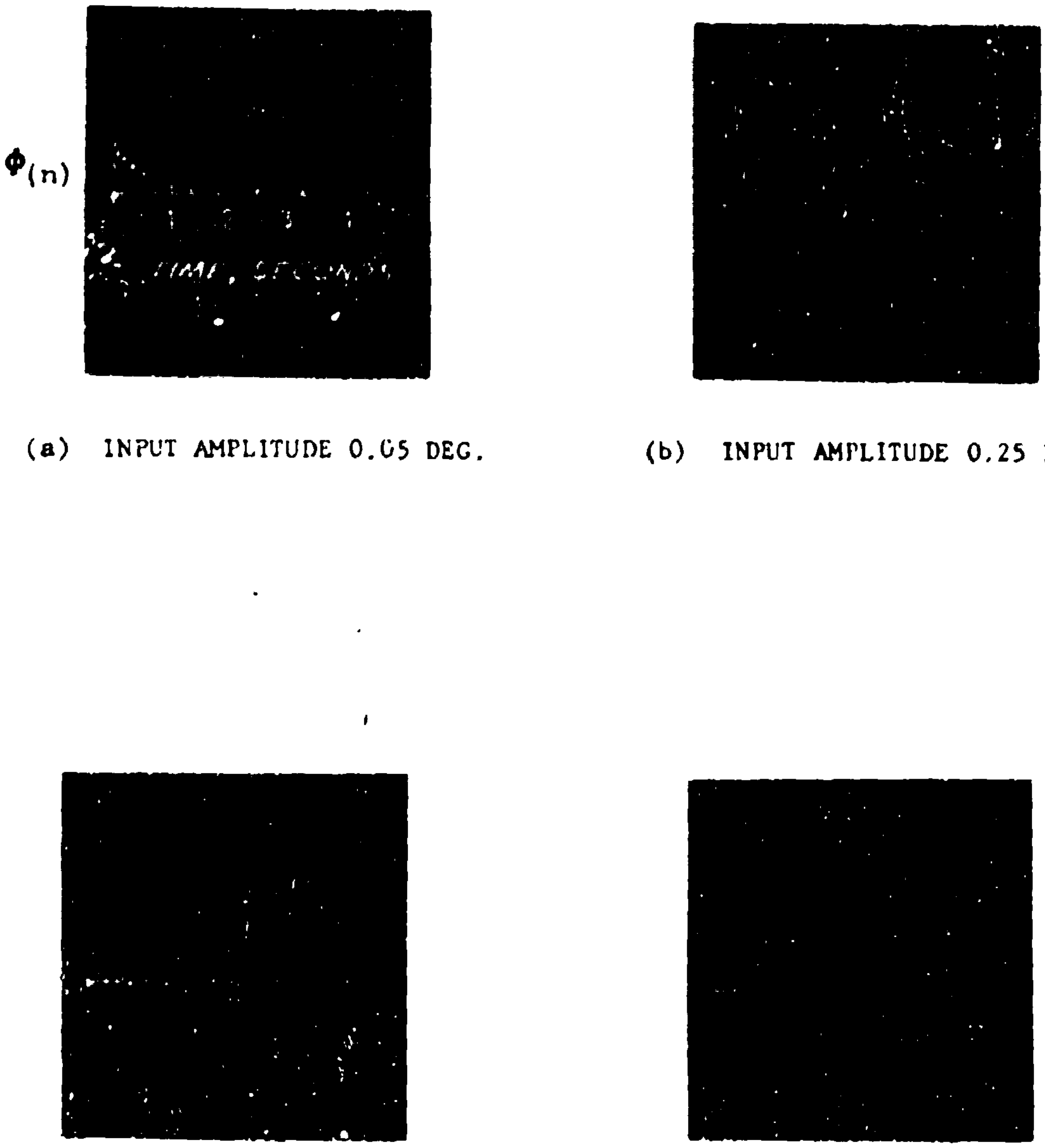
* 1958 IRE National Convention Record, Part 4 "A Self-Adjusting System for Optimum Dynamic Performance"



(a) INPUT AMPLITUDE 0.05 DEG.

(b) INPUT AMPLITUDE 0.25 DEG.

(c) INPUT AMPLITUDE 0.5 DEG.

(d) INPUT AMPLITUDE 1.0 DEG.

FIGURE 4-8. CROSSCORRELATION OF NONLINEAR ACTUATOR SYSTEM

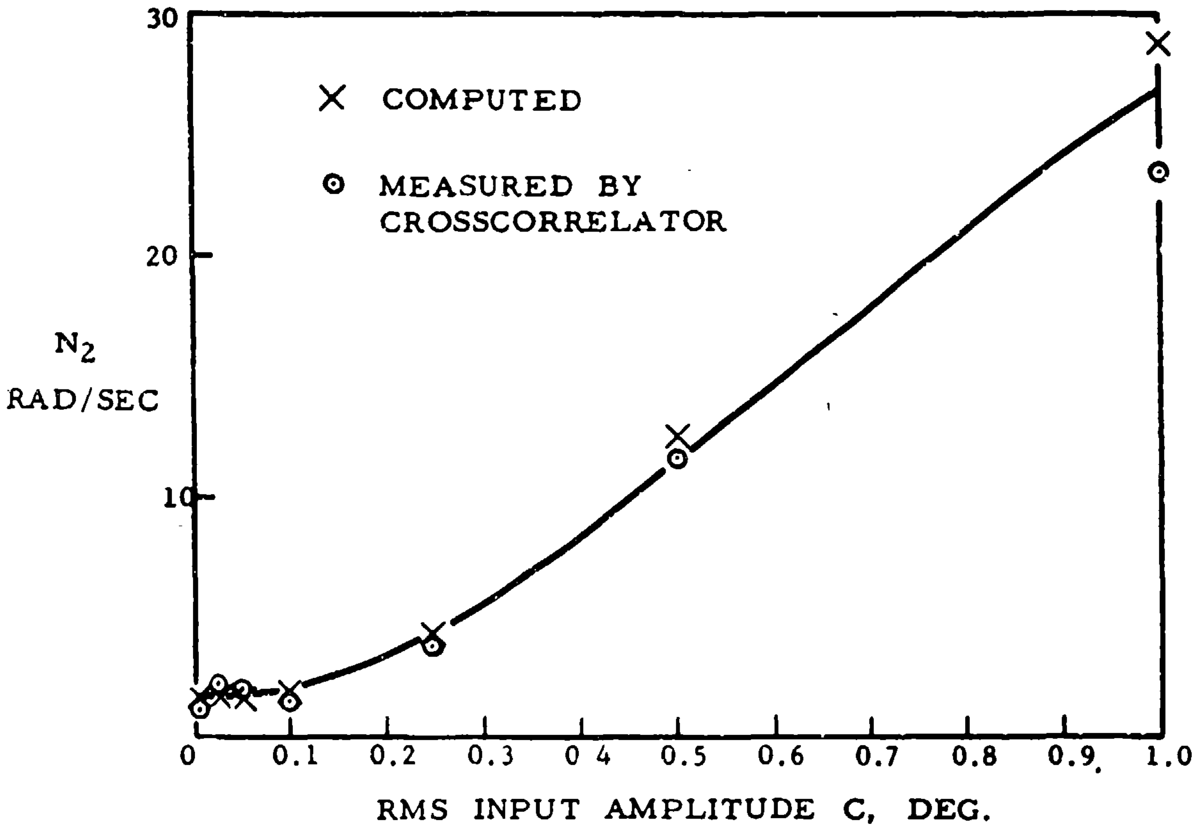


FIGURE 4-9. ACTUATOR TIME CONSTANT VS. INPUT NOISE AMPLITUDE

ADAPTIVE CONTROL SYSTEMS PHILOSOPHY

Dr. Yao Tzu Li
Associate Professor of Aeronautical Engineering
Massachusetts Institute of Technology

INTRODUCTION

Literally, an adaptive control system or self-adapting system is a control system which can produce a desirable performance under adverse changes in the operating conditions. To be sure, all control systems are designed to have some degree of adaptive ability. The glorified title of this new interest in the instrumentation field is, therefore, to emphasize the need for more adaptive ability in a system which did not have enough.

To accomplish this object, within the capability of the primary components, one may have to use several known schemes instead of only one, as in the case of simple control systems. When combination schemes are used in a system, a knowledge is required of the exact role and proportion for each scheme so that the overall performance may be improved while the physical components of the system may actually be simplified. The object of the present paper, therefore, is to establish from a basic philosophical point of view most of the possible schemes and arrange them in a general pattern. Some of the schemes are devised in existing works. By inquiring into basic philosophy several new schemes are thereby generated. With the classification of the design philosophy the true merits and possible limitations of each scheme can also be compared.

OBJECTS AND METHODS OF ADAPTIVE CONTROL SYSTEMS

Control systems may be classified according to the objective for which they will be used. For instance, a control system may be desinged to yield an output following an input with a minimum steady state error as in the case of an ordinary positional servo. It may also be designed to make some other physical operating system yield an optimum efficiency index as in the case of the cruise control of an aircraft. All these systems are designed to accomplish specified objectives under changing operating conditions and are therefore adaptive systems or self-adapting systems. But the current practice is to associate the name " adaptive control" only with the dynamic response of a control system. This is because most of the simple feedback systems we now have can provide enough self-adaptive capability for static performance, but not enough for dynamic response. At the same time, with the advance of modern aircraft and missiles, the system must go through an environmental change much larger than in conventional situations so that tighter specifications regarding the dynamic response of any individual system involved are

needed. Application to aircraft and missiles, in fact, has stimulated the growth of current interest in adaptive systems. People in this field recognize that an adaptive system is identified with the dynamic response of a control system. But to people in many other fields some confusion does exist as to the exact meaning of an adaptive system. For this reason, Table I is prepared to show the classification of adaptive control systems based upon their objectives and the possible methods that may be used to accomplish these objectives.

The objectives as listed in Table I are three, but these are neither exhaustive nor mutually exclusive. The first objective is to obtain the static performance and this is usually accomplished by feedback. Comparable results may be obtained by programming the interferences and the input. The second objective is to make some other physical operating system yield an optimum performance as measured by such factors as efficiency or economic index. For this type of control, the optimum performance is often related to the control inputs in a nonlinear manner and exhibits an optimum condition which changes with environmental effects. Programming and optimalizing systems are two possible methods to accomplish this objective. The third objective is to control the dynamic response of a system. Table I shows a few possible schemes that can be used to improve the self-adaptive ability as applied to dynamic response. A more detailed treatment is discussed after a brief review of the characteristics of the reference system.

REFERENCE SYSTEMS

Once the objective is fixed in an adaptive system, the next thing to be determined is the reference standard and the associated specifications regarding the deviation of the actual performance from the reference standard. For static performance, the reference is usually the input. For economic index, the reference is the best possible performance. For dynamic response, the reference may be described in the form of a reference system either analytically or represented by an analogue model. In addition to the reference system, a typical input test function and a method to measure the deviation of the actual system output from the reference system output may be needed. For example, by the use of a random signal as a test input and the use of correlation methods, one can establish the dynamic characteristics of the actual system or the reference system. For most analytical works the performance of the reference system is often taken as unity. But for the operation of a dynamic response adaptive system, the reference system should represent the most practical and feasible performance consistent with the consideration of the types of the disturbance input signal. Generally speaking, there are two types of disturbance input. The first type of disturbance input produces an output which may be considered as noise. The second type of disturbance may change the response of the system to the command input, but produces no output directly. When the disturbance is an active input signal, then the reference system should be one which yields the minimum combined error due

to the dynamic effect of the input signal plus the noise output. If the dynamic characteristics of the input and the disturbance are known in a convenient form such as a frequency spectrum then the desirable characteristics of the reference system are theoretically known. For instance when signal and noise are random in natural order, then the reference system would be the ideal filter formulated by Wiener.

When the disturbance is not a direct signal but a modifying input which changes the parameters of the actuating system, then two desirable conditions may exist. In one case, the system is forced to produce the best possible dynamic response for each environmental condition. In the other, the reference system is chosen according to the capability of the system and the characteristics of the output function under the worst environmental conditions.

ADAPTIVE CONTROL WITH LINEAR FEEDBACK SYSTEMS

Feedback is the most powerful scheme for increasing the adaptive capability of the static performance. The same principle can certainly be applied to systems for improving the dynamic response adaptability with somewhat more difficulty. For static performance, adaptive ability is achieved when the forward loop sensitivity is very high so that the overall performance is dominated by the characteristics of the feedback component. To extend this principle to dynamic response one should have a forward loop with a high sensitivity and low phase shift over the desirable operating frequency range. In the feedback branch, a system equivalent to the reciprocal of the reference system is installed as shown in Figure 1-a. Thus the overall system performance is forced to be equal to the reference system despite the possible change in characteristic of the variant parameters in the forward loop. As a variation of this scheme one may have a reference system in cascade with a unit feedback system as shown in Figure 1-b. In this latter arrangement, the feedback system should yield a performance function of unity over the entire range of operation in order to make the complete system behave like an adaptive system. This can only be accomplished when the forward loop continues to give high gain and low phase shift under the entire range of environmental conditions. One possible method to enforce this property is to use a type of compensation which introduces signals as a function of the rate or acceleration of the input to balance the forward loop lag. In doing this, one must bear in mind that if the lag is due to a certain parameter which is affected by environmental conditions, then only the part which is not affected can be balanced. For instance, if the mass of an airplane constitutes a certain forward loop lag, and a large part of the mass remains unchanged, then the lag due to this unchanged part of mass can be compensated for. When the lag due to invariant parameters is all properly compensated for, then a much faster system with a much higher forward loop gain can be established. With this modification a practical adaptive system utilizing linear feedback may be realized.

ADAPTIVE SYSTEM WITH RELAY SYSTEMS IN THE FORWARD LOOP AND A FEEDBACK TO OVERPOWER THE VARIANT PARAMETERS

As described in the last section, the system shown in Figure (1) would operate as a dynamic response adaptive system only when the forward loop gain is high. This may be accomplished by compensation. Another method is through the use of relay control. A relay control can provide large amplification of power at relatively high speed. One basic drawback of this type of controller is the inherent exaggerating of oscillation when the power drive system has second or higher order performance characteristics. The oscillation may be illustrated by Figure (2-a) and Figure (2-b). In Figure (2-a), a first order drive system is assumed. The output signal for this system is a saw tooth function bounded by the switching zone. This type of oscillation is, in general, considered satisfactory because the amplitude is no larger than the switching zone. In Figure (2-b) the output drive is assumed to be a second order system. Now the amplitude of the oscillation is larger than the switching zone. Since an airplane is usually a higher order system, it would show a rather exaggerated oscillation when controlled by simple two-way relay system. For this reason when a relay system is used some scheme must be incorporated to limit the oscillation; and the oscillation as shown in Figure (2-a), with the switching zone as a limit, may be considered as the ideal condition. When this is fulfilled, assuming the switching zone can be squeezed down smaller than a given tolerance, we have an adaptive system for all environmental conditions in which the output drive system has enough power to maintain the small oscillation straddling the ideal output.

The above discussion shows that for higher order drive systems, such as in airplanes, the use of relays as a means to get fast power amplification must incorporate schemes to minimize the oscillation. The ideal result is to make the relays approach a fast first order system. By emphasizing fast speed, it insures the ability for the output to straddle the ideal output when the latter is making a fast change.

To illustrate the possible schemes for minimizing the oscillation of a relay control used with higher order drive systems, a concept block diagram of this control system is shown in Figure (3). In this diagram, the command and the output is fed into a computer which in the simplest form would be a comparator. The output of this unit controls the timing of the relay. In the simplest form, a two position relay is usually used. The power is supplied from a source through a magnitude adjusting device. The output from the drive system is split to show an average output per cycle and the associated oscillatory component.

The concept block diagram of Figure (3) illustrates the basic characteristics and ingredients involved in the operation of a relay servo. One primary requirement is to keep the average output within the switching zone.

The other is to minimize the oscillation. Oscillation is caused by the dynamic characteristics of the drive system and is affected by the magnitude of the drive power and the relay switching timings. To achieve the design objective of a closely controlled average output and minimized oscillation, the following schemes may be used.

1. Programming

a. Compensate for the invariant parameters of the drive system to make it behave like a fast first order system. Compensation may be done by feedback from the output signal to modify the drive power magnitude as shown by the dotted line of Figure (4). Equivalent drive power magnitude modification may also be achieved with constant drive power but gated to give an adjustable duration.

b. Use multiple relay having switch timing of fixed relationships with error functions. As a typical example, the relay action may take place when the error is equal to zero and when the error rate is equal to zero. For each relay engagement the drive power may have fixed magnitudes at various levels. The purpose of the multiple control can be shown by Figure (5). In this figure one typical cycle of the oscillation is shown. The desired characteristics are to make section " a" of the output to be driven back toward the ideal output with highest power, and section " b" to coast over with sufficient speed to minimize the time and yet without too high an approaching speed at the crossing point. A similar situation is repeated at section " c" and " d" . The overall criterion is to minimize the integrated error with some constraint such as the maximum acceleration of certain components.

c. Fixed multiple relay timing with magnitude of the drive power is programmed according to input function as shown in Figure (6). The sensitivity used in the programming is based upon the drive system invariant parameters. Figure (7) illustrates the situation for three hypothetical cases. Case " a" - the input is moving slowly and a normal output oscillation results. Case " b" shows the output lagging behind when the input experiences a fast acceleration and when only normal drive power is used. Case " c" shows the desirable output condition for similar inputs as shown in case " b" when the drive power level is properly adjusted.

d. Use fixed multiple relay timing with the drive power programmed according to the output, as shown in Figure (8).

Schemes " c" and " d" aim at the same problem, namely to make the system follow a certain command signal faster. In doing so, the power drive for the various sections of output function, as shown in Figure (5), would be asymmetrical with respect to the average output. When scheme " d" is used, the output signal used to actuate the programming should be the average

output instead of the instantaneous output. This is one reason why scheme " d" is not as easy to execute as scheme " c" .

The logic of the relay servo developed by Flugge-Lotz and Taylor, involving the switching of the feedback sensitivity, seems to be based upon an intuition to achieve the goal as outlined in scheme " b" . In their system, however, a certain part of scheme " d" is also involved, and yet is not blended in the most desirable proportion.

2. Non-Linear System with Feedback Regulation to Limit the Oscillation Amplitude

The undesirable oscillation of a relay system depends upon the magnitude of the drive power and the timing. It is possible to arrange a feedback system to regulate the oscillation amplitude to a tolerable limit by means of the following two schemes.

a. Oscillation amplitude limited by controlling the magnitude of the drive power, as shown in Figure (9).

b. Oscillation amplitude limited by controlling the relay engaging timing with respect to the error signal as shown in Figure (10). Figure 11 shows that the relay engaging timing may be shifted ahead by the following three methods.

(1) By a controlled bias added to the zero error.

(2) By a controlled bias added to the zero error rate.

(3) By a controlled delay time applied to the zero error rate.

All the above feedback schemes must derive the feedback signal from the oscillatory output amplitude. Some suitable scheme would be needed to yield a signal proportional to the amplitude. Other constraining factors such as acceleration limit of certain control members may also be added when desired.

Like all feedback systems there is a dynamic stability problem that requires limiting of the loop gain and therefore the response speeds; whereas the programmed types, as outlined in Scheme 1, are not subject to this limit. Since programmed schemes can only be applied to the nonvarient parameters, a combination of the two schemes may prove to be most advantageous.

In all the systems thus far presented, the adaptive ability of a system is achieved by a strong forward loop paired with a feedback to render the effects of the variant parameters in the drive system relatively unimportant.

Programming of the nonvariant parameters is recommended to strengthen the forward loop whenever necessary. The schemes to be described in the following sections approach the problem in a different manner. In these systems the effects of the variant parameters upon the dynamic response of the system is counterbalanced by the adjustment of some compensating parameters in the controller. The first necessary condition is the realization of such an adjustable parameter or a set of parameters which can offset the effects of the variant parameter. The next problem is to find a suitable scheme to execute the adjustment. The easiest one to be compensated for is the sensitivity of the system. This is because the overall sensitivity of the system is equal to the product of the sensitivities of various components; and a change of the sensitivity of one component can be compensated for by the adjustment of the sensitivity of some other component. But the physical parameters of a component usually effect all the dynamic characteristic parameters such as natural frequencies and damping ratios. In other words, a change of one physical parameter may shift the position of all the poles and zeros of a physical system. To compensate exactly for these effects the controller should create a reciprocal dynamic characteristic function of the system to be compensated for, so that the shifting of the corresponding parameter of the controller may counterbalance exactly the effect of the variant parameter in the controlled system. In practice this exact compensation is too complicated to be useful and some sort of approximation is always necessary. For instance, the transient response of a second order system with normal damping ratio may be approximated by the transient response of a low damped system cascaded with a first order system. Generally speaking, removing the effects of a variant parameter in a higher order system by parameters of a lower order system can usually be accomplished under the following two conditions: (1) when some of the poles of the higher order system are not effected significantly by the variant parameter, and (2) when the total effects of the shifting of the poles of the higher order system yield approximately the same effects, as the shifting of a set of poles in the controller of lower order. The cascaded first and second order system, described before, illustrates this latter condition.

ADAPTIVE SYSTEM BY PROGRAMMING OF THE COMPENSATING PARAMETER

The first possible scheme to make the necessary adjustment of the compensating parameter described in the previous section involves the use of a programmed control system which makes the adjustment based upon the actual measurement of the variant parameter or the environment conditions which affect the variant parameter. The programming control is designed based upon knowledge of the relationship between the variant parameter and the compensation parameter. Generally speaking, a precise programming control is difficult to realize; but a moderately accurate system is quite practical.

ADAPTIVE CONTROL SYSTEM BY ADJUSTING THE COMPENSATING PARAMETER THROUGH FEEDBACK

The adjustment of the compensating parameter would change the system dynamic response. If a device is available to interpret the dynamic response in terms of a performance index, and if a particular value of the performance index is known to represent the desired dynamic response, then it is simple to design a feedback system to adjust the compensating parameter to force the system performance index equal to the desired performance index. The key to the realization of this scheme depends upon the performance index generating system. Generally speaking, it is difficult to express the dynamic response in terms of a performance index on an absolute scale upon which the reference system corresponds to a fixed value. On the other hand, a reference system can be represented by an analogue model. The difference of the output between the actual system and that of the model, when both are subjected to a suitable test input, may be minimized through the adjustment of the compensating parameter. Thus an actual model is used to represent the reference system instead of a performance index. For practical reasons, the model can be made corresponding to the dominating poles of the reference system. This simplification is subjected to the same qualification as discussed before for the adjusting of the parameters. Despite this simplification, a model can represent the reference system much better than a performance index. The use of a model requires an optimalizing controller instead of a simple feedback controller.

ADAPTIVE CONTROL SYSTEM BY ADJUSTING THE COMPENSATING PARAMETER THROUGH OPTIMALIZING CONTROLLER

An optimalizing controller is one type of feedback control which makes adjustment of certain parameters of a controlled system, to force a certain output of the controlled system to reach the optimum level. The difference between an optimalizing control system and ordinary feedback lies in the fact that the optimum output level does not represent a definite output level. Without a definite desired output level, it is impossible to generate an error signal to be used in ordinary feedback control system. An optimalizing system incorporates some suitable form of test adjustment and is followed by a waiting period to observe its effect upon the controlled system and to make further adjustment after the correct direction of proper adjustment is established. For this reason, an optimalizing controller may be regarded as possessing one degree higher of intelligence than an ordinary feedback system. For dynamic response control system, an optimalizing controller may be used to adjust the compensating parameter under one of the three following conditions.

(1) When the desirable dynamic response is the best possible dynamic response a system can produce under a given environment. This

means the reference system dynamic characteristic function is unity. If a performance index signal generating system is available to convert the dynamic response of the controlled system into the performance index, then the optimalizing system may receive the performance index as the input signal to control the compensating parameter and thus close the loop. A typical functional block diagram is shown in Figure 12.

(2) When the reference system represents a compromised performance and the purpose is to make the controlled system behave exactly the same as the reference system, but, because of practical reasons the compensating parameter is not a perfect one, the performance index generating method can only give an approximate value. For these reasons, the relationship between the performance index and the compensating parameter may become quite nonlinear so that the optimum value of the performance index represents the desirable operating condition. An optimalizing controller should therefore be used for this situation instead of the ordinary feedback as described earlier.

(3) When the reference system is represented by a model and the controlled system is matched with the reference system by comparing the difference of the output function subject to some suitable inputs. Several methods can be used to compare the two output functions. All involve some sort of integration of the function of the absolute value of the difference. As a result, the best match is obtained when the difference between the functions of the two outputs is minimized. For this reason an optimalizing controller is needed as shown in Figure 13. The adaptive system designed by M.I.T. is a typical example of this type. This system seems to be the most promising type among all systems involving compensating parameter adjustment.

CONCLUSION

The above analysis shows that a large number of possible schemes may be used to accomplish dynamic response adaptability. Since the characteristics of an actual controlled system and its environment may vary very much from one to the other, it is rather difficult to draw a conclusion to say which system is definitely better than the other. A well-engineered feedback adaptive system with relay control in the forward loop, as developed by a Minneapolis-Honeywell group, and an equally recommended system utilizing an optimalizing controller, as developed by Massachusetts Institute of Technology group, have both produced satisfactory results. It is true that when changes in environmental conditions are moderately severe and the desirable output is moderately fast, many schemes can yield satisfactory results. But when the output demand is high and the environmental conditions are severe, then a real test of the capability of a given scheme is on hand.

A suitable figure of merit may possibly be established to assess the

achievement of a design with different types of input functions, various response speeds of the reference system, specified tolerance of the deviation of the dynamic response, and variation of the operating conditions. A good design is one which can achieve a high figure of merit and yet not be too complicated in construction through the use of a well balanced scheme of the various principles.

TABLE I

CLASSIFICATION OF ADAPTIVE CONTROL SYSTEMS

I. According to objectives

A. A constant static performance between command and output.

B. Optimum performance in the form of an economic index.

C. Matching the dynamic response of an operating system against that of a reference system, with the reference system dynamic response established as following:

1. A fixed reference system representing the best possible performance for the entire range of operating conditions.

2. A fixed reference system representing the best possible performance for the worst operating conditions.

3. A reference system with performance programmed according to the maneuver requirements of the command signal and the capability of the drive system.

4. A reference system with performance programmed according to the types of command signal and active interference signal.

II. Possible schemes for the matching of the dynamic response of an operating system against that of a reference system.

A. Use feedback to overcome the effects of the variant parameters.

1. Linear system with compensation for invariant parameters.

2. Use relay control to manipulate the drive power.

a. Use various programming methods to minimize the inherent oscillations.

b. Use various schemes of feedback to minimize the amplitude of oscillation of the limit cycles.

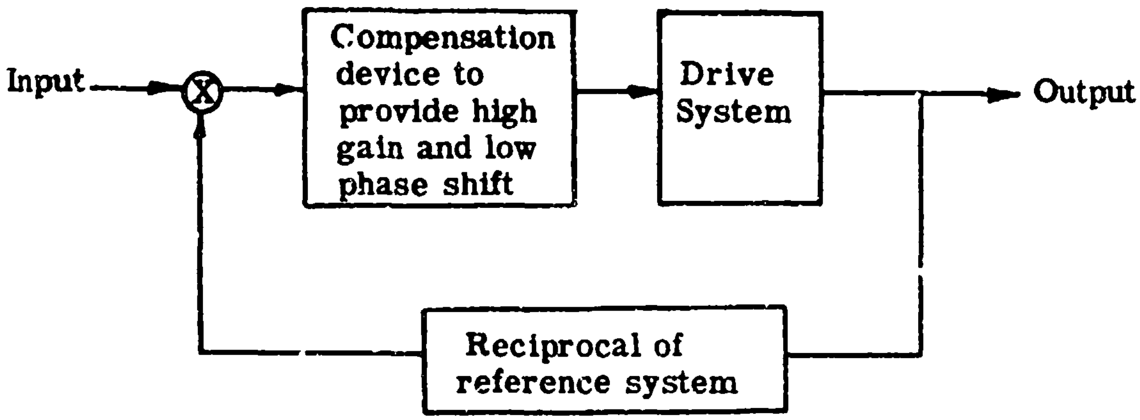
B. Use adjustable parameters to compensate for the effect of the variant parameters.

TABLE I (CONT'D)

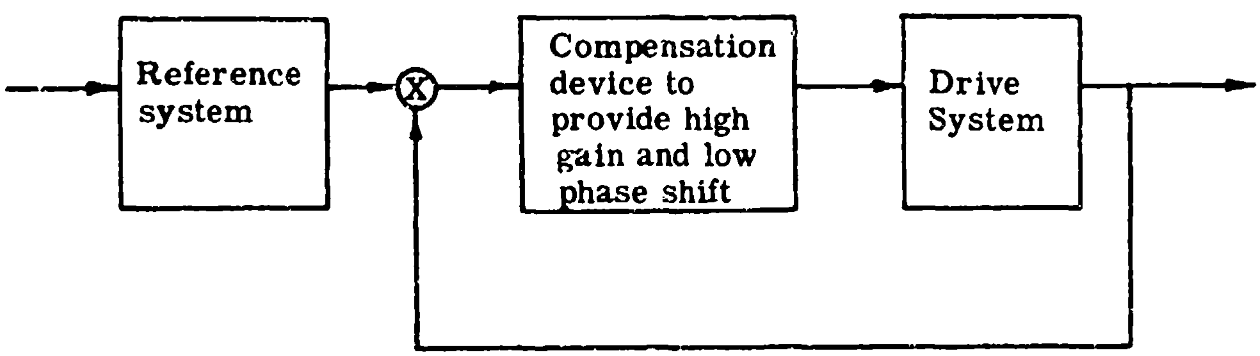
1. By programming with suitable computor.

2. By linear feedback with suitable dynamic performance index indicator.

3. By optimalizing system, with either a suitable dynamic performance index indicator or a reference system model.

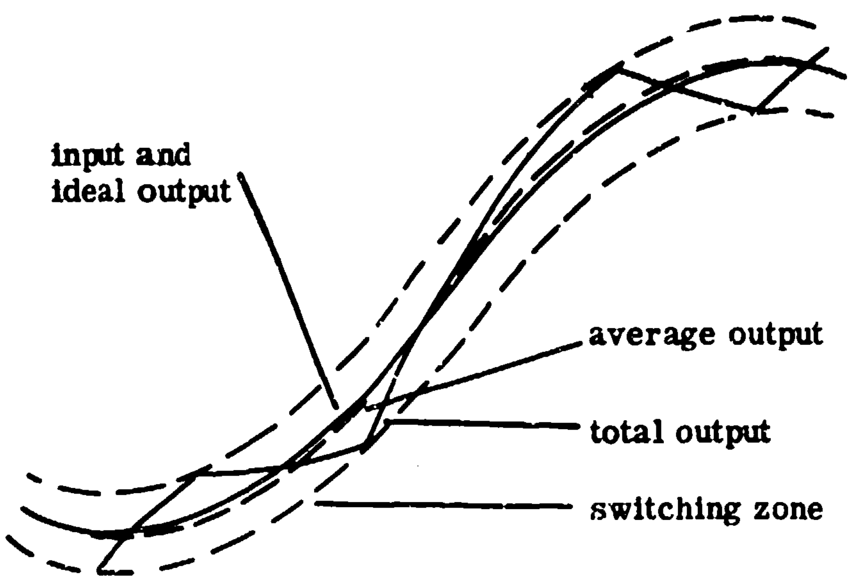


a. Use reference system in feedback loop.

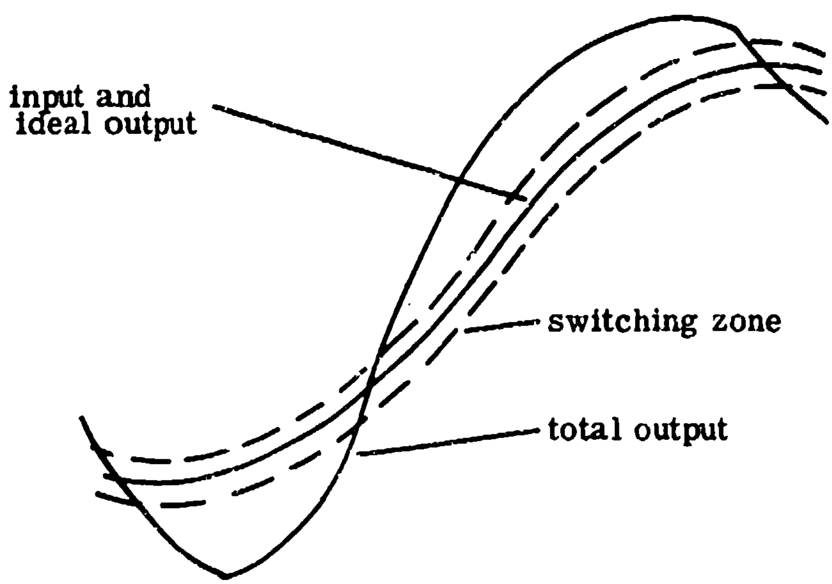


b. Use reference system in cascade with a unit feedback system.

Fig 1. Basic schemes using feedback to overcome effects of the variant parameters in the drive system.



a. Relay control with first order drive system



b. Relay control with high order drive system

Fig. 2 Input-output relationship with relay control system for different types of drive systems.

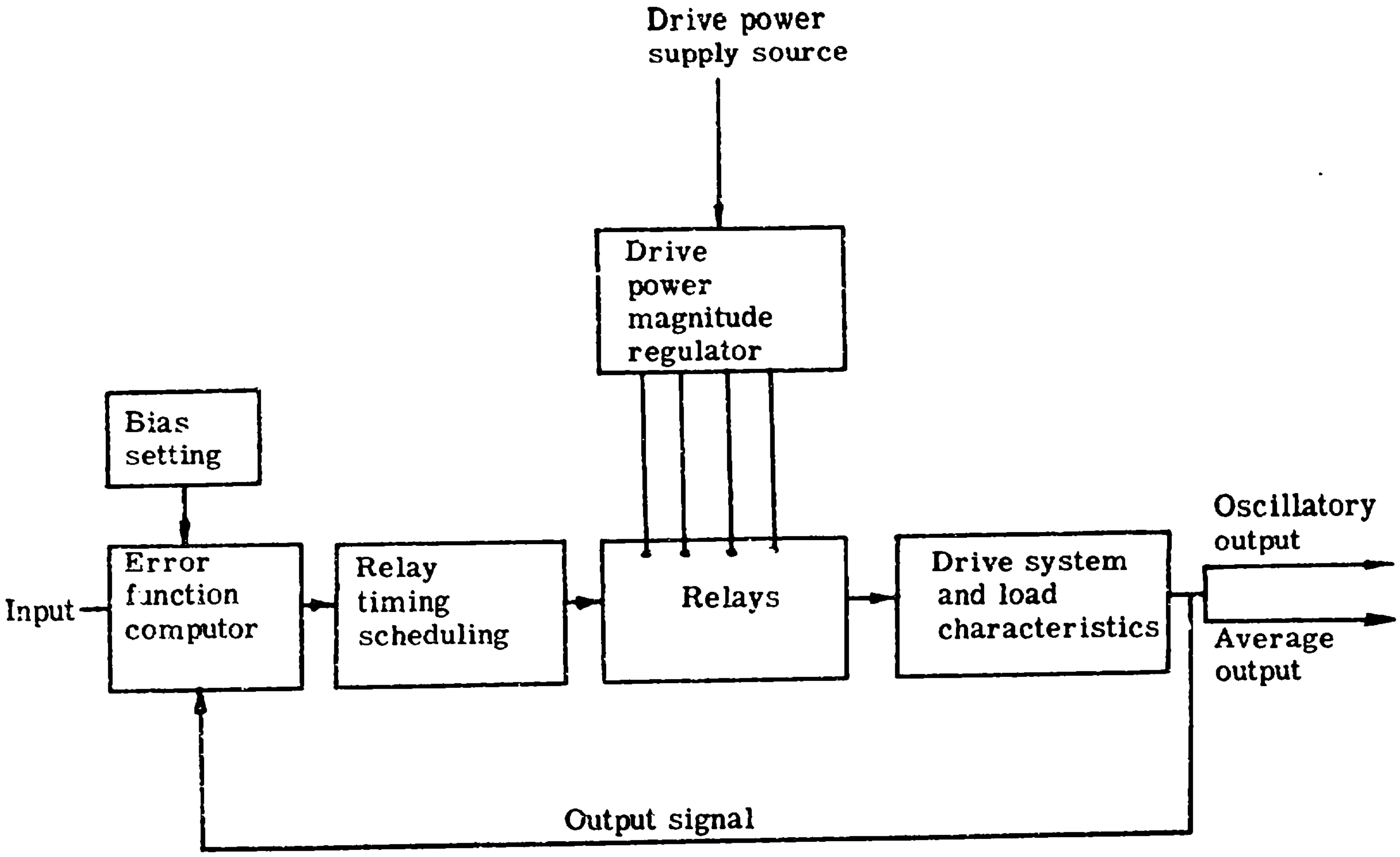


Fig. 3 Basic concept block diagram of relay control system

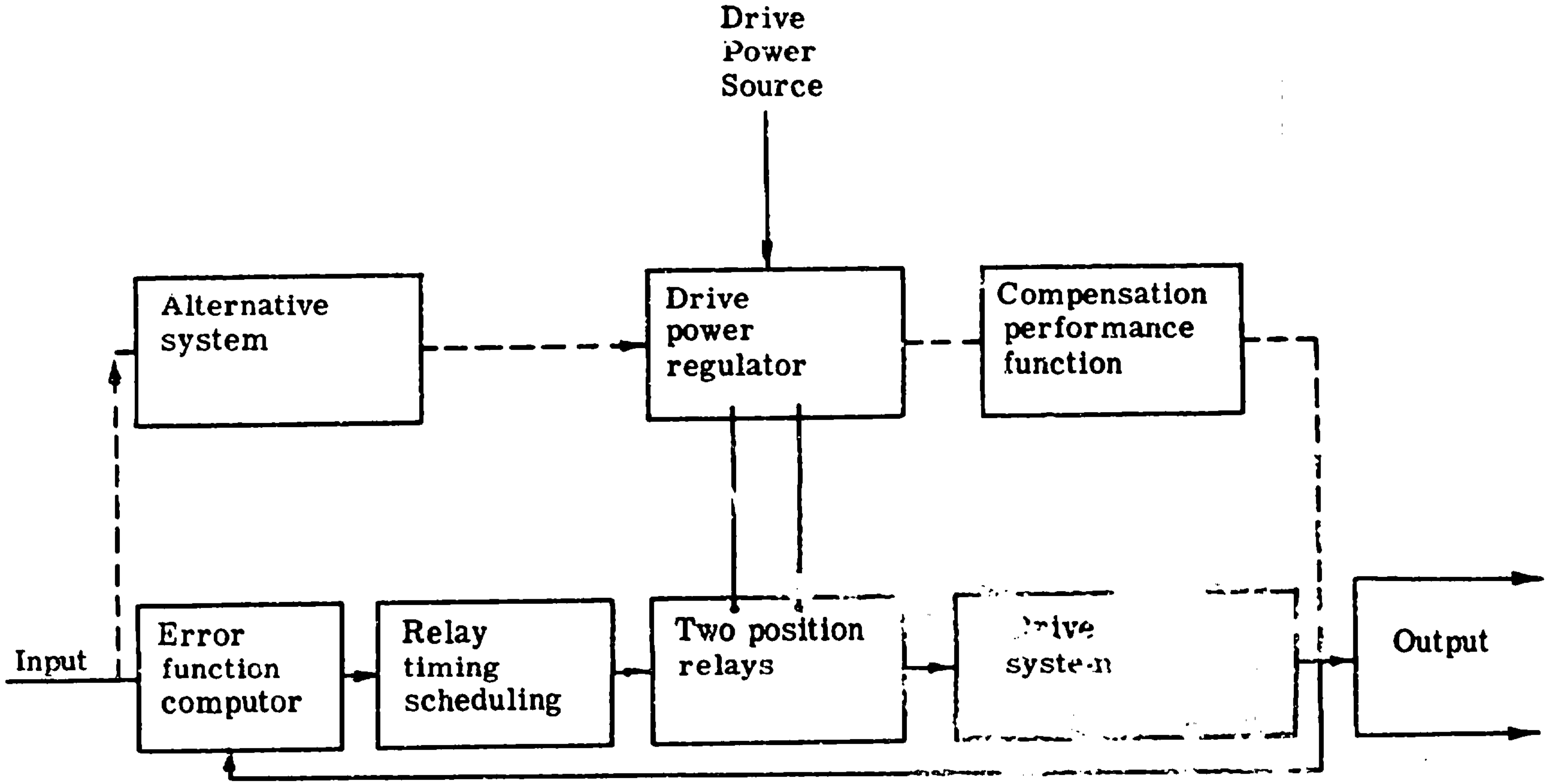


Fig. 4 Block diagram to show the use of feedback from output or input to compensate for invariant parameter and thereby make the system performance function approach that of first order system between switching

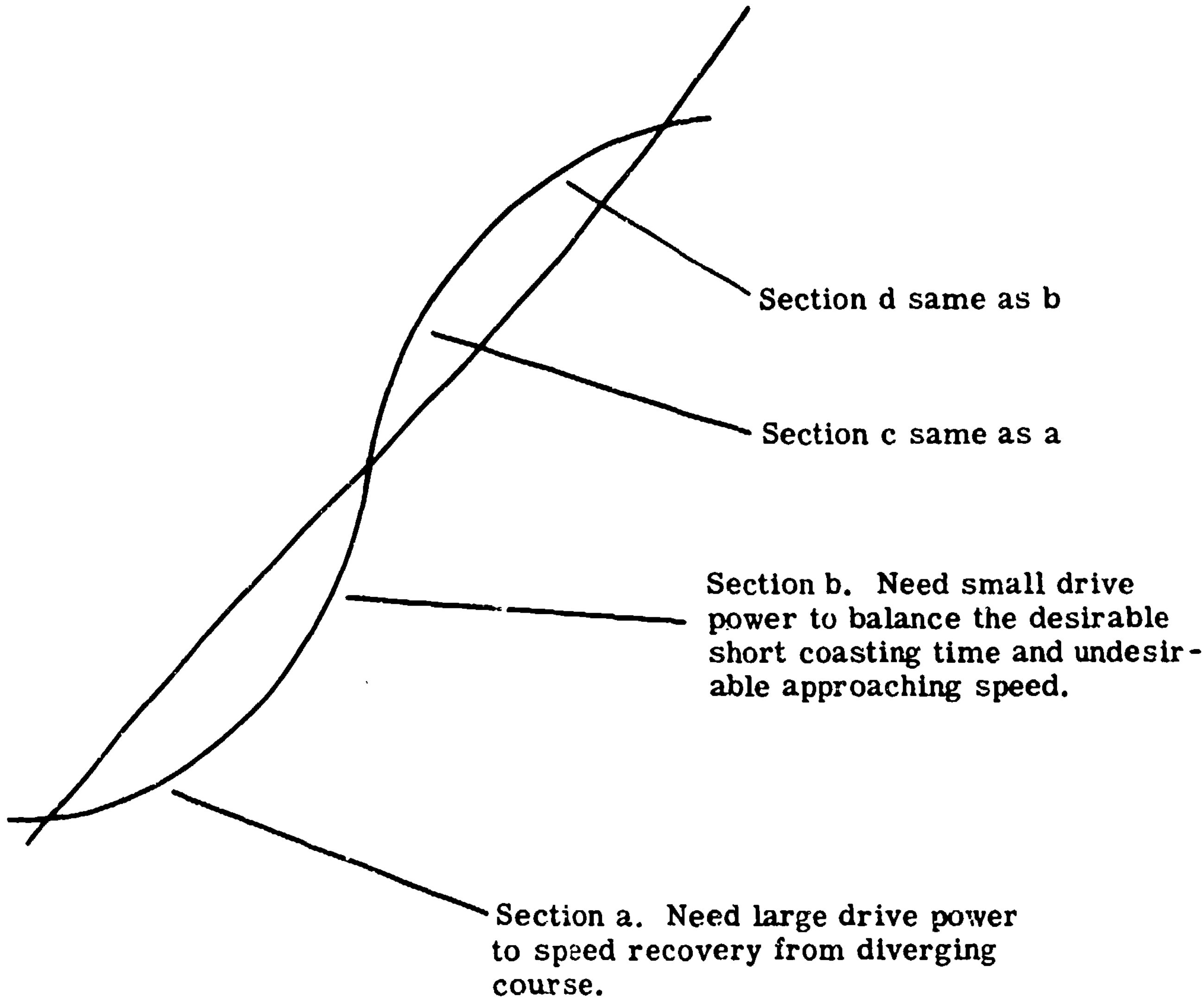


Fig. 5 Power requirement at different section when four way relay is used.

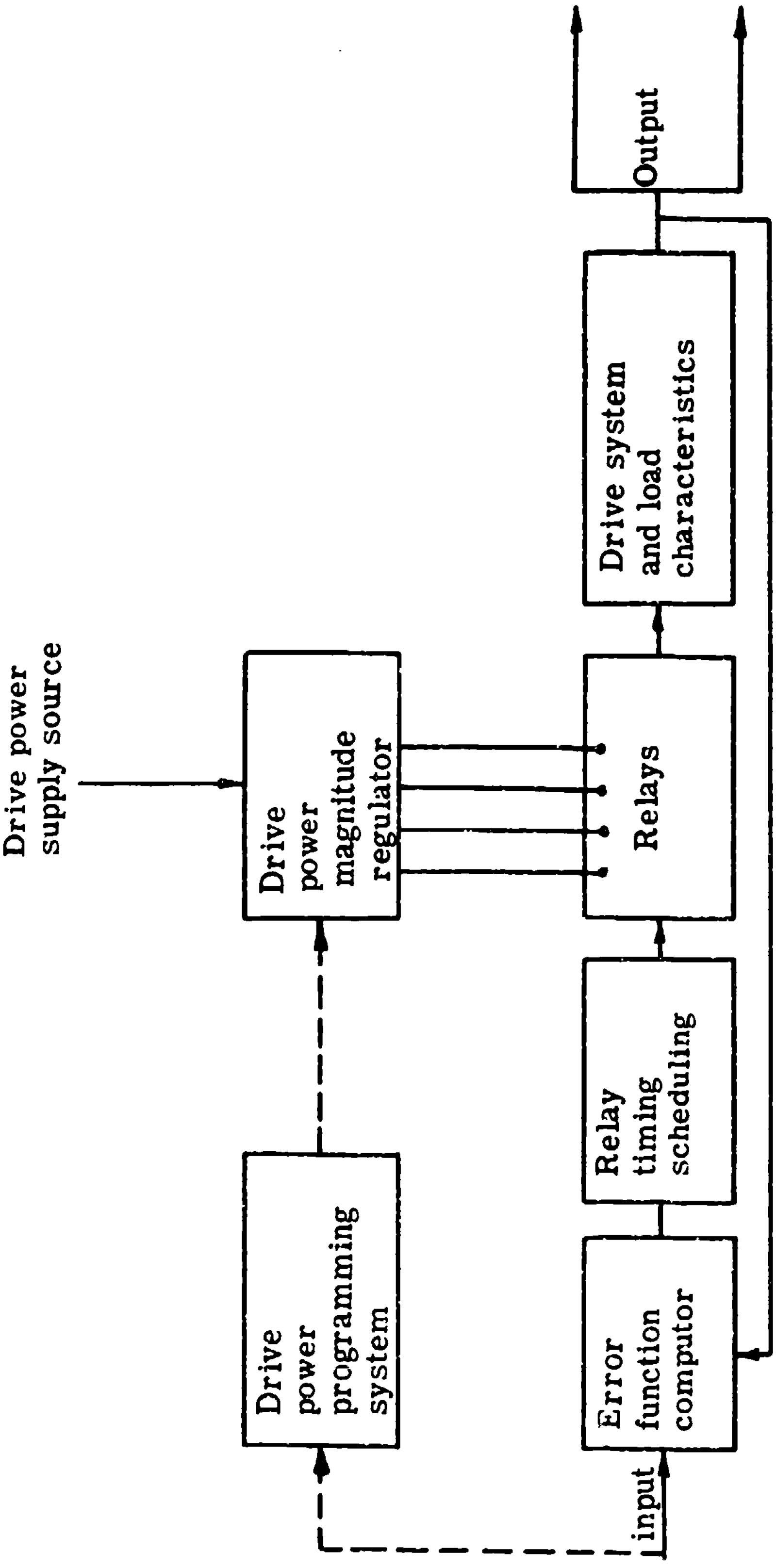
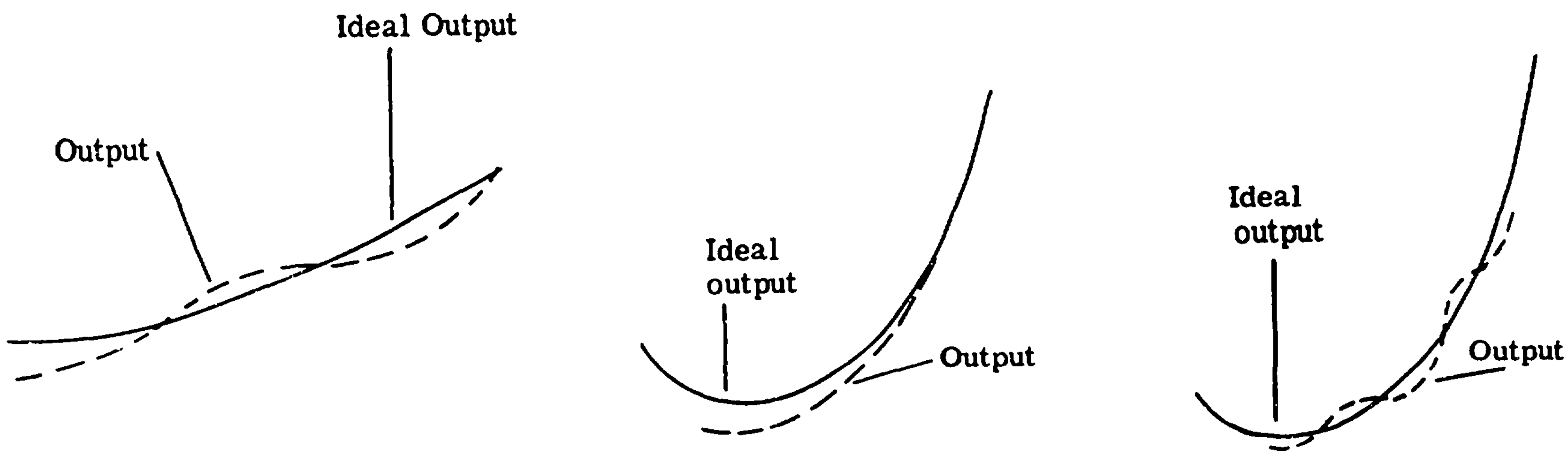


Fig. 6 Multiple relay with drive power magnitude programmed according to the input function.



a. Low load demand and low drive power
b. High load demand and low drive power
c. High load demand with programmed drive power.

Fig. 7 Output oscillation with drive power programmed according to input function.

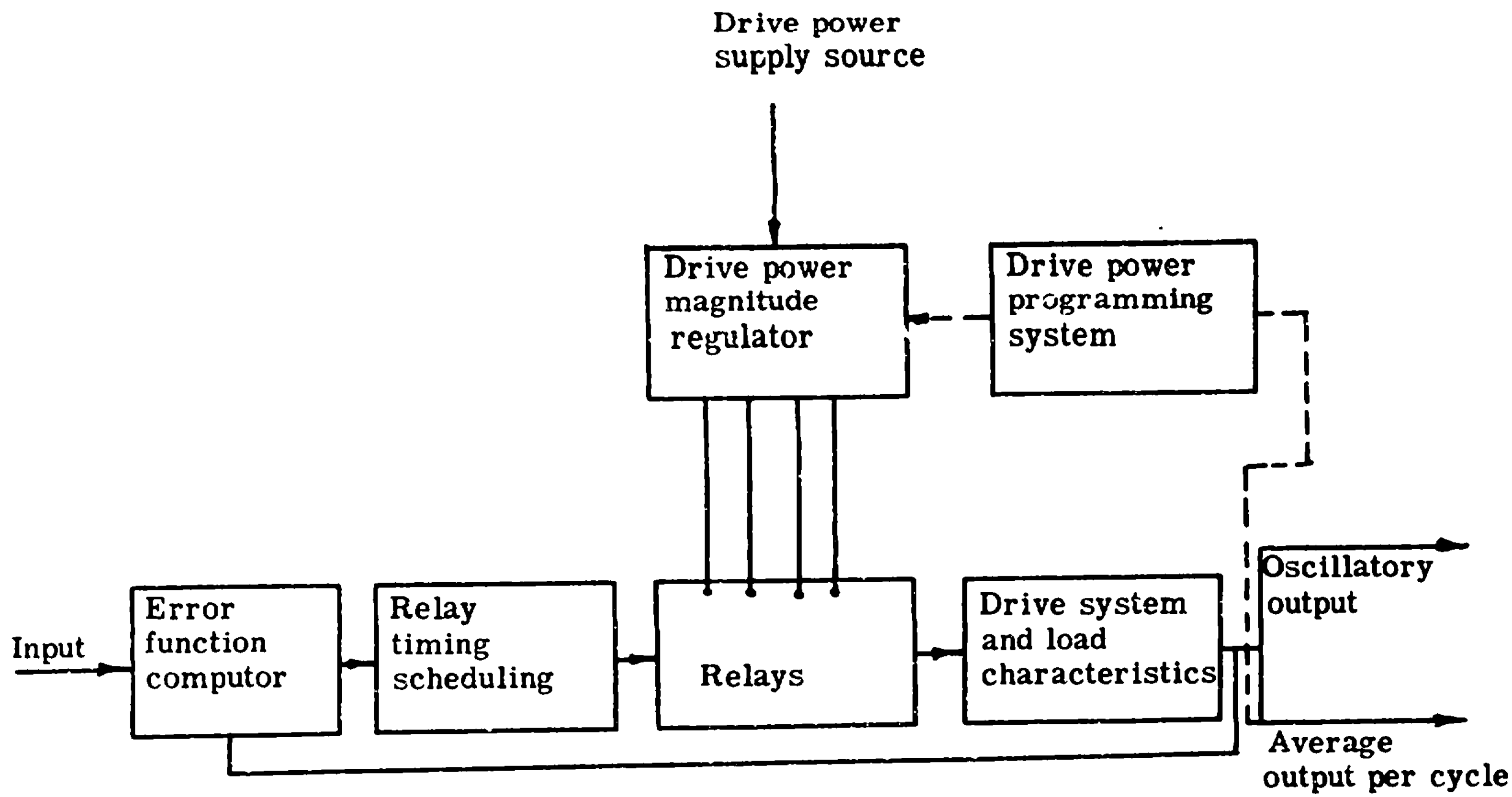


Fig. 8 Multiple relay with drive power programmed according to the average power per cycle.

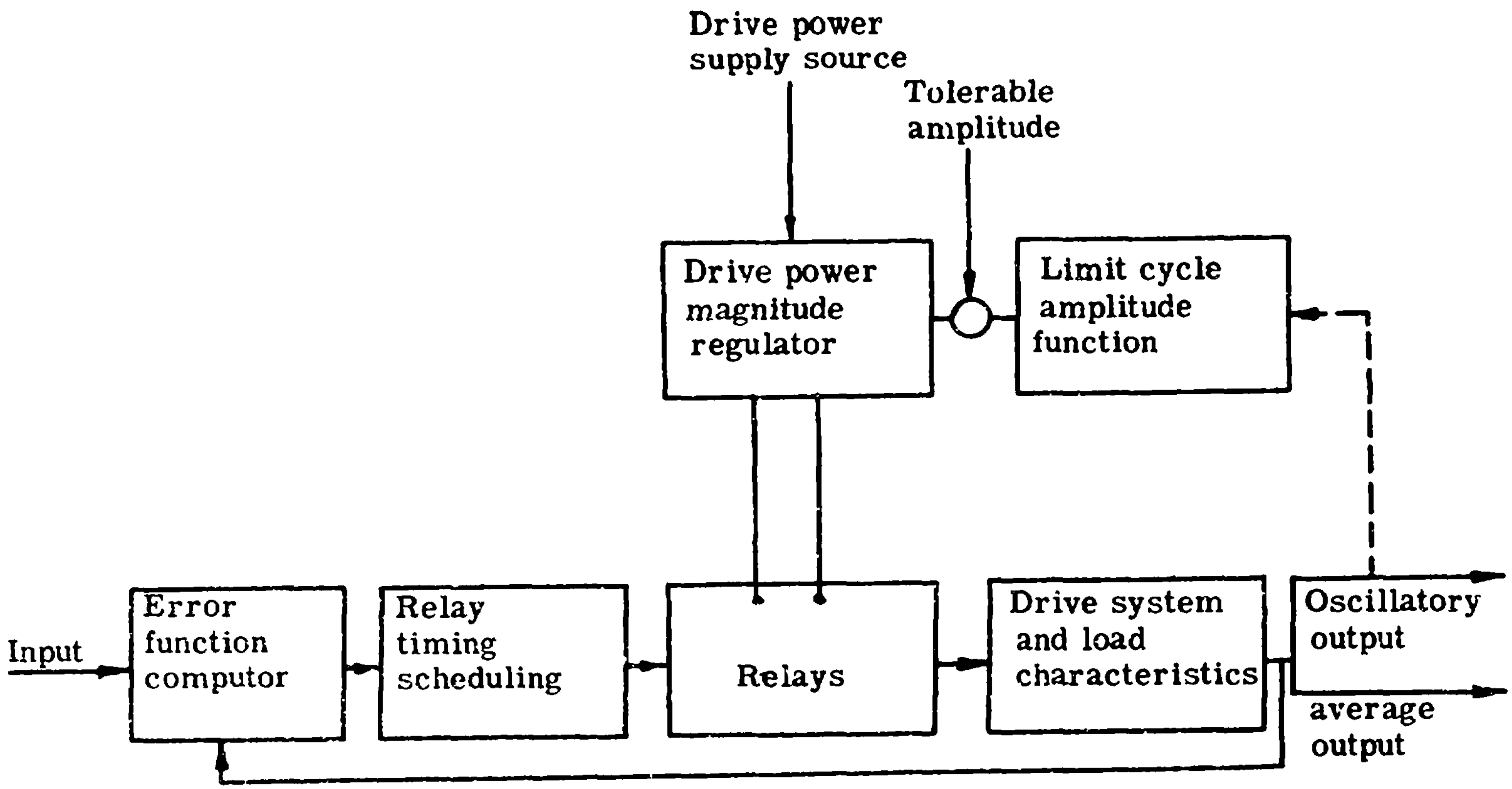


Fig. 9 Drive power magnitude adjustment through feedback of amplitude of oscillatory output.

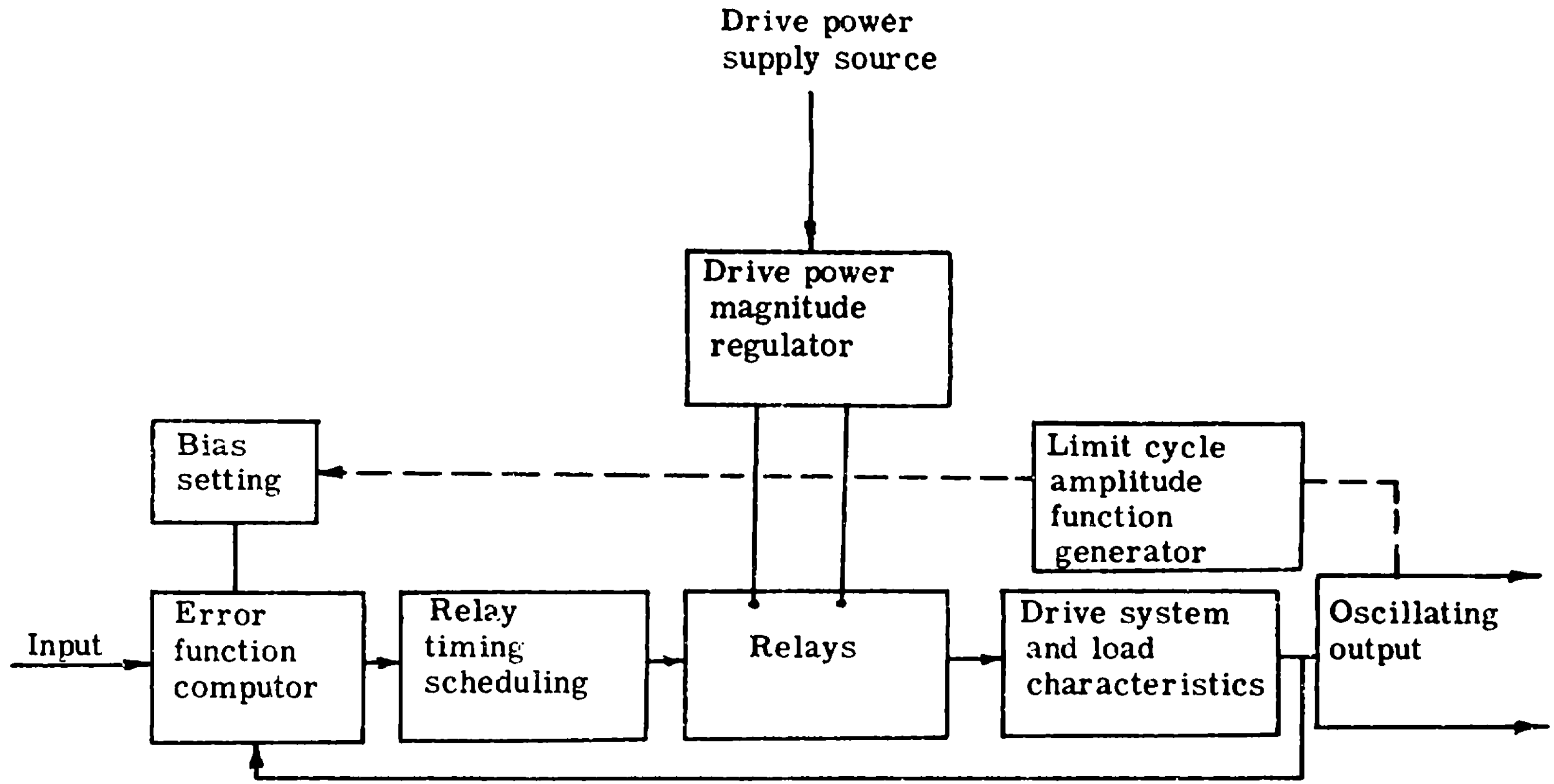


Fig. 10 Drive power switch timing adjustment through feedback of amplitude of oscillatory output.

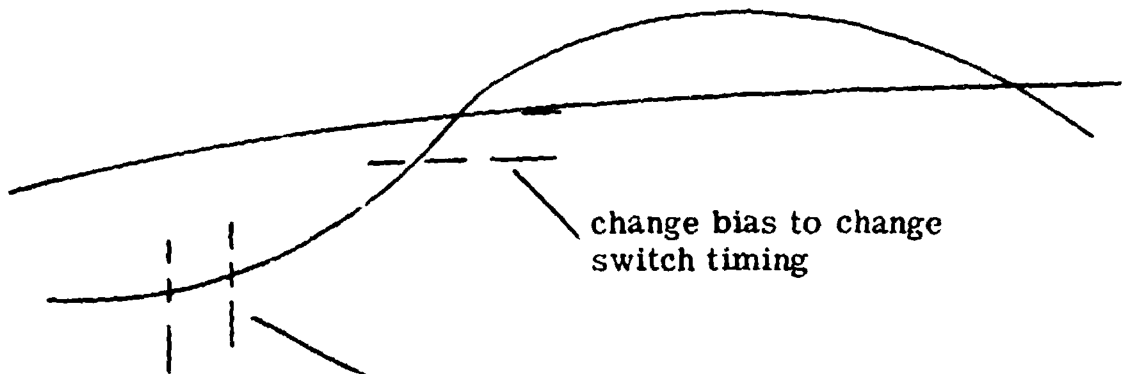


Fig. 11 Methods to change switch timing.

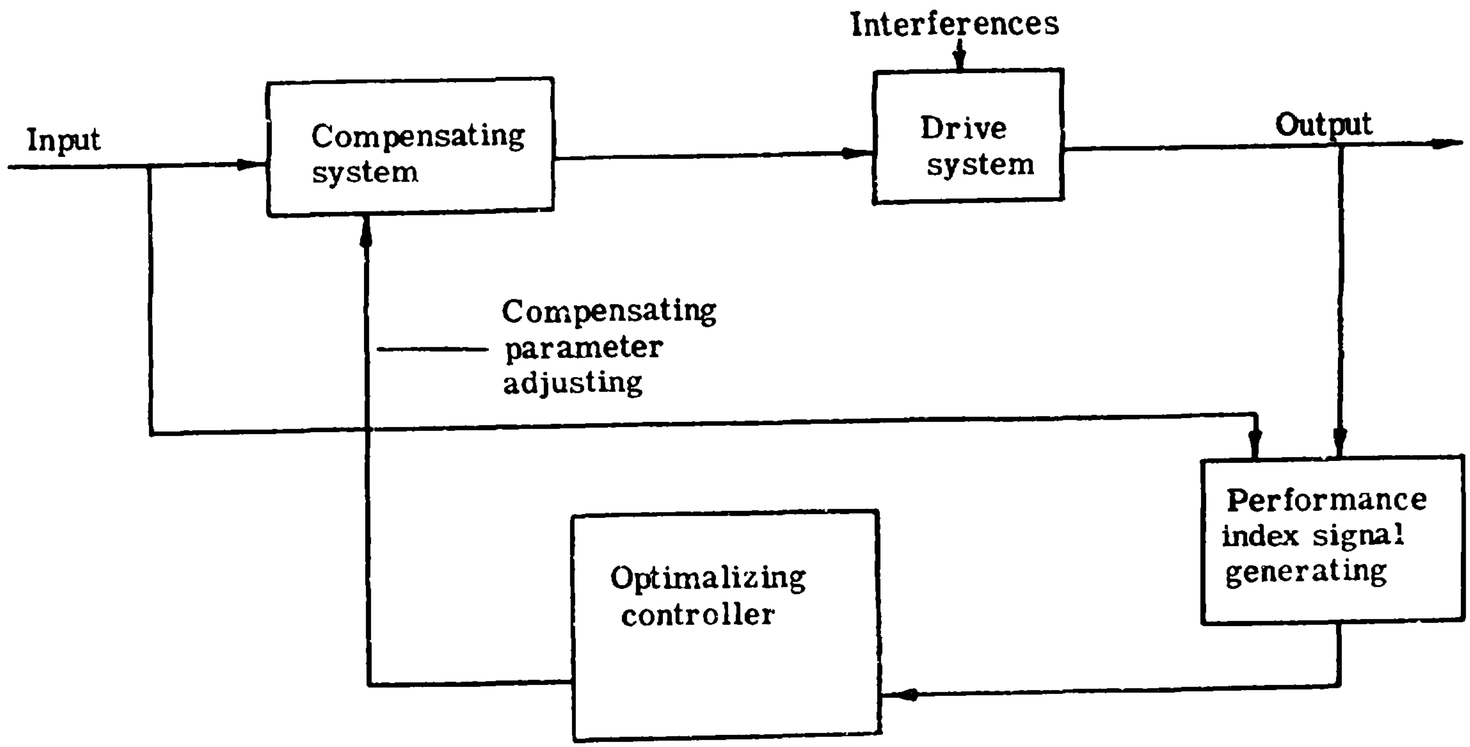


Fig. 12 Typical dynamic response optimalizing system with the use of a performance index signal generating system.

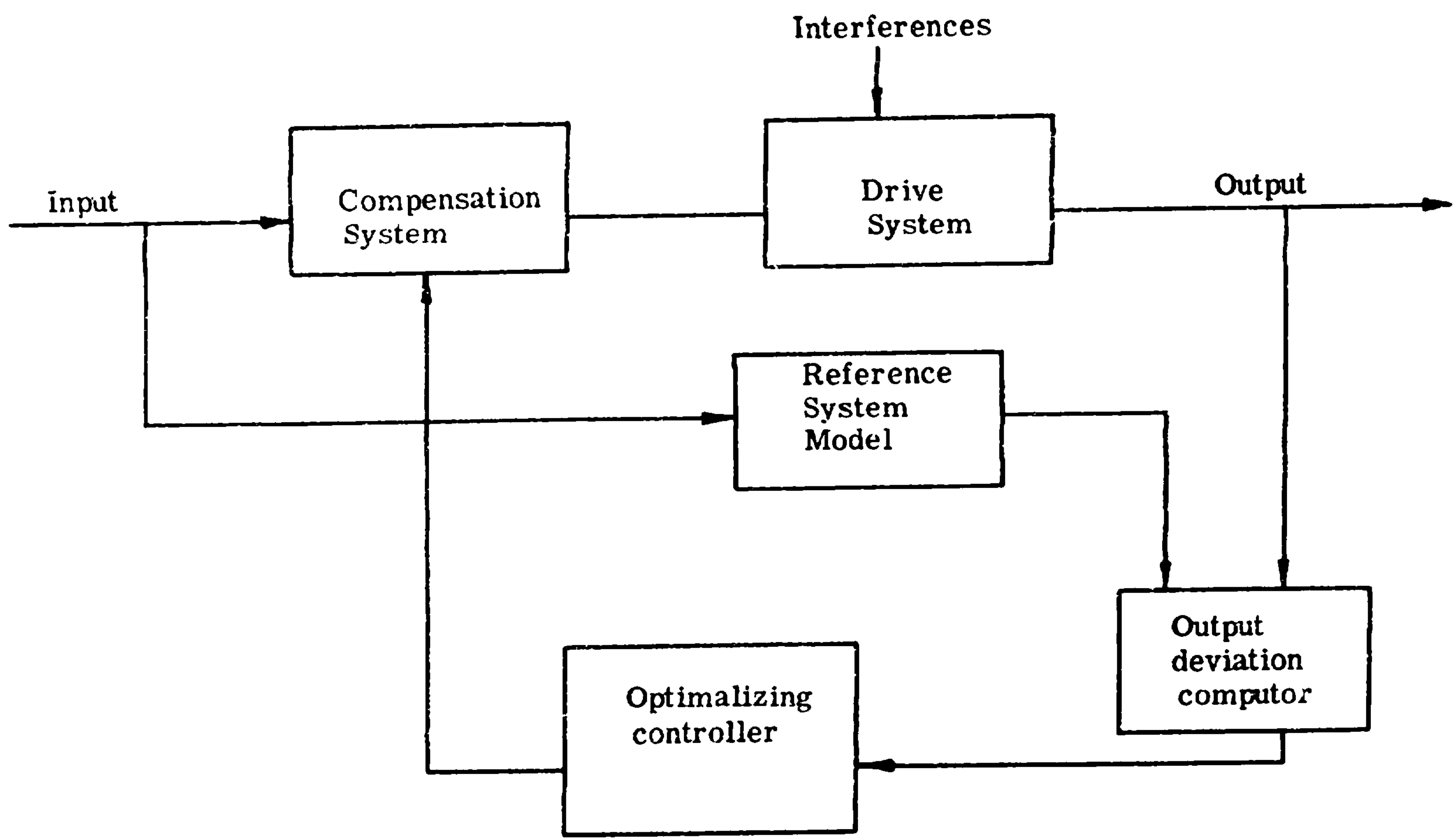


Fig. 13 Typical dynamic response optimalizing system with the use of a reference system model and an output deviation computor.

OPEN FORUM

SELF-ADAPTIVE CONTROLS

CHAIRMAN: Dr. Y. T. Li, Massachusetts Institute of Technology

PANEL: Dr. J. Aseltine, Space Technology Laboratories

Mr. R. Bretoi, Minneapolis-Honeywell Regulator Company

Mr. E. R. Buxton, Autonetics Division

Mr. M. Dandois, Convair, Fort Worth

Mr. M. Marx, General Electric Company

Mr. S. S. Osder, Sperry Gyroscope Company

Captain R. R. Rath, Wright Air Development Center

Dr. J. G. Truxal, Brooklyn Polytechnic Institute

Mr. W. P. Whitaker, Massachusetts Institute of Technology

OPEN FORUM

Dr. Li: Gentlemen, this is a panel discussion. Several of the members have not made presentations; therefore, each of them will be allowed three minutes to express his views. Rather than limit individual presentations, the three minutes will be taken as a statistical average time for the speakers to give their point of view about the subject matter. After the presentations, anyone who has a question, please stand. You will then be recognized by the chairman and the military guides will bring you one of the four microphones located on the floor. Please state your name and association before asking your question. I will ask one or more panel members to answer. I will now ask Captain Rath to start this panel discussion.

Captain Rath: Thank you, Doctor Li. I think most people are familiar with my viewpoint on adaptive systems. I have been with the program for three years now and I would like to feel that the present state of the art is such that these adaptive systems have demonstrated that there are adaptive techniques which are capable of being utilized by control system manufacturers to provide a control system which gives us the desired response throughout the flight regime. The using people, airplane manufacturers, the Air Force, and so forth, should have enough confidence in the capabilities of adaptive techniques so that they will use them in their airplanes. They will not have to go through a long component development, because, after all, the mechanization of adaptive techniques, as pointed out in the last few days does not involve anything radically new. It is only the idea that is new.

We are still using feedback techniques. We may have come to a point where we are using a more sophisticated type of feedback technique, but it doesn't require anything new in the way of product development to utilize them at the present time. I personally hope that in the very near future these types of techniques are utilized to give control systems the capability that they should have at the present time. Thank you.

Doctor Li: Thank you Capt Rath. Doctor Truxal.

Doctor Truxal: This will be the low side of the statistical three minutes. My only reaction is, I think this is a very interesting, enthusing, and exciting way to look at the design of the feedback system, to try to learn how we might design systems that we could not have designed before. As a teacher who competes with my colleagues for graduate students for research contracts, it is real nice to have something to go into the students and talk about that will compete with plasma electronics and some of these other glamour terms. I think this is a wonderful development.

Dr. Li: Thank you, Dr. Truxal.

Dr. Aseltine: I think I have already said pretty much what I wanted to here, but let me say it again just to emphasize it a little. I feel rather strongly that the term adaptive ought to be defined. I have a fear that we are tending to define it in such a manner that it becomes all inclusive and I really don't think that this is of any particular help to those of us who are working in the field. I would like to tie it down a little bit more. As far as what is going to happen in the future, it is a little harder for me to comment on this. I don't see any immediate remarkable progress in sight. In fact, we seem to be moving rather slowly toward an objective which is yet to be clearly defined. I do feel that work should be done on basic limitations. Once we know what we want to do, we ought to be able to say more clearly just what we can do. The things that I have in mind here are considerations wuch as the time necessary to measure a system performance before anything can be done. I suspect maybe there is an uncertainty principle operating in an adaptive system which changes the performance to do a task and must use some finite time to do this. I think these limitations must be tied down. I don't see why theoretical work in this direction cannot be done. These two points, definition, and some way to bound the performance of an ideal adaptive system have a particular interest for me. Thank you.

Dr. Li: Thank you, Dr. Aseltine; Mr. Bretoi.

Mr. Bretoi: There are two or three aspects of this conference that have interested me particularly. One of them relates to the interest and attitude of the aviation industry toward adaptive systems and non-linear techniques. One of the big stumbling blocks which many of the control engineers have - I shouldn't say stumbling block, it is more of a mental block really, relates to their hesitance to consider non-linear systems. In my past experience there have been many engineers who were reluctant to consider something other than a linear system because they tended to assume that a linear system was, indeed, the best way of accomplishing a control mission. Probably this stems from the fact that the non-linear parts of the control system are those parts which have given them very much trouble in their past experiences. For example, such things as control dead-spots, thresholds and so on. However, more recently I have gotten the impression that we are adopting the philosophy of Christine Jorgensen that if you can't fight it, join it. Well, somehow, I feel that we are making a rapid rate of progress now in the control field. There was much interest in automatic controls and a rapid build up of effort about ten or fifteen years ago and in the early phases of this effort there was rapid progress being made. I don't know exactly the rate of progress but it has slowed down in the past few years. Somehow, I feel that we are just about to begin a period in our control development techniques and so on, where we will probably have another rapid rate of progress. One of the significant aspects of adaptive controls, among many of the other mentioned here, relates to the fact that now we are at a point where we can achieve the desired performance goals more conveniently and more easily than in the past through the elimination of dependence on accurate dynamic data and air data

scheduling. I think the significance of our ability to eliminate the dependence on these quantities is that we can start concentrating on design criteria relating more along the line of such things as reliability and simplicity of mechanization. As for future efforts, I think that Dr. Li touched on those very nicely - I would just like to mention that I think some of our future programs should be conducted, or at least people should be thinking seriously about, what kind of performance we do actually want and what kind of criteria we should use to get this performance.

An effort should be directed to study the effects of such things as noise sources, parathetic and non-linear effects, and control of higher order systems. We have been concentrating pretty heavily on the lower order systems and assuming we can separate second order roots from the influences of the other system roots. However, when we try to control vehicles of higher order that have roots which are more nearly equal to each other in some applications, we will have problems. I think that we will be encountering vehicles which will require this consideration. The design trend is in a direction where the structural modes of the aircrafts have natural frequencies which are near the control modes. So that we are not able conveniently to separate the roots of the systems.

Dr. Li: Thank you Mr. Bretoi, Mr. Buxton.

Mr. Buxton: So much has already been said that I'm afraid I can add very little to the ideas that have already been presented. I am on this panel as a sort of consolation prize. We did submit a paper; however, Capt Rath looked it over and thought it was fine but that it covered some areas that were already being covered and so here I am. As a matter of fact, though, it is very interesting point that the systems that we have been working with represented actually a combination of two or three of the systems that were presented. We find after looking at a great many of these systems that there are possibilities of combining to an advantage a number of the principles of the basic techniques or solutions, which recently have been presented. If one is ruthlessly objective, he is usually identified as a negative thinker. There has been much discussion regarding the advantages of these systems, but there has been little said about the disadvantages that might accompany a few of the techniques involved. I am sure we will get into this area on questions from the floor, but I would like to add one more advantage that nobody has yet mentioned and that is the auto-pilot salesman must recognize that these adaptive gimmicks are the best sales pitch in twenty years.

Dr. Li: Thank you very much, Mr. Buxton. Are there any questions from the floor:

Question from the Floor: My name is William O'Neil, Douglas Aircraft Company. Instead of putting these in the form of a question I will put

these in for form of a statement to be commented on. First of all, without disrespect to the speakers or the quality of the papers, I feel a good deal of what has been said here in the past few days has not dealt with the fundamental theories of adaptation, such as the general process information required, and the solution time which must occur after acquisition of information before action is taken by the control system. In other words, the really basic items which we need to design any adaptive system, not just an adaptive airplane control system. So I would state that I feel that investigation of fundamentals is important and necessary. That is item one. Item two, no one has agreed to mention the effect of changes internal to the control system. We allow the gain, aerodynamic or otherwise external to the control system,to vary by large factors, ten, one hundred, a thousand; occasionally this happens inside too! A complete adaptive control system program should look at every element of the system as being the same in this sense. Every element can have a variable gain, including, by the way, the germanistic models which we built. The inputs should also be considered. That is item two.

Item three is people want or have given definitions of adaptation and I would like to break them down - one is program adaptation such as we have in present flight control systems; the second is logic adaptation including non-linearity, attempting to achieve some specified condition; and the third, a random method which has been discussed mainly by Ashby. I think the general theory fundamentals that I discussed should be emphasized more strongly regarding all three methods in the future.

Dr. Li: Thank you, Mr. O'Neil. I wonder whether Captain Rath would like to comment on these statements?

Captain Rath: Well, I am the first to agree with Mr. O'Neil that more work in the basic theory is required. However, we started out with the idea that we wanted to get techniques that could be applied immediately and then after we had demonstrated that adaptive controls are feasible, to support these techniques with the basic theory so that you can actually synthesize systems rather than build them on an experimental basis using the empherical data that we had gotten from experiments. I think a review of the work that we in the Flight Control Laboratory have planned would show that we are going to do more basic research. There have been questions raised about whether our plans are really for basic research. The best I can say is this. We welcome having people or organizations that feel that they have legitimate approaches that should be pursued to send in proposals to us. We intend to evaluate these fairly and try to prevent duplication of effort, if possible. Then, based on what comes out of that, those that can be funded are funded, so I would say then that if you have proposals that you think will help us, please send them in.

Dr. Aseltine: I hate to let Ashby get by without taking a crack at him. The homeostat that Ashby built is perhaps a pioneering step in a long range direction; however, there are a couple of things about it that I think perhaps

explain why it has not been applied on the programs that we have been hearing about. In the first place this is a device which searches in a random way for a stable operating condition and in general the steps that it goes through in order to get to a stable condition may lead it violently unstable for a sizeable time. This is one of the problems that Ashby was worried about and he does have some solutions, but the system may be badly behaved and objectively so if it were an airplane. The other point about Ashby's system is that it serves no useful function and I think I am quoting him almost directly, " Except to run to a state of equilibrium, it demonstrates purposeful behavior but it does not do anything else". I have thought a lot about how you could make it do something else, I'm not deprecating this, I think it is a device that is nice to look at and it is a nice device to get ideas from but we shouldn't be mistaken in thinking that it can be put in an airplane, yet. I agree that this is a good area to look at.

Dr. Truxal: To go back to the comment on the program itself, the basic theoretical questions that are involved here are exceedingly diffficult. I think if you look at other fields of science you don't find general theories on engineering that are built up early in the game. We didn't have a feedback theory until we had a lot of feedback systems operating. We didn't have communication theories until after a lot of radios were operating. It seems to me we need a lot of different types of adaptive systems operating before the poor theorist can decide what the theory ought to provide. You can't go to a university person or a basic research person and ask hom to develop a general theory on the grounds we have so far established. That doesn't mean that we shouldn't work on it but I think this will be a longer time coming than the applications of these techniques.

Mr. Whitaker: I might only say that in answer to the second part of your question certainly no one believes more in Murphy's law of random perversity than I do. Anything that can go wrong will go wrong, but I didn't gather from any of the presentations that anyone was excluding the failure inside the loop either. Particularly, if you have a measurable criteria performance of some kind then no matter what is upsetting the performance then presumably that index is going to change. You aren't limited just to environmental changes.

I might also comment on the third one. There has been some philosophy that we could make an aircraft system completely self organizing; however, I feel that this requires too much predetermined mechanics and it seems to me this is unwarranted complexity. I think in most cases you definitely know what you want the airplane to do. To make the system decide what its loop configuration is is unwarranted complexity.

Mr. Buxton: I might comment on the first one. Mr. Barron from Dodco gave a very interesting and very fundamental approach I thought in some of the basic principles. Variational calculus is a powerful tool to attack at least the

optimization problem and surely represents at least one area where some fundamental development work is being done. I, for one, would be anxious to follow the work that these people are doing at Dodco.

Dr. Li: Mr. O'Neil's opinion about inside and outside is all relative. All your trouble comes from the outside. You may always name some region which is not inherent in the system. And if you draw a black diagram, depending upon where you put an error it can be outside.

Question from the Floor: Mr. Frank Barnes, Missile Electronics and Controls Department of RCA. One of the things that I haven't heard very much about here is the effect of disturbances on adaptive control systems. In particular, I am concerned about the kind of system that adjusts itself on the basis of the difference between the model performance and system's performance in the presence of a continuous series of disturbances at a time when there may not have been much of a form of a desired input to the model. Since the model doesn't know about these disturbances I am wondering if the error criteria that is used to optimalize the system will drive the parameters of the system in a direction to optimize the system in these cases. I feel that there should be some attention put on what kind of performance the system has when affected by these disturbances, especially in the simulation and computer work.

Captain Rath: I think I understand the question. The systems under consideration are those in which the adjustment takes place based on some criteria applied to an error between the model and the actual output. He feels that in those cases where you have a disturbance into the airplane other than through the model you would generate an error signal which, in turn, would call for some adjustments, although the system may already be at an optimum setting. Actually, I think there are two types of system that we have considered to date. One is where we do actually have a physically adjustment of a gain based on this error. I think Mr. Whitaker could best answer the question regarding this type of system. You do not get any adjustment unless there is a command input to the model. He will probably amplify that if there is any further question. The second type of system could be represented by the Minneapolis-Honeywell technique where, except for the gain change based on the amplitude of the limit cycle, there is no physical adjustment in the system other than the more or less spontaneous gain change due to the non-linear characteristics of it. Therefore, when there is an error between the model and the actual airplane response the performance parameters do not change in the respect that the question referred to. Is this right?

Mr. Bretoi: As long as the disturbance is not at the same frequency as the characteristic frequency of the limit cycle oscillations then the effect of noise is not expected to have a strong influence on the performance of the system except as it might relate to providing so much noise input to the system that the signal is too small relative to that noise. I might mention that work has been done in another area which is called an adaptive noise

filter which we are using in connection with automatic beam following systems and this work does relate to the question posed here. The problem was to follow a beam, in this case an ILS beam, to an airport. This beam, however, is not necessarily straight. It contains bends and it varies with respect to time in many cases. The best gains for the system depend upon the bends in the beam and the time variation of the beam center. If you can measure the noise characteristics of the beam it is possible to adjust the lead and gain parameters of the system in such a way that you do get a satisfactory performance. Some work has been done along these lines. It will be interesting to see how some of these same ideas could be adapted or applied in conjunction with some of the adaptive techniques which are being investigated at this time.

Mr. Whitaker: I think I had better comment on this one. One of the fundamental features of the type of system that we are proposing was that you were making adjustments to an existing control system to optimalize its performance. Now, I was somewhat crushed by Dr. Truxal's comment much earlier in the day when he said that he had to redraw the diagram in order to show that we in fact had a linear system. I tried to draw the diagram so that this was obvious in the first place. The only thing that we are doing in this particular application is changing the parameters in the system on a basis that is closed loop with respect to its performance index. Now we had exactly the same system and we changed exactly the same parameters before only they were changed on a program basis. In relation to this particular question, we were happy with the response of the programmed flight control system before when it was flying through gusts; there is no reason why the system has changed. All that has changed is the manner in which you adjust to the optimum state. Now in getting to the flight condition in which you encounter a particular turbulence then presumably you have put inputs into the system that have caused the system to seek the optimum response. From then on the controller is there as a closed loop system and still has the same characteristics of service as the programmed system. Does this answer the question?

Mr. Osder: I think I ought to comment on that too because one of the first problems we considered was the effects of turbulence or noise on our performance computor which attempts to measure our so-called high frequency oscillations. From the very beginning we applied atmospheric turbulence to the problem and tried to develop techniques which could discriminate between the impulse we were applying and the turbulence. Actually, there is some slight effect in the system we worked with. It becomes more appreciable at high speeds where the spectural densities of turbulence appear in a higher frequency range. The worst effect that we ever got due to the turbulence at a high gain level was a slight change in gain of about two DB. Actually, it lowered the flight control system gain two DB's in turbulence and some people think it may be a good idea to lower the gain of the control system in turbulence. So from that point of view, as far as turbulence is concerned, we have not encountered any problem.

Dr. Li: Thank you very much.

Question from the Floor: Lieutenant Hoffman, Dynasoar Project Office. I had three questions but you have covered two already. That makes it very simple for me. I wondered if any of you gentlemen have any feeling regarding the relationship between the inner loop or stabilization loop adaption and the outer loop adaption. For example, the flight path or guidance loop adaption and what the relative payoff might be.

Mr. Marx: I think the reason why most of the people here have considered inner loop adaptation rather than outer loop adaption is that for the most part, your gain changes or variations are in the inner loop. For instance, if you are controlling attitude and you make the attitude or the rate loop adaptive you have included the effects of elasticity, change of c.q., Mach Number, and what have you on the performance of the system. If you extend this to the flight path loop, really, the only parameters which change are the airplane path-time constant and an airspeed term. The path time constant usually can be neglected if you have complete control of your inner loop. The airspeed term can best be provided by a program. The only payoff you you would get by using adaptive control then in the outer loop would be compensation for this airspeed variation. Now, it is necessary to answer a question here, can you actually measure the airspeed or do you have to resort to adaptive techniques?

Mr. Bretoi: Our particular approach to this aspect of the problem is to provide an inner loop which has the kind of dynamics which we desire. Then, depending upon what type of outer loop control that is desired, the inner loop is either an acceleration control or it is a pitch rate control. The closing of loops about either one of these is straightforward if you choose the right one. In the case of pitch attitude control, pitch rate inner loop is a good one to use because pitch rate is displaced just 90 degrees from pitch attitude and there is no gain compensation required if you have the same pitch rate response throughout the entire flight regime of the airplane. In the case of normal acceleration control, normal acceleration is quire directly related to flight on a vertical flight path, depending on the units with which you measure or define the flight path. If you define your flight path in terms of linear quantities such as feet per second or feet per second squared, the normal acceleration does have a constant relationship between these quantities. If you define your flight path in terms of an angle, then it would be more convenient to use an angular inner loop such as pitch rate.

Dr. Aseltine: I have just a short comment. I suggest that, if I understand the question properly, in many cases the guidance loop is almost an adaptive system by my definition already, because it measures performance of the vehicle regarding its orbit by comparisons through terrestial or inertial means and it does something about correcting errors. Perhaps, if you get too far off you might tighten the loop up a little bit and then this would be purely

an adaptive system in my estimation. I think we are rather close to this now in an outer loop guidance system. This is what I understand the question to be.

Mr. Bretoi: I just want to point out that when you consider outer loop control, together with the inner loop control, as a part of an adaptive system, or when you try to make the outer loop part of a mechanism adaptive in addition to the inner loop control, you are dealing with a higher order system. This is an area where we should do more basic work. The question is, how do you make a higher order system adaptive?

Mr. Osder: I think that this is a very important point because I think it might indicate an area where possibly we try to push our adaptive systems too far. There are certain gain schedules that we use, such as the airspeed schedules mentioned before that are invarient; that is, they don't depend upon unknown aircraft characteristics. For example, take the case of the altitude outer loop. Basically, you have a true airspeed requirement on a parameter control if you have an inner loop which is a pitch attitude system. If, however, you convert your inner loop to a normal acceleration system you possibly can get away from needing true airspeed but the question is do we really gain simplicity. We never had a problem if we had available the airspeed data and it seems that in a manned aircraft the true airspeed doesn't have to be too accurate; as a matter of fact, Mach number always did the job quite well. If the true airspeed data is available or the Mach number data is available, you have a fairly simple system whereby you can vary the gain of your altitude error and retain an inner loop pitch attitude system. If you use a normal acceleration system, you will have problems such as how do you hold altitude in a turn? Unless you compensate the normal acceleration data to make it true vertical acceleration you are going to go into a dive. If you try to inertially compensate your altitude error data you will use h error and h. If h is derived from a normal accelerometer then it has to be compensated for the one minus cosine function. Here in the compensation problem you run into a lot of difficulties regarding accuracies. Small errors in the balance of the circuit can give you errors to either make you dive or climb and it usually results in complexities. This might look very clever and you might possibly get away with it but the point here is that this has never been a problem before. There are certain dangers in switching outer loops and inner loops. We have the problem of reliability in switching and we have transient problems. The transient problem occurs when we switch from a normal acceleration loop to a pitch rate loop with an outer loop such as altitude control engaged. So the word of caution that I would like to offer here is that we ought to try to apply the adaptive techniques primarily to those areas where there is some uncertainty in our gain programming or in those areas where there is absolutely no chance of getting the kind of gain control information required. As long as there are indicators that display speed to the pilot, such things as Mach numbers, for example, are being computed and they should be utilized to the best advantage in the flight control system.

Mr. Dandois: I would just like to add one comment. One type of self adaptive control that might be used as an outer loop would be a self optimizing type which seeks a maximum or minimum. For example, in guiding an airplane or a missile from one place to another we might want to maximize the range or minimize the fuel consumption. Usually this is programmed but it might be possible to devise some means of controlling the aircraft so that the maximum range is obtained automatically by maximizing his climb.

Captain Rath: I would like to comment on Steve Osder's remark. It is true that for certain applications where we do have available air data that the mechanization can be simpler and therefore more reliable and should be used. However, it is necessary to look to those applications which are not too distant in the future where we do not have the capabilities of getting accurate air data information and we should now begin to try to determine ways of operating without programming. I think although we may not use these techniques immediately in the future they will definitely have application and we should be working on them.

Mr. Buxton: I want to amplify on Captain Rath's comment. Basically, we are seeking a greater reliability in these systems. In most cases we have to use adaptive techniques when we don't have anything with which to schedule our knowledge at all. I would say a design consideration should be that if you can establish any reliability at all over a system that formerly used scheduling techniques then you should use adaptive techniques rather than try to use something that we haven't had trouble with before. I can't help but feel that one of the major causes of our control difficulties has been our scheduling devises. If we can improve the reliability I would sincerely vote for adaptive techniques in the outer loops as well as in the inner loops.

Dr. Li: All right, I think we have had enough answers to this question. I might add a little bit; that is, right now we are trying very hard to introduce adaptive techniques to this outer loop. As a matter of fact, when the Russians try to shoot at the moon, or we try to shoot at the moon, we schedule everything. Why do we schedule everything? Because there is no feedback. Why don't we have the feedback? The reason may be that we don't have a way of measuring the signals. Just like in the old days, when the house is too cold we put a shovel of coal in the stove. Later on we invented the thermostat and so we had feedback. If you don't invent the thermostat then you don't have feedback. If we can measure the so called ballistic missile and close up the loop we certainly would be glad to do it. Sometimes measurements are very important.

Question from the Floor: R. N. Clark, University of Washington. I would like to say three things: First, I think this conference has provoked a lot of material for discussion, but I will spare you gentlemen my opinions on most of this material and limit my remarks to two things. First, I would like to say if anyone is in Seattle, I would be very happy to continue this discussion

in the electrical engineering department there and could probably arrange for a seminar type of discussion. Secondly, I would like to direct a question to Mr. Whitaker. Would you discuss the problems which you have encountered, if any, in making your system work with inputs other than step inputs, perhaps ramp or sine wave or statistical type inputs? A third thing that I would like to mention concerns performance criteria. Performance criteria have not been discussed here and I feel we have need for work in this area in the future. I think that we have to search for a criteria beyond that which is found in our text books now. In particular, we have to find a criteria that is compatible with realistic type input signals, in the aircraft business, the kind of signals our auto pilots really do get. I think we should make an attempt also to incorporate in our performance criteria the same sort of subjective evaluation to which we subject our control systems on the analogue computor. In particular we look at the response and say this is better than that. I think the attitude with which the Dodco people have approached their work is a good indication that we are getting out of the text books for a criteria.

Dr. Truxal: I agree we need more knowledge. I agree with that wholeheartedly.

Mr. Whitaker: I am not sure how things are on the frontier in Seattle but we would be happy to have anyone come up to the hub of civilization in Boston. In regard to the second question, the questions of input, I knew that if we tried to show you something here that, as Dr. Li says, has some sex appeal - that we would be accused of having a system that only works with step functions and I would like to assure you that this is not the case at all. In probably the most typical operational use, environment is changing in a continuous manner and the parameters would also change in a continuous manner. Actually, it isn't very glamorous to see the same response all the time. In order to have something that would show what would happen if the parameters were way off, we are using the step function to define a parameter called convergence time. You could define convergence time as the time it would take to come back to the optimum state from some non-optimum condition. Similarly, you could define a similar time for changing an environment if the conditions were fixed and then the environment was changed, and the system allowed to converge. It is not necessary to only have step function inputs. This is the prime reason why the sampling for the system is keyed to the input and output relations of the model. If the input changes the sampling also changes. Now our flight test program had some of the characteristics of an Egyptian Mummy, we were pressed for time. We weren't able to present you with complete quantitative data on the other types of input. However, we have some flights in which we just let the pilot fly around and do normal maneuvering. I can't give you quantitative information on these flights; however, the system appeared to give satisfactory flight response as he did that. Another type of test was one in which we simulated a dive bombing mission. If we just asked the pilot to nose over wings level and not touch the stick, then, of course, there would not be any input in and the system would not change, but if we give him the task of

tracking a target on the ground, then his tracking inputs will be sufficient to give the system enough information to find its own optimum point and to keep the gains changing as the environment changes. A dive bombing mission is the most severe type of environmental change you will get. It involves starting from high altitude at a low Q and getting down to sea level in a hurry. Unfortunately, dive bombing isn't very popular anymore and I am not quite sure what the most stringent environment change would be or what the most stringent requirement for this system would be. In summary, it is not necessary to have step inputs and what input data we do have indicates the system works quite well on the basis of just the normal flying input that you would have in traversing the sky. We did do some simulator work in which we used a cockpit simulator and scope presentations, and had someone sitting in the cockpit trying to track the scope in an effort to simulate the same type of thing.

Question from the Floor: G. G. Moss, Convair, Pomona. Something has impressed me about the design criteria. It seems that we all want something that is adaptive but there is a slight difference in some of the approaches. One that Minneapolis-Honeywell has taken is where the switching occurs and you try to get a sort of a least time optimization. The approach by Dr. Aseltine and others is to adjust the system rather slowly so that we have perhaps a least square criteria. Now the two different approaches seem to have different merits. For instance, in the least square criteria, where we slowly change the parameters of the system, you don't introduce erratic signals and perhaps excite modes that you don't know anything about, but at the same time you don't take full advantage of the least time approach where you can respond more quickly to large errors. I would think that these differences should be emphasized and something said in defense of the particular approaches. Also, it seems that more could be said about the possibilities of improving the reliability and dependence of the system on components, but I suppose there is no use in doing that now. I prefer the contest.

Dr. Li: This is a very broad and general question and touches on almost every type of system that we have tried to analyze and classify; however, I will turn this over to the panel for three answers and ask that the answers be limited so that we can get another question in.

Dr. Aseltine: I might remark that in reference to what was called " least square" , this was really a method of evaluation which was based on looking at the transient response and measuring areas above and below the line that was obtained by noise variance and " least squares" played no part in it. Incidentally, it is also a method which is not in any text book. It was an attempt to find some description of the system. It is based on a property of the system which is a measure of performance and that is impulse response. In special cases it probably is true that the Flugge-Lotz method would be faster and I think this is important. I think also that one ought to realize that the general application of the switching method to an unknown system is something I don't believe has been studied. When you get into a second order system,

such as the studies that Flugge-Lotz and Taylor have made, you can probably get very fast results, but I am not sure that with an unknown system this would be possible. These responses are not based on anything that you know about the system itself; they are based on the way it works. I think the speed is good and I think Flugge-Lotz has a good way of going after it but a lot more work is needed there. I just wonder again if there isn't some minimum time that it takes to do this job.

Question from the Floor: Hugo Shuck, Minneapolis-Honeywell. I would like to comment in that general vein and sort of second the motion that Dr. Aseltine has indicated. We need to get down to a fundamental thing like Heisenburg's uncertainty principle to establish what kind of accuracy can be obtained as a limit and as a function of the length of time required. There are several techniques that have been mentioned in which integration is used, and in a way, we are sort of shaping down to integration versus no integration, cross corrolation and so forth all implying that there is a time integration. In Fourier transforms you have an integration that must go on out into infinity which isn't very useful because you want very recent information. So you have to truncate to some extent. Now it is this truncation process which is significant in this application as in many others. There hasn't been too much study on it and certainly the suggestion that we ought to try to get something definite down is very much in order. In fact, to put a question tinge on to this, I would like to ask if any of the panel members know of any optimization of form of the time waiting function which is applied to this integration process? In other words, do you cut it off exactly on a square chop as you do when you try to average breakfast food weight or do you use the exponential draw up that you can get out of a simple RC circuit or is there some other optimum method?

Dr. Li: The heart of his question is that he would like to know in a general way what is the best way of chopping off the so-called pulse response in order to give you enough information. I wonder if any of the panel members want to answer this question?

Dr. Aseltine: One brief comment is that in the particular method I just talked about this thing is sort of taken care of because the thing you are integrating tails off generally exponentially, at least if you are anywhere near the operating point that you would like to be near, so there isn't any necessity to worry about it.

Mr. Whitaker: I would like to make one comment. When using a model the characteristics of the model such as its rise time and exclusion time may be used to regulate any sampling. Integration in that case actually is performed in potentiometer servos. When you are integrating you are changing the gain at that time and the change in the potentiometer position is the value of the integral. The criteria of the integral should be zero so that when you reach your final state over any sampling time the actual change in the potentiometer is zero. You are changing the gain while you are doing the integration.

I don't know whether this is quite what you had in mind or not. I know the effect of the sampling time is to weigh the error. I think Graham and Lathrop suggested as a criterion the integral of time times the absolute value of the error. In a sense, using a definite sampling time is also using a weighing function. It is a weighing function at a constant level for a given time and then zero from then on.

Question from the Floor: Leo Chattler, Chance Vought Aircraft. I would like to make some statements which can also end up as a question in regard to the self adaptive systems that were discussed at this meeting. For one, I think we have to divide our thinking between a manned vehicle and an unmanned vehicle, because for one I don't believe that any pilot in an aircraft of the type where a pilot will hold the controlling lever in his hand, will accept any feedback into his controller. That has been proven time in and time out and has been the point of many discussions with many military flyers. So for those systems where you contemplate that there will be feedback into the controller, I would recommend that you not seriously consider their use for fighter or attack type airplanes. To use the systems that give you a saw tooth output or residual amplitude at the output of the actuator to the degree that was discussed here, I think we will have to get a new line of bearings for our aircraft. I think if people will remember back when the acceleration switching servo valve first came into the picture one of the problems with it - I am speaking of the time modulated valve - was the fact that before you would even get the system tested, if you had significant time modulation you wore all the bearings out.

Now the other point of consideration here is the fact that you still must look at the mission of your vehicle. Do you really want to put all this equipment into a vehicle that may have a transient gain change and then spend a great deal of time at a fixed gain that would give the vehicle the response and stability that is desired for a point of the mission? It may be better to try to devise a scheme that will affect a gain change from an optimum within a certain area of the flight envelope to an optimum at another point of the envelope. I think Mr. Osder can allay his fears about people going too far with the systems. I proposed this on one of our vehicles. I said, " Wouldn't it be wonderful, we don't need air data" The answer was, " Who won't, the pilot will still need air data" . So this will still have to be in the airplane although the order of magnitude is different.

Actually as was pointed out here previously the vehicles that we are looking at today can use this adaptive system and one very pointed question that I would like to have answered is, are people designing some one of these schemes into the vehicles that they are working with today? Thank you.

Mr. Bretoi: There are several questions, Mr. Chattler brought up the fact that you have to consider manned aircraft in a different light than you do unmanned aircraft in many respects. One aspect of this relates to the fact that you do have a pilot aboard and your performance criteria for the system will depend on how you use that pilot in the system. He could be used in a control

stick steering mode or used to identify targets or track targets, or used to provide a navigational function, so the performance of your system does depend upon your pilot and it suggests that what we need to do is to know more about what the pilot's transfer function is. Work is being done by a number of agencies under military sponsorship to find out this transfer function; and maybe one of the values of non-linear techniques is that these techniques will give the engineers a better feeling for non-linear systems and possibly permit them to devise better approximations to pilot performance in terms of non-linear characteristics. The pilot is, of course, a non-linear mechanism.

Another question related to the amplitude of a limit cycle motion when you are using a system with a discontinuous signal. We, of course, have been worried about this problem also and we have looked into it. One of our approaches is to make this amplitude as small as possible and the amplitude of the motion which we are shooting for is just barely above the threshold of our instrumentation which picks up or senses the motion of the control servo. The amplitude which we are shooting for is in the order of a tenth of a degree. To find out how objectionable these limit cycle amplitudes are we compared them with the amplitude of motions that we would get from a conventional system. This comparison was made from flight test results of both systems. We have found, and this is somewhat surprising, that the amplitude of the surface motions using the non-linear system are no greater than the amplitude of the motions that you get with a linear system and in many cases they are smaller. The linear systems we investigated included the control systems in the F-94, F-100, and F-101 airplanes. We have quite a bit of data on these. As far as the amplitudes of residual motions are concerned, the non-linear systems do compare favorably with the linear systems.

Dr. Li: Thank you. Well, gentlemen, as the moderator of this last session, I would like to represent you, the audience and the participants, to thank our host, the Wright Air Development Center and especially Captain Rath and Lieutenant Gregory for this opportunity to gather together. I am looking forward to other opportunities to have this same type of meeting. I wonder if Captain Rath has something else to say?

Captain Rath: I certainly want to thank everyone for their patience and cooperation in making this two day symposium a success. Thank you very much.

ATTENDEES

ADAPTIVE FLIGHT CONTROL SYSTEMS SYMPOSIUM

AERONAUTICAL RESEARCH ASSOCIATES OF PRINCETON - Bernard Paiewonsky, Dr. W. H. Surber

AERONCA MANUFACTURING CORPORATION - William Ronci

AERONUTRONICS SYSTEMS, INC - G. W. Anderson, R. N. Buland, Dr. G. R. Cooper

AIRBORNE ACCESSORIES CORP - Dr. J. L. Zar, Mr. Wolfgang Merel

ALLIED RESEARCH ASSOCIATES - Robert A. Summers, William B. Bryant

AMERICAN BOSCH ARMA CORP, ARMA Div - Dr. D. Zarwin, Dr. J. S. Smith

AMERICAN POWER JET COMPANY - George Chernowitz

AMERICAN RESEARCH & MANUFACTURING CORP - George A. Beiser, H. L. Goda

ARMOR RESEARCH FOUNDATION - Dr. Shizuo Hori, Martin Kinnary

AVCO, Crosley Div - D. C. Maxwell, W. H. Doerr, V. L. Lambert

BATELLE MEMORIAL INSTITUTE - Herbert S. Kirschbaum, Magnus Moll

BELL AIRCRAFT CORP, Airplane Div - Borah Popovich, Jim Madden

BELL AIRCRAFT CORP - Dr. J. Goerner, G. Halsted

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